Learning with an educational mathematical game at a science center

A case study exploring visitor interaction with a collaborative educational game illustrating the mathematical model of Voronoi diagrams

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This master's thesis was executed in collaboration with Universeum Science Center in Gothenburg, Sweden

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Abstract

Science centers are used as a way of creating easily relatable and authentic learning settings. The science center of Universeum in Gothenburg has developed their new mathematical exhibition *Mathrix* with the purpose of lowering visitor thresholds to mathematics. This study investigates one of the exhibits of *Mathrix*, the exhibit named Voronoi, which is designed as a collaborative and exploratory educational game based on the mathematical model describing the formation of Voronoi diagrams. The questions investigated are: What types of embodied conversations emerge when visitors are interacting with the exhibit? What learning opportunities can be identified during these conversations? And, do the identified learning opportunities align with the intentions of the exhibit designers?

The results show that 73,0% of the utterances made by the studied visitors are connected to learning talk, either to explicit mathematical talk or to talk concerning the mechanics of the game. 11,8% of the utterances are connected to different kinds of problems in relation to the exhibit system, and 15,2% are connected to affective talk. The study concludes that the exhibit nurtured fruitful conversations and learning processes where the participants were given the opportunities to practice and assimilate the knowledge and the skills that the game was designed to foster. The study also concludes that the results align with the intentions of the exhibit designers, where the aim was to create a successful educational game where the mathematical content was well integrated into the game without interrupting the fun.

Keywords: informal learning, exploratory learning, learning talk, science center, interactive exhibit, embodied learning conversations, illustrating mathematical models, educational games

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Thesis outline

Chapter 1 presents the introduction to the study including relevant background information, purpose and aims, the three specific research questions and delimitations to the case study.

Chapter 2 presents the theoretical framework used as the foundation of how learning may be described and understood in the empirical setting where the case study is executed.

Chapter 3 presents previous research and related coding systems on learning talk in informal learning settings. The presented studies are used as inspiration for the methodology and the analysis used in the study.

Chapter 4 describes the methodology of the study, including the research approach and strategy, the empirical setting of the case study, the data collection and the data analysis.

Chapter 5 presents the results of the study, including the participant and session characteristics, the developed coding system, the utterances statistics and the design and learning objectives of the exhibit designers.

Chapter 6 analyzes the findings in relation to the three research questions. This includes analyses of the participant and session characteristics, the conversation characteristics, the learning opportunities and the alignment with the intentions of the exhibit designers.

Chapter 7 discusses the validity of the study, the limitations to the study, suggestions of future research and the applicability of the developed methodology.

Chapter 8 is the final chapter and presents the conclusions of the study.

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1. Introduction

1.1 Background

Mathematics is by many considered to be the 'mother of all sciences' as it works as a tool to solve problems in every other field of science. It is the basic language of science, and all other fields of science would be hard to imagine without the presence and the use of mathematics (Shah et al., 2023).

Research tend to indicate that students' attitudes towards the subject of mathematics are predominantly negative, to the extent that many students sustain an aversion and an anxiety towards the subject (Fenyvesi, Koskimaa & Lavicza, 2015; Shah et al., 2023). These negative attitudes play a crucial role in students' learning processes and accordingly in their learning success (Farooq & Shah, 2008). Research also shows that students generally have little knowledge of how deeply imbedded mathematics is in the everyday world around them. In our constantly developing digital society the importance of mathematically structured systems is increasing, affecting our daily lives more and more. However, the abstractness of the subject of mathematics makes it perceived as something very detached from reality. By using easily relatable and authentic learning methods, connecting mathematics to something the students have experience of and are interested in, their natural curiosities can be triggered and the education found more enjoyable (Fenyvesi, Koskimaa & Lavicza, 2015; Shah et al., 2023).

One way of creating these easily relatable and authentic learning settings has been by using so called science centers. Science centers are impartial institutions with the aim of introducing individuals to science, triggering their scientific curiosity and offering them to learn about science in an experimental and practical environment. In comparison to museums, visitors are encouraged to touch, test and interact with the exhibitions (Gürsoy, 2020). Science centers should be open to visitors of all ages, learning styles and cultures, focus on the relationships between science, technology and society, work as lifelong learning centers and be able to provide an environment where current issues can be presented and discussed (Koster, 1999).

During the past decades, there has been an increasing interest in teaching science and technology in informal settings such as science centers and by using cross-disciplinary methods to illustrate the wide range of uses and applications of the subjects (Sasson, 2014; Vainikainen, 2015). Sasson (2014) and Vainikainen (2015) also states that these out-of-school learning environments are thought to be particularly useful when it comes to creating engagement, interest and

motivation and thereby also having positive effects on learning. However, unlike traditional educational settings, there is much less guidance and no clearly defined curriculum and learning objectives in these informal settings, such as science centers. On the contrary, in science centers, the visitor herself usually bears the main responsibility of her learning process (Allen, 2004; Rogoff et al., 2016). This makes the design and implementation of interactive exhibits at science centers as well as the assessment of their success a difficult task.

Universeum in Gothenburg is the national science center of Sweden and the following text explaining their focus can be found on their website: "Our mission is to offer a public arena for lifelong learning where children and adults can explore the world through science and technology. Our goal is to create experiences enhancing the creativity and innovation capacity, increasing the awareness and knowledge and activating the critical thinking. Using science as a foundation and engaging learning methods, Universeum aims at challenging people to enrichen their lives and act for a sustainable future" (Universeum, 2024a).

In February 2023 a new exhibition was opened to the public, focusing solely on the subject of mathematics: *Mathrix*. The main target group of the exhibition is visitors between the ages of 13 and 18 years old, however, families of various ages are seen as the secondary target group. The exhibition introduces the visitors to mathematics in all kinds of ways with the aim of lowering the threshold to mathematics. With the help of more than 20 interactive exhibits, *Mathrix* challenges the view of mathematics as something too complicated and irrelevant. The visitors are shown the existence and usability of mathematics everywhere in their everyday life. For example, visitors can create their own music, explore the Gothenburg skyline, learn about myths about mathematics, compete against each other in different mind games and puzzles and discover some fields of application of artificial intelligence (Universeum, 2024b).

This study will focus on one of these interactive exhibits; one that is named *"Voronoi – Natural phenomena and mathematical model"*. The purpose of this station is to introduce the visitors to something called the Voronoi diagram. This is a visual pattern that appear naturally in a wide range of contexts in nature, but the pattern can also be explained or created with a mathematical model. The visitors are invited to explore Voronoi diagrams by playing a game based on the creation of the diagrams. The station is hence designed as large display where the visitors are supposed to compete against each other in a game of four, trying to conquer as much display area as possible. A more elaborate

description of the chosen exhibit and how it works, including photographs from the exhibition, is presented in chapter 4.3.

1.2 Purpose and aims

The overall aim of this study is to investigate what types of conversations that emerge during a typical gameplay and exploration of the exhibit by the general public. This includes the overall types of conversations as well as the content details. The aim is also to be able to draw conclusions about what learning opportunities arise and how these take place. The final aim is to investigate to what extent the presented findings align with the intentions and the learning objectives of the exhibit designers. This is to help Universeum understand how their exhibitions are used, what learning and exploration opportunities actually take place and if they align with their intentions. Furthermore, this might lead to useful insights for the creations of future exhibition designs.

1.3 Research questions

RQ1: What types of embodied conversations emerge when visitors are interacting with the exhibit?

RQ2: What learning opportunities can be identified during these conversations?

RQ3: Do the identified learning opportunities align with the intentions of the exhibit designers?

1.4 Delimitations

This study focuses on investigating what happens during the sessions where visitors are interacting with the chosen exhibit. This means recording and noting all embodied conversations and relevant actions of the visitors. However, the study does not include mapping what the visitors have learned or memorized after the session is over. The learning will be investigated solely based on analyses of the embodied conversations and drawing conclusions about possible learning opportunities. This is why the term ‘learning opportunities’, rather than actual learning outcomes, is used in the second research question. It would not be possible to determine any kinds of actual learning outcomes without testing the visitors prior to and after the sessions. This would have been too time consuming and also, more importantly, it would have affected the experience of the participants and thus would not have represented an authentic informal learning situation. Asking the visitors about their learning would instead only uncover the

perceived learning and not the actual learning outcomes. This difference is further explained by Bacon (2016), where he defines perceived learning as a self-report of knowledge gain done by the learner and generally based on reflection and introspection. This cannot be seen as the same thing as actual learning, which reflects a change in knowledge defined by a thorough measurement of learning. Therefore, the study focuses on the learning opportunities.

The initial aim of this study was that it would only focus on completely authentic learning situations where visitors acted on their own initiatives and where the sessions were completely unguided. However, during the course of the study, it became evident that some instruction was needed in most of the sessions in order for the visitors to start interacting with the exhibit. These instructions only included the basics of the game and the interface to enable the visitors to start acting on their own.

2. Theoretical framework

The theoretical framework that is presented in this chapter is focused on two learning theories and two other aspects of or approaches to learning that are used as the foundation of how learning may be described and understood in the empirical setting where the case study is executed. The analysis in chapter 6 will be based on and related to the theoretical framework that is presented here.

Firstly, the extensive learning theory of constructivism is introduced and the two types of constructivism, cognitive and social constructivism, are further differentiated. Secondly, the learning theory of experiential learning is introduced. Thirdly, the more specific approach to learning related to game-based learning is introduced. The final part of the chapter presents a brief description of learning with interactive exhibitions in informal settings such as science centers, also mentioning the difficulty of creating and understanding these opportunities for learning.

2.1 Learning theories

2.1.1 Constructivism

The first learning theory that may describe some of the learning processes taking place in the empirical setting where the case study is executed is the learning theory of constructivism. The fundamental idea of constructivism is that learners actively construct or build their own knowledge rather than just passively receive information as an entity from a source such as a teacher or a book (Amineh & Asl, 2015; Olusegun, 2015). This coincides with the fundamental idea of science centers as they are designed to engage visitors, encouraging them to touch, test and interact with the exhibitions (Gürsoy, 2020). In these active learning processes, learners use their previous knowledge as a foundation and continuously construct new knowledge from the new things that they learn. Hence the new knowledge is built upon the old knowledge (Amineh & Asl, 2015; Olusegun, 2015). Each learner takes pieces and puts them together in their own way, which means that no learner learns in the exact same manner but creates their own systems of meaning that works best for them. A part of this learning process is therefore also learning how to learn. Constructivism is also based on the idea that learning requires sensory input, which means that learners need to do something themselves in order to learn. Just listening or watching someone else is not enough to construct knowledge (Olusegun, 2015). This is another reason as to why the theories of constructivism are relevant when analyzing the learning opportunities that may arise in a science center.

2.1.1.1 Cognitive constructivism

The cognitive constructivism is mainly based on the ideas of the Swiss theorist Jean Piaget (1896-1980). The cognitive constructivism is primarily highlighted in this context because of the four stages of cognitive development that were described by Piaget as they can be used as a tool to further understand when children of different ages may be receptive to different problems and learning situations.

Piaget's four stages are divided into the sensorimotor stage, from birth to 2 years old, the preoperational stage, from 3 to 7 years old, the concrete operational stage, from 8 to 11 years old, and the formal operational stage, from 12 years and up. The sensorimotor stage involves mastering physical activities such as grabbing things and bringing them to the mouth as well as understanding the world through movements and sensations. When children are in the preoperational stage they often struggle with understanding abstract situations, having a strong need for thinking in concrete terms. During the concrete operational stage, the understanding for abstract situations strengthens. For example, children in the preoperational stage usually have to learn to count by using specific objects, while children in the concrete operational stage begin to be able to simply use numbers. Piaget calls this the development of logic structures, where children's thinking becomes more logical and organized. During the last stage, the formal operational stage, the logical structures of children start to become more and more equal to the logical structures of adults. They obtain the ability to solve abstract problems and to think in conceptual terms. The focus of cognitive constructivism is that learning should always be related to the learners' stages of cognitive development, scaffolding the learners' own building processes (Phillips & Soltis, 2020).

2.1.1.2 Social constructivism

A common critique of the work of Piaget is that he underestimated the meaning of the societal and peer influences and that all descriptions of learning that do not address these influences cannot be defined as complete. The theory of cognitive constructivism focuses on how learning takes place among individuals and how their inner cognitive structures are built and constructed. The learner is hence described as a lone explorer, actively engaging with its environment but acting on its own (Amineh & Asl, 2015; Phillips & Soltis, 2020). To be able to better understand the learning opportunities that may arise in a science center, and particularly in relation to the chosen interactive exhibit as it is designed as a collaborative game, the importance of the social context has to be addressed.

The social constructivism was developed by the Soviet psychologist Lev Vygotskij (1896-1934). Vygotskij agreed on many of the ideas of the cognitive constructivism but not the assumption that it was possible to separate learning from its social context. He described learning as knowledge developing based on how people interact with each other, their culture and society at large. This means that learners rely on each other, and learning from others together with others helps them build their own cognitive structures (Amineh & Asl, 2015).

Vygotskij was also critical to the different stages of cognitive development described by Piaget. According to him, the stages of cognitive development were quite statical, simply stating what kinds of intellectual activities children are able to conduct on their own. He was more interested in the learning potentials of children, meaning what kinds of intellectual activities children are able to conduct with the help of adults or older peers, also called 'more knowledgeable others'. On the basis of these ideas he developed a theory called 'the zone of proximal development' (ZPD) (Phillips & Soltis, 2020). Vygotskij defines the zone of proximal development as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers" (Vygotskij, 1978, p. 86). What Vygotskij believed is that when a learner is in the zone of proximal development for a particular task and therefore tries to learn something new that they do not already have knowledge of, providing the learner with the suitable support will be the key to conquering the knowledge (Phillips & Soltis, 2020).

Another theorist being critical to the works of Piaget is the American philosopher John Dewey (1859-1952). He claimed that the best way of learning a new concept is through 'normal communication with others' in a communication process where the learner interacts with others engaging in suitable activities or through exploration of common interests. Dewey was also critical to the common structuring of learning and teaching in school settings, where teachers mostly let students work separately with individual assignments rather than involving the students in activities which require collaboration to solve problems (Phillips & Soltis, 2020).

2.1.2 Experiential learning

A learning aspect that also needs to be addressed in relation to the empirical setting where the case study is executed, is the fact that the learning takes place in an experiential setting where the learners use their own experiences as a foundation of their learning. Therefore, the learning theory of experiential learning is elaborated as it provides a

model of the learning process where the central role of experience is emphasized. The theory of experiential learning was developed by the American theorist David Kolb (1939-) and the term 'experiential' is used to differentiate the theory both from cognitive learning theories such as the constructivism and from behavioural learning theories such as the behaviourism. Kolb states that cognitive learning theories tend to focus on cognition rather than affect and that behavioural learning theories deny the role of the learner's subjective experience in the learning process (Kolb, Boyatzis & Mainemelis, 1999). Kolb defines learning through the experiential learning theory as "the process whereby knowledge is created through the transformation of experience. Knowledge results from the combination of grasping and transforming experience" (Kolb, 1984, p. 41).

Kolb described the experiential learning process with a four-step learning cycle. The first step is defined as experiencing, also called concrete experience (CE), and this is where the learner uses its senses and perceptions to engage in the current situation. The second step is defined as reflecting, also called reflective observation (RO), where the learner uses the experiences as the basis for observations and reflections. The third step is defined as thinking, also called abstract conceptualization (AC), where the observations and reflections from step two are assimilated and distilled into abstract concepts and theories that can be tested and acted on. The fourth and last step is then defined as acting, also called active experimentation (AE) where the concepts and theories from step three can be actively tested, the learners can get feedback and create new experiences. From the last step, where new experiences have been created, the cycle can begin from the first step once again (Kolb, Boyatzis & Mainemelis, 1999).

Different people tend to prefer using different learning abilities, either obtaining knowledge by experiencing the concrete or by abstract conceptualization. Similarly, some people tend to prefer carefully watching others involved in experiencing and reflecting on what happens from afar, while others prefer to jump right in and actively take part. The fact is that the four-step learning cycle consists of two pairs of opposite ways of grasping information and transforming it into new knowledge. The pairs are experiencing-thinking (grasping) and reflecting-acting (transforming). However, even though different people may prefer using different learning abilities more or less or in different orders, Kolb emphasizes that the deepest kind of learning appears when all four steps of the learning cycle are used. They necessarily do not have to be used in the exact order presented in figure 2.1 on page 9, as long as they are all actively engaged at some point of the learning process (Kolb, Boyatzis & Mainemelis, 1999).

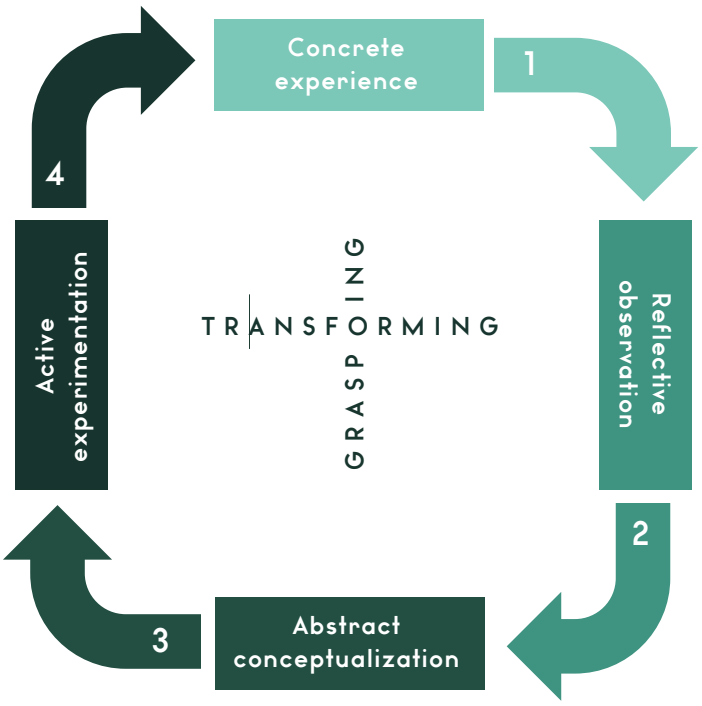

Figure 2.1: *An illustration of Kolb's experiential learning process in a four-step learning cycle.*

2.2 Other approaches to learning

2.2.1 Game-based learning

Lastly, as the chosen interactive exhibit is designed as a game that the visitors are supposed to play, the learning approach of game-based learning is further elaborated. The definition of game-based learning can be summarized as taking advantage of and utilizing the power of different kinds of digital games to captivate and engage learners for a specific purpose. This purpose should include some clearly defined learning outcomes where learners are supposed to develop new knowledge and skills. In short, games are essentially recreated environments or systems where the users are supposed to solve a problem or a series of problems, which is what makes them the perfect setup for different learning situations (Corti, 2006; Felicia, 2014; Pan et al., 2021). By definition, game-based learning does not have the aspect of entertainment and fun as the primary purpose as the main objective is for the learners to learn something. However, if possible, the learners should also have fun while learning, and preferably the fun should be collaboratively shared (Michael & Chen, 2005).

The ideas of game-based learning are not solely focused on collaborative gameplay, even though many argue that one of the strengths of game-based learning is the possibilities for collective learning. Games usually require some type of interaction, in most cases with the content of the game but often also with teammates or opponents. Collaborative gameplay where learners play in groups creates an effective learning environment where learners are allowed to help each other when playing, accommodating their talents and insights and learning from each other. Even when learners are not playing in teams together but rather against each other, the aspect of playing together in a group can foster learning through the communication with others (McCall, 2009).

2.2.2 Learning with interactive exhibitions in science centers

As mentioned in chapter 1.1, the interest in teaching science and technology in informal settings such as science centers has increased during the past decades (Sasson, 2014; Vainikainen, 2015). Designing these interactive experiences can be driven by many different objectives depending on the focus of the science center, however, visitor learning of some sort is usually the main priority (Barriault & Pearson, 2010). Falk (2001) describes the learning taking place at informal settings as ‘free-choice learning’. He argues that the degree of self-direction and self-selection involved when visitors choose to interact with exhibitions in these informal settings is very high. When visitors have chosen to interact with a specific exhibition, their learning tend to be non-linear and personally motivated where the learner herself chooses what to

learn and where and when to participate in the learning (Falk & Dierking, 2000).

As science centers seldom offer clear guidance and learning objectives, in a way the direct result of them being informal which evidently is their intention, it is not easy to investigate and understand what kind of learning opportunities actually has taken place and how. The methods being used in formal settings to evaluate learning are not as applicable in these informal settings which means that other types of methods needs to be used (Allen, 2004; Barriault & Pearson, 2010; Rogoff et al., 2016). Barriault and Pearson (2010) argues that learning in science centers is multi-dimensional and that the understanding of the learning that takes place in these settings needs to include affective impacts as well as the understanding of how each experience is highly personal and contextualized.

It is clear that getting an understanding of the learning in informal settings such as science centers is not as straight forward as in formal settings as the learning itself is a self-regulated and multi-dimensional process taking place in a partly unguided and unsupervised environment. It is also more difficult to express the actual learning objectives which leads to the expectations being unclear. In the next chapter, chapter 3, previous research on learning in informal settings will be presented which will then form the foundation for creating a coding system to further get an understanding of the conversations and the associated learning opportunities.

3. Previous research

This chapter presents nine different case studies where learning talk has been investigated in different kinds of informal settings. The case studies were chosen based on the criteria that they were conducted from the year of 2000 and onwards, that they were investigating some kind of science-based learning in relation to conversations and utterances and that the learning settings were not strictly formal. During the process of finding these relevant studies, many case studies were found by looking at the references from another study as many of them are referring to each other.

The nine case studies are used as inspiration, mainly in relation to the development of a coding system through which the participants' utterances and actions will be categorized. Inspiration has also been taken in relation to the data collection, where most of the case studies presented below are using audio recordings or audio-visual recordings to collect the data as the participants are taking part in the different learning situations.

Each case study is briefly presented and then the related coding system is described. The final part of the chapter discusses which coding system characteristics seen in the different case studies are most relevant as inspiration and why in relation to the research questions of this master's thesis.

3.1 Previous research and related coding systems on learning talk in informal learning settings

The first example of studying visitor conversations and learning talk is the study done by Scalfi et al. (2022) investigating what families visiting a zoo in Brazil are talking about in order to characterize the visitor experiences. The study develops and uses a bottom-up encoding system, meaning that a system of codes are created iteratively based on the analysis of the data itself. The final codes used in the study are: *Superficial conversations about animals, Science-based conversations, Conversations about the exhibition, Conversations including associations with previous experiences, Conversations with emotional responses* and *Conversations involving reading*.

Another similar example is the study done by Tunnicliffe and Reiss (2000) where the conversations of children relating to three-dimensional representations of animals are investigated. The coding into categories is here done according to a systemic network developed from the work of Bliss, Monk and Ogborn (1983). The categories are structured in a hierarchical manner and the major categories of the

study are: *Management and social comments, Exhibition-focused comments, Exhibit access comments* and *Animal-focused comments*. The animal-focused comments are then divided into six subordinate groups according to: *Interpretative comments, Affective comments, Environmental comments, Body part comments, Comments about the animals' behaviours* and *Comments about the animals' names*.

Allen (2003) has done a study looking for learning in visitor talk at an Exploratorium in San Francisco, focusing on a frog exhibition including elements typical for a science museum, a zoo and a natural history museum. Allen states that most of the methods used at that time to study visitors' experiences rely on the responses of individuals rather than groups. She is also critical to using the visitors' feedback after they have left the exhibitions rather than looking at their conversations and behaviours during the visits. The study thereby develops a system of categories and subcategories to analyze the visitors' talk while experiencing the different elements of the exhibit. The five main categories are: *Perceptual talk, Conceptual talk, Connecting talk, Strategic talk* and *Affective talk*. Perceptual talk is divided into *Identification, Naming, Feature* and *Quotation*. Conceptual talk is divided into *Simple inference, Complex inference, Prediction* and *Metacognition*. Connecting talk is divided into *Life-connection, Knowledge-connection* and *Inter-exhibit connection*. Strategic talk is divided into *Use* and *Meta performance* and Affective talk into *Pleasure, Displeasure* and *Intrigue/Surprise*.

Two other studies executed at about the same time as the study of Allen (2003) are the two connecting studies done by Ash (2002; 2003) exploring family conversations during museum visits in Santa Cruz. These studies focuses on collaborative scientific sense-making based on these family conversations. Ash (2002; 2003) does not use coding systems as straight forward as the previously mentioned studies as she is studying the conversations in terms of longer representative dialogic segments, meaning several sentences belonging together. However, some of the coding categories in relation to the visitor talk used that might be relevant to the scope of this master's thesis are: *Observing, Questioning, Interpreting, Comparing, Explaining, Hypothesizing, Identifying* and *Contrasting*.

In the study of DeWitt and Hohenstein (2010), they are investigating and comparing student discussions on different scientific topics being presented to the students in museums and then in classrooms. They are focusing on the discourses between only children, where no adults are taking part. Also, they are aiming at highlighting not only the content or the topics of the conversations, but also the nature of the interactions. Their coding system in relation to the nature of the discourses include the categories of *Cumulative talk, Disputational talk,*

Parallel talk and *Exploratory talk*. They also categorize the type of talk according to *Content-related talk* and *Procedural talk*, where the Content-related talk has been divided into the categories of *Explanation, Fit, Description, Read, Description (visual), Content-superficial, Affective, Attention* and *Other*.

The following two studies are very different from the previous ones, however, they define and use coding systems that are of relevance. In the study done by Saraiya et al. (2005), several bioinformatic visualization tools are evaluated letting recruited subjects with no prior experience use the tools. The purpose of the visualization tools are to generate insight in relation to the data that they are visualizing, and the study is trying to determine whether this has occurred successfully or not. The categories used in the coding system of this study are: *Observation, Time, Domain Value, Hypotheses, Directed/Unexpected insights, Correctness, Breadth/Depth* and *Category*.

Liu and Heer (2014) uses the categories from the study by Saraiya et al. (2005) to further develop a coding system to be used when evaluating the performance of another exploratory visual analysis tool. In their coding system, they are using the categories *Observation, Generalization, Hypothesis, Question, Recall, Interface* and *Simulation (Mental visualization)*.

The final study presented here is the study conducted by Isaksson and Söderberg (2022), also at the science center of Universeum in Gothenburg but focusing on another exhibition called the OpenSpace exhibit were visitors can explore open research data from space that requires visualizations to be accessible. In this study, Isaksson and Söderberg took inspiration from Saraiya et al. (2015) and Liu and Heer (2014) when they created their own coding system. Their coding system consisted of three main categories related to *Learning, The visualization system* and *Experience*. Learning included the subcategories of *Observation, Comparison, Shallow question and answer, Deep question and answer, Recall, Mental visualization, Quotation, Interpretation of written information, Interpretation of visual information* and *Exploration*. The visualization system included the subcategories of *Interface, Instruction, Orientation and Planning* and Experience included the subcategory of *Indication of experience*. The findings from this study has later been summarized and published together with Pareto (Pareto et al., 2023).

The nine above presented studies are different in their nature. Some focusing more on the specific content and topics of the conversations rather than the conversation processes and the interactions between the participants, wherein this category some studies focused more on the child-child relations and others on the child-adult relations. Some differentiating units or segments based on single words or sentences

rather than complete conversations or several sentences. Also, they are different in how they choose to summarize the findings. Some count frequencies in numbers, other by total absence or presence. Some do not even present the findings in a quantitative way at all, instead focusing on figures and maps illustrating the analyses and the results.

Reflecting back to the research questions of this master's thesis in relation to what kind of coding system would fit best, a coding system focused on revealing the specific content and topics of the conversations would be most relevant as this is what will be the foundation of the analyses. This coding system should preferably also categorize this conversational content according to larger categories to be able to determine what parts of the conversations are related to learning and not. The process of developing the coding system, with the inspiration from these previous studies, is further elaborated in chapter 4.5.1.1.3. The final coding system is presented in chapter 5.3.

4. Methodology

4.1 Research approach

This study was executed on the basis of an *inductive research approach*. Thomas (2006) states that "the primary purpose of the inductive research approach is to allow research findings to emerge from the frequent, dominant, or significant themes inherent in raw data, without the restraints imposed by structured methodologies". Inductive research represents the reverse procedure of deductive research, where key themes and categories are usually based on the preconceptions imposed by the researchers meaning that some findings might be overlooked or left invisible. Thomas (2006) also explains that "the inductive approach is intended to clarify the data reduction process by describing a set of procedures for creating meaning in complex data through the development of summary themes or categories from the raw data".

The inductive research approach was chosen based on the nature of the research questions. To be able to fully explore what types of conversations, actions and learning opportunities emerged, the analyses had to be unhindered by any preconceptions. At the beginning of the process of the study, there were no expected ideas of what the results might turn out to be.

4.2 Research strategy

The main aim of this study was to investigate authentic informal learning situations in a science center, where participants were using an interactive mathematical exhibit. To ensure the validity of the study, the authentic experiences of the participants had to be preserved. This resulted in the research strategy of a *case study* being the natural choice.

A case study can be defined as an empirical investigation of any phenomenon in its natural setting. Multiple methods of data collection can be used depending on what type of phenomena is to be studied. The definition of a case study also includes the fact that it is bound by time and activity, where the researcher executes the data collection over a sustained period of time (Creswell, 2014; Priya, 2020; Yin, 2009). Case studies are usually divided into three different categories: descriptive, explanatory and exploratory case studies. This case study belongs to the category of exploratory case studies, meaning that the aim is to gain a deeper understanding of a particular phenomenon or topic. Exploratory case studies involve detailed investigations of specific cases to explore and generate new insights, theories or hypotheses (Yin, 2014).

In the case study, the methods of *audio-visual observations* and *semi-structured interviews* were used. The audio-visual observations were done through recordings executed at the interactive mathematical exhibit at the science center to be able to capture the embodied conversations and the learning opportunities of the participants. The semi-structured interviews were used to be able to understand the intentions of the exhibit designers and to compare those to the findings from the audio-visual observations.

A semi-structured interview is defined as a qualitative data collection method where the investigator asks the interviewees a number of predetermined but also open-ended questions. Semi-structured interviews are the middle ground between unstructured interviews, mainly general verbal communication, and structured interviews, where template questions are used and asked in a specific order (Ayres, 2008). Semi-structured interviews are suitable when the goal is to better understand the unique perspectives and opinions of the interviewees (Adeoye-Olatunde & Olenik, 2021). Adeoye-Olatunde & Olenik (2021) also argues that "a primary benefit of the semi-structured interview is that it permits interviews to be focused while still giving the investigator the autonomy to explore pertinent ideas that may come up in the course of the interview".

4.3 Empirical setting

The audio-visual observations were conducted at the mathematical exhibition *Mathrix* at the science center Universeum in the city of Gothenburg. The exhibition consists of 23 interactive exhibits that are divided into four different zones: 'The self", "The world", "The creation" and "The nature". Each of the themes are meant to describe how mathematics are related to things that the visitors encounter in their everyday lives.

The case study focuses on one of these interactive exhibits which belongs to the theme related to nature. The station is called "*Voronoi – Natural phenomena and mathematical model*" and at this station the visitors are introduced to something called a Voronoi diagram. As explained in chapter 1.1, a Voronoi diagram is a visual pattern that appears naturally in a wide range of contexts in nature, but the pattern can also be explained mathematically. A Voronoi diagram is constituted by a plane with a given number of dots in the plane, called seeds or generators. For each seed there is a corresponding region, consisting of all points of the plane that are closer to that seed than to any other seed on the plane. Voronoi diagrams are, among other things, used to help understand the proximity and distance of different features (Wolfram MathWorld, 2024). See figure 4.2 on page 20 and figure A.1 in Appendix

I for the full information text that can be found at the station, explaining the mathematical model as well as several areas of application.

The station is designed as large horizonal digital display surrounded by a green pattern influenced by the Voronoi diagram and accompanied by three chairs, see figure 4.1 on page 19. Directly in front of the display there are four separate buttons in the colours of yellow, green, red and blue. The visitors are invited to explore the Voronoi diagram by playing a game based on the formations of different Voronoi diagrams. Before any participants have started the game, the display shows the text “Can you capture the largest area?”. The goal of the game is hence to try and conquer as much display area as possible. This is done by placing three different dots for each player, each participant in its turn, on the display. These dots will, when all dots are placed, be the originating points, or seeds, of a Voronoi diagram, filling the whole display area with regions of the four different colours. When the whole display is filled with the different colours, the game tells the players which player has conquered the largest area and how many percent that area conquers. See figure 4.3 on page 21 for an example of the display after the completion of a game round. The game can only be played with four different players. If the participants choose to play with fewer than four players, the computer will act as the remaining player or players.

As far as the gameplay is concerned, the game interface itself does not offer any instructions on how the formation of the pattern works. The instructions from the game interface are: “Press the button to play”, which means that the visitors need to press the different coloured buttons to enter the game, and “Press the yellow/green/red/blue button” and “Press anywhere on the screen to place your dot” which means that the visitors always have to press the physical coloured button first and then the display to place their dots. These last two hints are repeated during the course of the whole game round. To be able to understand how the formation of the pattern works, the visitors need to read the information that is provided on the information board, next to the horizontal digital display.

Figure 4.1: *The setup of the exhibit Voronoi, including the digital display, the three chairs and the information board.*

Figure 4.2: *The information board presented to the visitors next to the exhibit. The full text of the information board can be found in Appendix I.*

Figure 4.3: *Close-up of the exhibit including the placements of the video camera (1) and the microphone (2). This figure also shows an example of a complete Voronoi diagram at the end of a game round.*

Figure 4.4: *The surroundings behind the exhibit.*

Figure 4.5: *The surroundings behind the exhibit.*

4.4 Data collection

The data collection was based on two empirical sources: use and conversations (audio-visual observations) and design and learning objectives (semi-structured interviews). In order to produce reliable results, the interviews with the exhibit designers were conducted after the completion of the data collection and the analysis of the data. In that way, there would be no preconceptions that might affect the findings from the case study, in accordance with the inductive research approach.

4.4.1 Use and conversations (audio-visual observations)

The main empirical source of the data collection was the audio-visual observations executed on site at the exhibition *Mathrix* at the science center of Universeum in Gothenburg. The observations were executed during two days of a week of school holiday when the exhibition was more crowded than usual with visitors between approximately the ages of 1 and 70 years old. This was an intentional choice to be able to have multiple visitors to choose from.

4.4.1.1 Recruitment of participants

As the main target group of the exhibition is children of the ages between 13 and 18 years old, the main focus was to find participants in that particular age range. Groups of two participants or more were aimed for as a crucial part of the study includes the verbal conversations between the participants when using and exploring the interactive exhibit. Also, only pairs or groups of participants speaking the languages of Swedish, English or Danish were selected so that the transcription process would be as easy as possible.

4.4.1.2 Procedure

The randomly chosen visitors of the science center was observed with the help of audio-visual recordings from a GoPro Hero8 Black camera discretely placed on top of the interactive exhibit, only capturing the display area and the four coloured buttons (see figure 4.3 on page 21). As the audio recordings provided by the camera itself were insufficient, a Zoom H1n microphone was placed just above the display area to be able to better capture the sound and conversations coming from multiple directions at once (see figure 4.3 on page 21).

In order to maintain a natural visit experience for the participants and to influence the result as little as possible, the initial aim was that the participants would be given no specific instructions. They were also not asked to communicate more than usual or to explain what they were

doing or thinking. However, as the interface, the rules and the purpose of the game turned out to be quite tricky for the participants to understand, some instructions had to be given to most of the participants before or during the different sessions to encourage them to interact with the station. The aim was then that these instructions would affect the results of the study as little as possible.

4.4.1.3 Chosen participants

In total 20 sessions with 57 different participants were recorded and they were all included in the analysis. The different sessions were given the anonymous numbers from 1 to 20. Table 4.1 on page 26 summarizes the ages, genders and nationalities of each participant belonging to each session. It also presents the number of actively talking or playing participants in each session.

4.4.1.4 Ethical considerations

As the data collection was executed in the form of audio-visual recordings, naturally there were ethical issues that had to be considered beforehand. Research ethics are not static nor straight forward, however the guidelines provided by the Swedish Research Council (2017) were used as foundation when designing the setup of the data collection.

During the data collection, all groups of participants were informed of their participation and what the collected data was going to be used for, in other words the main purpose of the overall study. They were informed that their participation was voluntary and anonymous and that they were always able to withdraw their participation after the collection of the data. Also, the participants were informed of how the data was going to be collected, who was going to have access to it and for how long it was going to be stored and where. Time was allocated for any kinds of questions from the participants prior to and after the data collection. The demographic information of age, gender and nationality was collected from the participants.

All aspects of the data collection were presented in the Participation form and Consent form & demographic information (see Appendix II, III, IV and V) which was written in simple language to enable all participants to understand what they were agreeing to. All participants younger than 15 years old were required to be accompanied by a parent or legal guardian who could fill in the Participation form and the Consent form & demographic information. However, the parent or the legal guardian did not have to take part in the data collection. Participants of age 15 or older were allowed to fill in the forms

ID	<13	13-15	16-18	19-21	22-25	26-35	>35	N	P
1	FF						M	SWE	3
2	MM						F	SWE	3
3	F						M	SWE	2
4	MM						F	SWE	3
5	FFF							SWE	3
6	FM						FM	GER	4
7	FM						F	SWE	3
8	FFM						F	SWE	4
9	M						M	DEN	2
10						FM		SWE	2
11	F						M	SWE	2
12	MM						F	SWE	3
13		F						SWE	1
14	F						M	SWE	2
15	FMM						F	SWE	4
16		FM					FM	DEN	4
17		F	M				M	SWE	3
18	M						F	SWE	2
19				M	MM			NED	3
20	M	M					FM	SWE	4

Table 4.1: *Overview of the 20 sessions with a total of 57 different participants. The columns represent the ages and nationalities of the participants as well as the number of participants in each session. The letters F and M represent whether the participants were female or male.*

themselves. The contact details of the researchers were included in the form to enable participants to retrieve from the case study or to ask questions risen long after the visit to the science center.

The most problematic ethical aspect was the fact that the data collection was made through audio-visual recordings. However, both the audio and the visual parts of the recordings were crucial to be able to fully conduct the data collection. The video recorder was placed so that only the display area of the exhibit was visible which means that only the hands of the participants were recorded. Even though hands might be enough to identify a person, the hands were not blurred as the video recordings were never used visually in the final report of the study. Whenever the participants used personal identification information in their conversations, such as the participants' own names, these parts were removed from the transcripts and replaced with the unique number of each participant (see chapter 4.5.1.1.1). Each group or pair of participants was given a number and the same number was used on their matching participation and consent forms, recordings and transcriptions.

4.4.2 Design and learning objectives (semi-structured interviews)

The second empirical source of the data collection was the semi-structured interviews with the exhibit designers. They were interviewed to identify and categorize their design and learning objectives.

4.4.2.1 Chosen interviewees

In total four people who were involved in the developing of the exhibit were interviewed and they were chosen based on the suggestion of the supervisor. These were Håkan Sigurdsson, Philip Gerlee, Mats Blysing and Lena Pareto.

Håkan Sigurdsson works as a scientific director at the science center of Universeum. He describes his role as being responsible for the bigger decisions in relation to the exhibition rather than focusing on individual exhibits. Together with a group of other colleagues, he developed the story and the concept of the exhibition as a whole. In the case of the exhibition of *Mathrix,* he was also further involved in the details of some of the exhibits where Voronoi was one of them.

Philip Gerlee works as a professor in applied mathematics and statistics at Chalmers University of Technology. He describes his main research interest as mathematical biology but he is also interested in applications of game theory to biology. In terms of the development of

the exhibition *Mathrix*, he was involved in the beginning phases where he and another colleague was invited to brainstorm different ideas of possible exhibits, including the idea of the exhibit Voronoi. His main role was to act as a so called 'external expert', providing Universeum with useful insights and knowledge on specific topics.

Mats Blysing works as a UX-designer and creative director and his role in the project of *Mathrix* mainly included the responsibility for the overall user experience of the exhibition. He made sure that all exhibits were connected, in relation to both graphics and tonality, to maintain the feeling of a unified exhibition even though the exhibits contain very different topics. He also describes his role as making sure that the experiences are similar and dissimilar enough, to stimulate movement through the exhibition space. He was specifically involved in the development of some of the exhibits where Voronoi was one of them.

The last interviewee, Lena Pareto, is also the supervisor of this master's thesis. She works as a professor in pedagogy with a special mission for Universeum and is based at the University of Gothenburg. She describes her main research interest as digital design for learning including game design. Similar to the rest of the interviewees, she has been involved in the process of the development of the exhibition *Mathrix* from the very beginning. She describes that she took the responsibility of keeping the level of mathematical content sufficiently high based on the chosen main target group. She also made sure that external experts were involved and that prototypes were created and tested throughout the process. She has also been further involved in the details of the exhibit Voronoi and that is the main reason as to why she has been included in the interviews. When the Voronoi interactive exhibit was chosen to be the focus of this master's thesis, it had not been revealed that it was one of the exhibits that she had been specifically involved in.

4.4.2.2 Procedure

The semi-structured interviews were conducted separately due to logistic reasons. The audio from the interviews was recorded. The interviews were based on an interview guide containing three sections:

1. The interviewees' roles in the project
2. The intended learning objectives associated with the exhibit
3. The intentions with the physical and digital design of the exhibit to support the intended learning objectives

See Appendix VI for the complete interview guide including all questions and themes that were asked and discussed.

The initial idea was to interview all four interviewees after the completion of the data collection. However, during the course of the study, it became evident that as one of the designers of the exhibit was the supervisor of the master's thesis, that interview had to be conducted in a way so that neither the supervisor nor the researcher would be coloured by one another. This was solved by not having an oral interview but rather a written one, where the questions from the interview guide were sent to the supervisor before any discussions of the data and the analysis were begun. The answers to these questions were not read and analyzed by the researcher until after the completion of the analysis of the data. This way the analysis would not be based on any preconceptions, neither from the supervisor nor the researcher, in accordance with the inductive research approach.

4.5 Data analysis

The data analysis consisted of three different parts. The categorization and classifying of the conversations, the identification of learning opportunities and the identification and comparing of design and learning objectives. The classifying of the conversations was only done to the subcategories belonging to the main category of Learning talk (mathematical talk).

4.5.1 Categorizing and classifying conversations

The categorization of the conversations was done by developing a coding system through which the participants' utterances and related actions were categorized. The creation of the coding system was done in three different steps where each session was thoroughly transcribed, relevant actions were added and lastly a categorization was done which then resulted in the final coding system.

4.5.1.1 Categorizing conversations

4.5.1.1.1 Transcription

Firstly, each session was transcribed by hand by only using the audio recordings. This means that all conversations that were possible to pick up were written down in the order that they were spoken.

Any utterances from the researcher were removed as the details of these utterances were assessed as irrelevant to the scope of the study. However, the nature of the utterance was noted which, at the end of the transcription process, resulted in four different themes of utterances from the researcher. These utterances were all related to different kinds of instructions to the participants. As mentioned in chapter 1.4, these instructions were only allowed to include the basics of the game and the

interface to enable the visitors to start acting on their own. This means that in the transcriptions, these utterances are presented as "*The researcher gives the instructions of theme 1/2/3/4*". Any comments from the participants only being reactions to these instructions were also removed. If any of the comments from the participants in relation to the instructions were evaluated as relevant, they were included in the transcriptions.

All other utterances were linked to each participant by using the names from **P1** to **P57**, meaning participant 1 to participant 57. As many of the participants had voices that sounded similar to each other, especially the youngest participants, and as the video recordings only showed the display and the hands of the participants, linking all quotes to the correct participant was not always a simple task. It was a process of listening to the sessions several times to try and understand the dynamics of the conversations and making sense of who must have said what. For example, trying to understand when a participant was answering herself or himself or when a participant was talking to another participant. Also, many of the participants used each other's names and nicknames, which worked as a support in identifying who was talking and who was answering.

During the transcription, a segmentation process was developed to determine the appropriate level of granularity. This means that many of the utterances were divided into parts where each part only had one specific focus. This segmentation process was not straight forward, however, during the course of the work it was quite evident which utterances had different characteristics or not. The segmentation process was mainly inspired by those of Liu and Heer (2014), Scalfi et al. (2022) and DeWitt and Hohenstein (2010), where the segments consisted of single sentences instead of words or longer conversations. The following utterance is expressed by the same participant but was segmented into three different parts as they were assessed to have different foci:

Original utterance

P5 *Yes, look at red! Yes, but look, it looks pretty good for yellow! I could have... I won! I thought I would try to do it very tight, it was just a test*

Segmented utterance

P5 *Yes, look at red! Yes, but look, it looks pretty good for yellow!*

P5 *I could have... I won!*

P5 *I thought I would try to do it very tight, it was just a test*

Choosing a level of granularity that singles out each word would have made the utterances lose their contextual meaning and would also have been too time consuming, whereas analyzing the data as continuous conversations would have been off-topic as the scope of this study is to focus on the content and topics of the conversations.

4.5.1.1.2 Addition of actions

Secondly, the video recordings were used to better understand the utterances and what the participants were referring to when they were talking. For example, many of the participants talked about or commented on actions of their co-participants or on things that happened on the display without actually saying what it was. These components would be impossible to analyze and understand without also being able to see what the participants themselves saw.

Whenever an action of this sort was conducted, this was noted as an addition to the utterance by using brackets and explaining what the participant did or what they meant. However, the video recordings were not used to make notes of exactly everything that the participants did. As long as the utterances themselves were enough to understand the context and meaning of the utterances, no actions were added.

4.5.1.1.3 Categorization

Lastly, the written transcripts consisting of clearly segmented utterances together with the notes of the relevant actions of the participants were used to categorize and structure the data. This was done by using the MaxQDA software for Mac, a qualitative research tool that can be used to easily code and analyze different source materials (MaxQDA, 2024). During the process of categorizing the data and developing a well-fitted coding system, the thematic analysis method of Braun and Clarke (2006) was used. This method is an iterative process consisting of six steps explained in the following paragraphs.

The first step (1) was to become familiar with the data, which was done by reading through all transcriptions several times to get a initial feel for the sense of the conversations.

The second step (2) was to generate codes, which was done as a linear process from beginning to end were all sessions were coded until all utterances were matched to a code. A code in this case study was a description of the content of the utterance on a very specific level, which means that during this step the number of codes was very high, close to 100. Whenever an utterance could be fitted into several codes, which happened seldom but a few times, the utterance was matched to

the code that fitted best with the assessed core meaning of the utterance. During this step, none of the coding systems of the previous research studies were reviewed yet. Up to this point, the code generation was only based on the conceptions of the researcher with no aim of trying to fit the codes into an already existing coding system.

During the third step (3), more general themes were generated from the set of codes. This means that the set of codes were reviewed with the purpose of finding similarities so that the number of codes could be narrowed down and combined into bigger themes. During this step, the coding systems of the previous case studies were used as inspiration. The set of codes were matched to different categories or themes that could be found in other coding systems. New categories were also created when there was no category that seemed to fit. At this point, the number of codes were narrowing down to about 50-60, and the categories to about 20-25. All nine example case studies were used as inspiration, however, as none of them are investigating the exact same thing with the exact same focus, no coding system could be used as a whole. Bits and pieces from the different coding systems were used to develop a new coding system. As these kinds of case studies are very specific, it is quite evident that most of them also need a specific coding system. In this case study, the coding system was supposed to investigate the actual content of the conversations, as well as the overall types or themes of the conversations, which means that different case studies with different foci could be used as inspiration. For example, the coding systems used to categorize what families visiting the zoo in Brazil (Scalfi et al., 2022) and what children seeing representations of three-dimensional animals (Tunnicliffe & Reiss, 2000) are talking about are more focused on the specific content of the conversations. The coding systems used to categorize what visitors of the frogs exhibition in San Francisco (Allen, 2002) and visitors at the museum in Santa Cruz (Ash, 2002; Ash, 2003) talk about, on the other hand, are much more focused simply on the type of conversation.

The fourth step (4) was then to start reviewing the themes, checking if they made sense in relation to the codes and to the data set as a whole. The fifth step (5) was to ultimately define and name the codes and the themes. At this stage in the process, during step 4 and 5, the codes and the categories were reviewed several times together with the supervisor to find possible overlaps, additions and general improvements. The purpose was to ensure that no code or theme was mentioned twice, was missing or was described in an unclear manner. This then resulted in several codes being combined, redefined and renamed. Quite early in the process, already during the second step, the aim was to distinguish which utterances were related to the process of learning and which were not. This means that some main themes or categories were already in

mind, where *Learning talk* was one of them. However, it was not until the fourth step of the process that all of these main categories were ultimately distinguished. The final main categories were also inspired by some of the previous case studies, mainly the case study conducted by Isaksson and Söderberg (2022) and Pareto et al. (2023). The three main categories that were defined in the end of the process were *Learning talk*, *System talk* and *Affective talk*. However, during the finalization of the main categories, it became evident that the category of Learning talk consisted of two types of talk that were not fully related to each other. On the one hand, the participants explicitly talked about the mathematical content of the game. On the other hand, they talked about the game by commenting on the game rules and the actions that they were taking. This second type of talk is defined as game mechanics (Fe, 2016). The main category of Learning talk was therefore divided into *mathematical talk* and *game mechanics*, to distinguish these different characteristics from each other. The characteristics of these two types of Learning talk, and why they are both considered to be Learning talk, are further elaborated in chapter 5.3.

The sixth and final step (6) was to locate exemplars which essentially meant finishing off the coding system and presenting it in a clear arrangement. The final coding system is presented in chapter 5.3 and the result is a hierarchical coding system containing three main categories, 16 subcategories and 37 subordinate groups. Examples from the data from the different main categories and subcategories are presented, together with a description of the characteristics of each main category.

The final analysis of the conversations is done in chapter 6.2, where the data is viewed from the perspective of the utterance statistics based on the final coding system.

4.5.1.2 Classifying conversations

The subcategories belonging to the main category of Learning talk (mathematical talk) were further analyzed by using the SOLO taxonomy as defined by Briggs and Collis (1982). The SOLO taxonomy, where SOLO is a shortening for The Structure of the Observed Learning Outcome, is used as a means of classifying different levels of understanding in terms of complexity, focusing on the quality of the learning outcome rather than quantitative measures. Five different stages of cognitive development are defined as Prestructural, Unistructural, Multistrucutral, Relational and Extended abstract. At the prestructural stage, the learner has not yet reached an understanding of the point of the task. At the unistructural stage, the learner has understood one relevant aspect of the task, while at the multistructural

stage, the learner has understood several but unconnected aspects of the task. When the learner reaches the relational stage, he or she can start to connect different ideas and aspects of the task. The final stage, the extended abstract stage, is reached when the learner is able to make connections beyond the original task (Biggs & Collis, 1982).

The analysis of the mathematical talk in relation to these different levels of complexity is also done in chapter 6.2.

4.5.2 Identifying learning opportunities

The identification process of the learning opportunities of the participants was made in four different steps.

Firstly, five of the 20 sessions were chosen to look further into where opportunities for learning were thought to be the most possible. The developed coding system and the analyses of the conversations were used as a tool to find these five sessions. The criteria for the selection was mainly based on the extent to which utterances belonging to the subcategories of game strategies, game management, connection and interpretation were present as these subcategories were classified as the parts of the mathematical talk that were the most complex. Session 2, 7, 15, 19 and 20 were chosen mainly based on these criteria but also based on the fact that they were some of the longest sessions with the most utterances. This decision was made so that there would be enough utterances to analyze. This means that these five sessions were the only ones being more closely analyzed. The analysis therefore does not represent the learning opportunities in a typical session, but rather the learning opportunities of some of the most 'successful' sessions.

The method of interaction analysis was then used to be able to draw conclusions about what learning opportunities actually arose. Jordan and Henderson (1995) defines the method of interaction analysis as "a method for the empirical investigation of the interaction of human beings with each other and with objects in their environment. It investigates human activities, such as talk, nonverbal interaction, and the use of artifacts and technologies, identifying routine practices and problems and the resources for their solution". Here, specific utterances and actions of each session were chronologically highlighted and described to get an understanding of what was said and what happened during these sessions.

The third and the fourth step involved generalizing the findings from the previous steps. The goal of the third step was to identify common patterns in the so called 'learning trajectories' of the participants. A learning trajectory can be explained as "ordered tendencies developed

through empirical research designed to identify highly probable steps students follow as they develop their initial ideas into formal concepts" (Maloney & Confrey, 2010). Simon (1995) expressed it more concisely as "the paths by which learning might proceed". This is presented as the different identified stages of the learning processes that the participants were engaging in during the course of the gameplays, and the mathematical abilities that were practiced during these stages. The four-step experiential learning process defined by Kolb (Kolb, Boyatzis & Mainemelis, 1999) was then used as a foundation onto which the identified stages were mapped.

The fourth step involved identifying possible success factors that could be found in all five sessions. The five identified success factors were used as a tool to understand and explain why the learning opportunities took place and why the sessions seemed to turn out so fruitful. During step three and four, the analyses and findings were closely linked to the theoretical framework presented in chapter 2.

The interaction analyses of the five chosen sessions, the identified learning trajectories and success factors are presented in chapter 6.3.

4.5.3 Identifying and comparing design and learning objectives

The data collected from the four semi-structured interviews were summarized based on the same six steps of the thematic analysis method of Braun and Clarke (2006), described in chapter 4.5.1.1.3. This analysis was, however, much more brief and executed without the support from the supervisor as she was one of the four interviewees. The aim was to find a few general themes that could be used to categorize the design and learning objectives of the exhibit designers. These objectives were then compared to the analyses in relation to the categorizing and classifying of the conversations and the identifying of the learning opportunities.

The summary of the design and learning objectives of the exhibit designers is presented in chapter 5.5 and the comparison of the intentions of the designers and the analyses is done in chapter 6.4.

5. Results

This chapter presents the results from the audio-visual observations conducted at the science center of Universeum and the semi-structured interviews with the four chosen exhibit designers. Chapter 5.1 and 5.2 presents the participant and session characteristics. Chapter 5.3 presents the final coding system where each main category is presented separately, followed by example utterances from the data belonging to the different subcategories of each main category, to further explain the characteristics of the different parts of the coding system. Chapter 5.4 presents a summary of the overall utterance statistics, in relation to all utterances and in relation to each session. The final chapter, chapter 5.5, consists of the results from the semi-structured interviews presenting the design and learning objectives from the exhibit designers. It also includes elements of improvement that were highlighted in the interviews. The first four chapters were completed before chapter 5.5 was started. In Appendix XII, a summary of the most common misconceptions of the gameplay is also presented.

5.1 Participant characteristics

In total, 57 different participants were actively playing or talking during the sessions. The vast majority of the participants were actively playing, however, there were a few participants that did not play but still took part in the discussions. These have also been included in the results.

The most represented age group was the participants under the age of 13. 27 out of 57 participants belonged to this age group, which corresponds to 47,4% of the participants belonged to this age group. The second most represented age group was the participants over the age of 35. 19 out of 57 participants belonged to this age group, which corresponds to 33,3% of the participants. The remaining age groups only corresponded to a total of 19,3% of the participants, where the participants between the ages of 13 and 15 represented 8,8% (5 participants), the participants between the ages of 16 and 18 represented 1,8% (1 participant), the participants between the ages of 19 and 21 represented 1,8% (1 participant), the participants between the ages of 22 and 25 represented 3,5% (2 participants) and the participants between the ages of 26 and 35 represented 3,5% (2 participants). Figure 5.1 on page 37 summarizes the distribution of the ages of the participants of the case study.

27 out of 57 participants were female, which corresponded to 47,4% of the participants. The remaining 30 participants were male, which

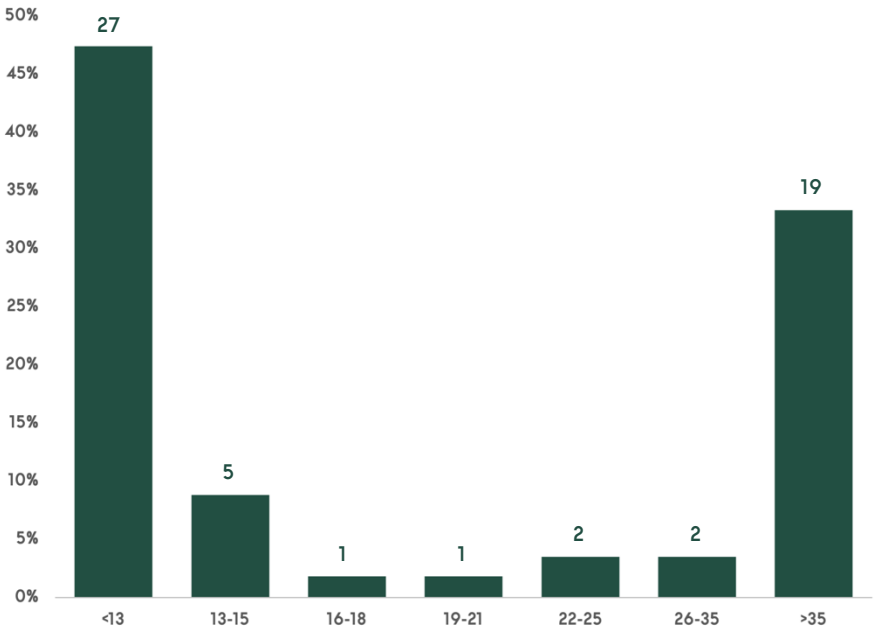

Figure 5.1: *Distribution of the ages of the participants of the study. The figure presents both the percentage and the number of participants belonging to each age interval.*

corresponded to 52,6% of the participants. The most common nationality which corresponded to 77,2% of the participants (44 participants) was Swedish. Apart from the Swedish participants, 10,5% were Danish (6 participants), 7,0% were German (4 participants) and 5,3% were Dutch (3 participants). The most common number of participants taking part in the session was three. 40,0% of the sessions (8 sessions) consisted of three actively playing or talking participants. 30,0% of the sessions (6 sessions) consisted of two participants, 25,0% of the sessions (5 sessions) consisted of four participants and 5,0% of the sessions (1 session) consisted of only one participant.

5.2 Session characteristics

In total, 20 different sessions were recorded. Table 5.1 on page 39 presents the number of game rounds, the number of utterances and the duration of each session. Both the number of utterances from the participants and from the researcher has been calculated, however summarized separately. This differentiation has been done so that the results from the conversations of the participants can be presented separately, without being affected by the number of utterances from the researcher.

A total of 86 different game rounds were played, which represented a mean of 4,3 game rounds per session. The participants made a total of 901 different utterances, while the researcher made 66. This represented a mean of 45,1 participant utterances and 3,3 researcher utterances per session. The total session time was 117 minutes and 42 seconds, which represented a mean of 5 minutes and 53 seconds per session.

ID	Game rounds	Utterances, participants (researcher)	Session duration
1	3	30 (7)	04:51
2	12	195 (6)	16:12
3	4	15 (3)	04:36
4	3	9 (3)	03:01
5	9	80 (9)	11:06
6	3	35 (3)	04:49
7	4	85 (4)	07:37
8	7	41 (7)	07:34
9	3	6 (2)	03:22
10	3	67 (1)	06:01
11	2	7 (1)	02:17
12	2	17 (2)	02:58
13	2	9 (3)	02:39
14	1	12 (1)	01:31
15	8	87 (6)	08:16
16	2	15 (2)	02:46
17	3	34 (3)	03:14
18	4	47 (2)	07:17
19	5	51 (0)	08:13
20	6	59 (1)	09:22
Total	**86**	**901 (66)**	**117:42**
Mean	**4,3**	**45,1 (3,3)**	**05:53**

Table 5.1: *Overview of the 20 sessions in relation to number of game rounds, number of utterances and duration of the sessions. The first number in the utterances column represents the utterances from the participants and the numbers inside the parentheses the utterances from the researcher.*

5.3 Coding system

The coding system is a hierarchical system containing three main categories, 16 subcategories and 37 subordinate groups. The first main category is named Learning talk and, as mentioned in chapter 4.5.1.1.3, the Learning talk is divided into mathematical talk and game mechanics. The mathematical talk consists of all the utterances from the participants that are explicitly connected to the mathematical content of the exhibit. The game mechanics consists of all the utterances where the participants are commenting on the rules of the game and the actions that they are taking, as defined by Fe (2016). In the most successful educational games, the educational content is placed at the heart of the gameplay which means that the participants of the game are engaging directly in the targeted thinking as they play the game (Fe, 2016; Fisch, 2005). The game presented in the exhibit of Voronoi can be seen as one of these successful educational games, as the participants' playing of the game draws directly on the mathematical knowledge and skills that the game is designed to foster. The educational content is not presented alongside the gameplay, but is rather integrated into the very game mechanics. Based on these arguments, both the mathematical talk and the game mechanics have been included in the main category of Learning talk as they are both related to the fundamental mathematical content of the game.

The following chapter presents each of these main categories, with their corresponding subcategories and subordinate groups. As the main category of Learning talk has been divided into Learning talk (mathematical talk) and Learning talk (game mechanics), these two are presented separately. Three typical utterances from each of the 16 subcategories used in the coding system is presented to demonstrate what kinds of utterances belong to the different subcategories. The 37 different subordinate groups are not demonstrated separately. The examples are taken from most of the different subordinate groups belonging to each subcategory.

Learning talk (mathematical talk)

Observation

Observing the formation of the pattern
Observing the placements of the dots

Inquiry

Wondering how the pattern will be formed
Wondering why the winner wins

Prediction

Predicting how the pattern will be formed

Interpretation

Interpreting the pattern to understand the outcome of the game

Connection

Making connections between the winner and the seating positions of the participants during the game

Strategy management

Realizing the need for a strategy
Reflecting on what strategy to use
Realizing a strategy is successful
Realizing a strategy is not successful

Strategy types

Using the strategy of placing the dots far from other participants' dots
Using the strategy of placing the dots close to other participants' dots
Using the strategy of placing the dots far from each other
Using the strategy of placing the dots close to each other
Using the strategy of placing the dots in the middle of the display
Using the strategy of placing the dots on the edges of the display

Figure 5.2: *The subcategories and subordinate groups of the main category of Learning talk (mathematical talk).*

5.3.1 Learning talk (mathematical talk)

The utterances belonging to the main category of Learning talk (mathematical talk) includes all utterances in relation to observing, predicting and interpreting the mathematical pattern on the display, making inquiries about how the pattern will be formed and why, identifying mathematical connections and identifying and using different mathematical strategies. These types of utterances corresponded to 34,4% of the total amount of utterances.

Example utterances from the data

Observation

P6 *Mine, look now, it grows [name of P4] [P6 points at a big blue area at the corner of the display]*

P41 *Oh wait, now it's mostly green... or blue*

P52 *Looks like I just won*

Inquiry

P51 *Who wins now?*

P2 *What will this become?*

P38 *Let's see if I will win one more time*

Interpretation

P19 *Okay, so that's how you were thinking, that you would get all of that, but I stole a bit from you there I think*

P4 *Look, if I hadn't placed my dot there, you would have come and taken all of this [P4 points at his blue areas next to the yellow areas of P5]*

P53 *I think we tried to go against each other too much here*

Connection

P53 *I think being in your spot is the best, I think the last dot is the best, you have... like in the last place you can just look at what dots give you the most area*

P56 *It was easiest for [name of P55] because he was the last one, your spot was the most difficult*

P29 *Yes, now I get to be the last one, that's great, that's really great [P29 thinks of what to do for a longer while]*

Prediction

P47 *I think it is the red one*

P14 *I bet yellow will win again, no, red?*

P43 *There will be very little of this colour, I'm completely sure*

Strategy management

P54 *I need to know if it's a good strategy to surround someone*

P51 *To like if the good spots... that's a really good strategy. Good spots go together*

P53 *I guess you have to consider who has got the most space currently*

Strategy types

P54 *But it is obviously a good thing to stay at the edges of the display*

P56 *I placed them where no one else was because then I get a bigger area*

P29 *So then you should be farthest away, kind of? So that you get a big area*

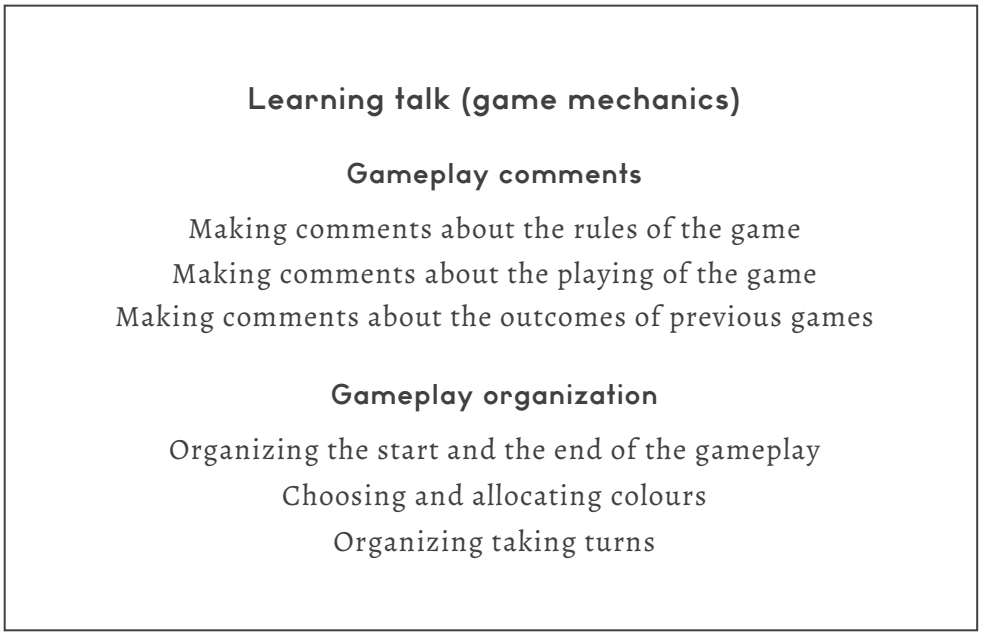

Figure 5.3: *The subcategories and subordinate groups of the main category of Learning talk (game mechanics).*

5.3.2 Learning talk (game mechanics)

The utterances belonging to the main category of Learning talk (game mechanics) includes all utterances in relation to commenting on the rules of the game, the playing of the game, recalling earlier events and outcomes and organizing and structuring the course of the gameplay. These types of utterances corresponded to 38,6% of the total amount of utterances.

Example utterances from the data

Gameplay comments

P39 *Then I will place my dot there*

P19 *Okay, so where will you place the green?*

P46 *Okay, so then it is the area that should... yeah, exactly*

Gameplay organization

P13 *I want to be blue*

P4 *Now it's your turn*

P32 *Do you want to play again?*

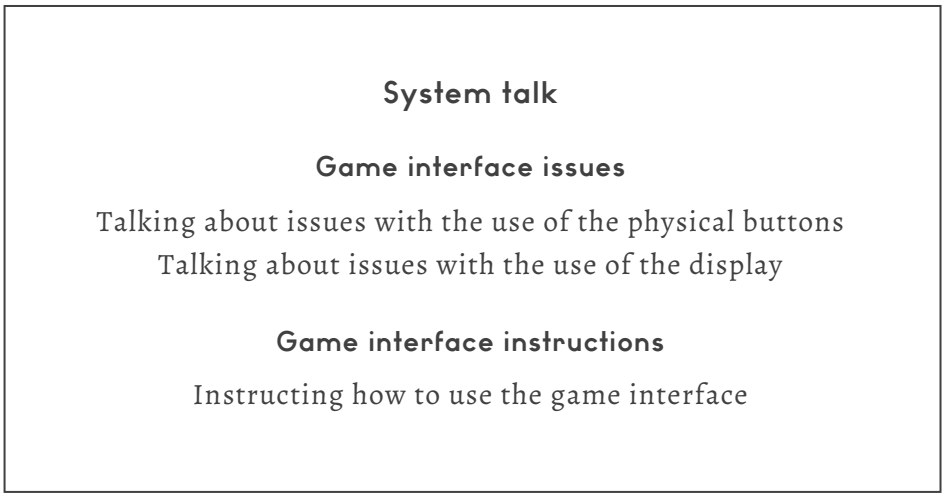

Figure 5.4: *The subcategories and subordinate groups of the main category of System talk.*

5.3.3 System talk

The second main category is named System talk. The system talk consists of all the utterances from the participants that are connected to different kinds of problems in relation to the exhibit. These utterances have no connection to the mathematical content of the game. This includes all utterances in relation to issues with using the game interface and utterances where participants are instructing each other when something is not working or being unclear. These types of utterances corresponded to 11,8% of the total amount of utterances.

Example utterances from the data

Game interface issues

P19 *One more time, try again [P21 has to press several times to place their dot]*

P1 *Maybe one at a time [P2 and P3 are pressing the display at the same time which results in no dots being placed]*

P8 *I need to press that one [P8 tries to place dots before the game has counted down to zero and started]*

Game interface instructions

P4 *It's your turn! It says down there [P4 points at the text at the bottom of the display]*

P2 *Okay, press that one, and then we are not pressing any of the other buttons*

P56 *[Name of P54], you have to press the yellow button*

Affective talk

Pleasure

Expressing pleasure of the outcome of the game
Expressing pleasure of the actions of the computer or the other participants
Expressing a will to play the game
Expressing pleasure of playing the game

Displeasure

Expressing displeasure of the outcome of the game
Expressing displeasure of the actions of the computer or the other participants
Expressing a reluctance to play the game

Surprise

Expressing surprise of the actions of the computer or the other participants
Expressing surprise of how the pattern is formed

Uncertainty

Expressing uncertainty of how to play the game

Praise

Expressing praise to oneself or to another participant

Figure 5.5: *The subcategories and subordinate groups of the main category of Affective talk.*

5.3.4 Affective talk

The third and final main category is called Affective talk. The affective talk consists of all the utterances from the participants that are connected to the feelings and thoughts of the participants as they are interacting with the exhibition station and playing the game. This includes all utterances in relation to expressing any kind of pleasure, displeasure, surprise, uncertainty or praise, either to the utterances or the actions of oneself, the other participants or the computer. These types of utterances corresponded to 15,2% of the total amount of utterances.

Example utterances from the data

Pleasure

P4 *This was nice, this was a nice game*

P22 *I want to play one more time*

P19 *It's quite fun when you understand how it works*

Displeasure

P14 *You took the placement I was going to take*

P28 *It's not fun to lose three times in a row*

P46 *Damnit! Right?*

Surprise

P14 *But what! [P14 is surprised that there are appearing dots that no one has placed on the display]*

P55 *How can the blue win!*

P8 *But how did the blue win?!*

Uncertainty

P4 *I don't understand anything*

P12 *I think this is a bit hard*

P7 *Should I press any of those? [P7 points at the coloured buttons at the interactive exhibit]*

Praise

P28 *Good job, good game!*

P50 *I'm pretty good at this!*

P5 *If you win again it's because you're so smart, you understand the game*

5.4 Utterance statistics

In this chapter, the overall statistics of the utterances of the participants is presented. The utterances from the researcher are not included in the total amount of utterances, used to calculate the different percentages.

Figure 5.6 on page 49 presents the percentage of the total amount of utterances belonging to each main category, as well as the percentage of the total amount of utterances per session belonging to each main category. The columns are organized according to the percentages of Learning talk (mathematical talk), where the sessions with the highest percentages are presented on top. The session ID for each column can be found to the left of each column. 34,4% of the total amount of utterances belonged to Learning talk (mathematical talk) and 38,6% to Learning talk (game mechanics). The total percentage of the utterances belonging to the main category of Learning talk corresponded to 73,0%. 11,8% of the utterances belonged to the main category of System talk and 15,2% to the main category of Affective talk. Looking at the utterances per session, these percentages differed significantly, where some sessions did not include all main categories at all. However, all sessions included at least 53,3% Learning talk, with session 16 being the session with the lowest percentage, taking into account both the mathematical talk and the game mechanics.

Figure 5.7 on page 50 presents a hierarchical chart of the frequencies and the percentages of each subcategory. The most common subcategory, by far, was the subcategory of Gameplay organization which corresponded to almost a fourth of all the utterances (224, utterances, 24,9%). The following four most common subcategories were Gameplay comments (124 utterances, 13,8%), Observation (105 utterances, 11,7%), Prediction (64 utterances, 7,1%) and Game interface instructions (63 utterances, 7,0%). The remaining 11 subcategories corresponded to a total of 35,5% of the utterances (321 utterances), ranging from 5,1% (46 utterances) to 1,3% (12 utterances).

Table 5.2 on page 51 presents the eleven most common utterances that are used by the participants, including their frequencies and percentages. The three most common utterances was “Observing the formation of the pattern” (100 utterances, 11,1%), “Choosing and allocating colours” (89 utterances, 9,9%) and “Organizing taking turns” (75 utterances, 8,3%).

The more detailed utterance statistics of each main category can be found in Appendix VIII, IX, X and XI. There, the frequencies and percentages of each subordinate group can be found.

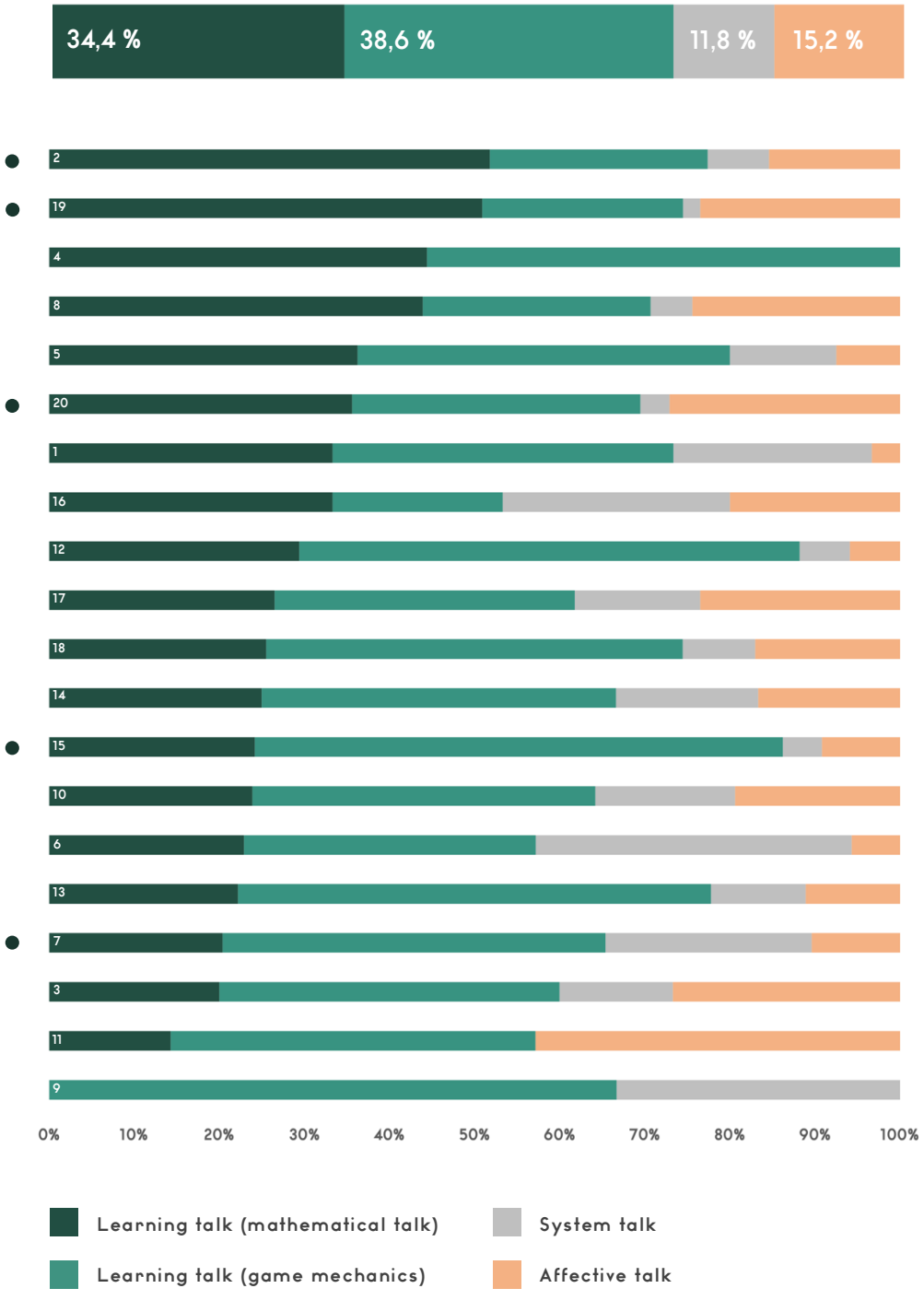

Figure 5.6: *Percentage of the total amount of utterances belonging to each main category and percentage of the total amount of utterances per session belonging to each main category. The numbers to the left of each column represent the corresponding session ID and the dots represent the five sessions that are analyzed more closely in chapter 6.*

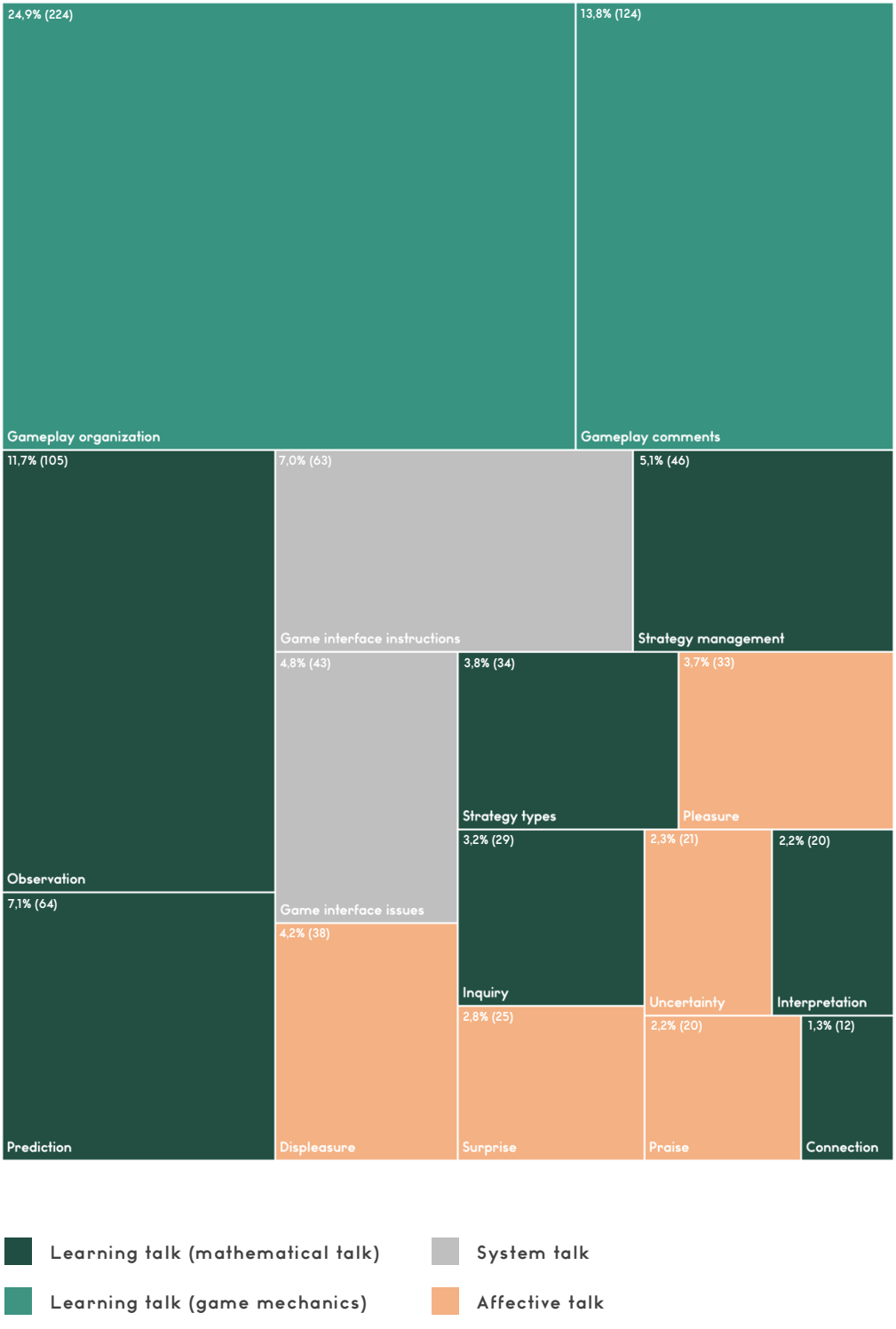

Figure 5.7: *A hierarchical chart presenting the frequencies and the percentages of the different subcategories, calculated from the total amount of utterances.*

Top 11 utterances	Main category	Frequency	%
Observing the formation of the pattern	Learning talk (mathematical talk)	100	11,1
Choosing and allocating colours	Learning talk (game mechanics)	89	9,9
Organizing taking turns	Learning talk (game mechanics)	75	8,3
Making comments about the playing of the game	Learning talk (game mechanics)	69	7,7
Predicting how the pattern will be formed	Learning talk (mathematical talk)	64	7,1
Instructing how to use the game interface	System talk	63	7,0
Organizing the start and the end of the gameplay	Learning talk (game mechanics)	60	6,7
Making comments about the rules of the game	Learning talk (game mechanics)	41	4,6
Talking about issues with the use of the display	System talk	32	3,6
Wondering how the pattern will be formed	Learning talk (mathematical talk)	26	2,9
Expressing displeasure of the actions of the computer or the other participants	Affective talk	26	2,9

Table 5.2: *Overview of the frequency and percentage of the eleven most common utterances.*

5.5 Design and learning objectives

The semi-structured interviews with the four exhibit designers are summarized according to the categories of learning objectives, design objectives and elements of improvement. The learning and design objectives are presented as single sentences whereas the elements of improvement are presented more elaborately. This is mainly due to the interviewees having different views on what could be improved and what was unwanted, and quite similar views on the learning and design objectives.

Learning objectives

To provide the visitors with a different view of mathematics as a mind-set and an integrated part of humanity.

To create awareness of patterns in nature in general and particularly Voronoi diagrams, including how the patterns can be explained with mathematical models and that they can be used in other contexts.

To create awareness of how powerful mathematical models are in general.

To foster strategical thinking, creating discussions on how to strategically conquer as much area as possible.

To foster mathematical thinking, creating discussions on how the model works and the properties of a Voronoi cell.

Design objectives

An appealing and attractive activity.

An engaging and interactive activity.

An exploratory activity.

An activity that quickly catches the visitors' interest.

A stimulating activity that retains the visitors' interest.

A visually striking activity.

An activity that is adequately fun and informative.

An activity catching people at different knowledge levels, from the most simple to the most qualified level.

Elements of improvement

The amount of formulas or animations presented to the visitors explaining the formation of the pattern and the mathematical model. Some of the exhibit designers expressed a will to have more formulas and animations included in the exhibit. Others stated that this would risk losing the fun and the pace of the game that stimulates people to play several game rounds. Some of them discussed that if a formula would have been used, each term would have needed to be explained. As there was not enough space and time to do that in this exhibit, a choice was then taken to exclude mathematical formulas and animations.

The fact that the game can only be played with four players. Some of the exhibit designers expressed a will to change the game so that it could be played with two or three players, whereas some thought it should not be changed. Some argued that the game might be even more strategic when there are only two players. Some thought that a game of two would not create a pattern interesting enough. They all stated that all exhibits are prototypes, specifically made for this exhibit, and that they have to be tested in action before any improvements can be considered. This was never done before the opening of the exhibition in the case of the Voronoi exhibit, which means that the exhibit used in the exhibition can be seen as the very prototype being tested.

The fact that when the computer acts as the remaining players (if the participants are fewer than four players), the dots are placed on the display quickly and randomly. Most exhibit designers expressed that it might create a feeling of confusion and that the interface could have been more clear about when it is the computer playing and not.

The initial idea for the exhibit was a free standing table in the middle of the room, where all players would occupy one side of the table each. This had to be changed to the current design, where the display is placed next to a wall and where all players are sitting on the same side, simply because there was not enough space in the exhibition hall. One of the exhibit designers mentioned that, with the design of a free standing table, the quite complicated procedure of having to press the physical buttons would not have been needed. If all participants were standing on separate sides of the table, the turn taking could have been organized more naturally simply by using the digital display.

Lastly, one of the exhibit designers expressed a will to improve the game by allowing the participants to see everyone's share of the display at the end of the game, adding the ability to add dots during the formation of the pattern and adding the ability to play tournaments instead of only separate game rounds.

6. Analysis

In this chapter, analyses of the collected data presented in chapter 5 in relation to the three different research questions are made respectively. Before any of the research questions are addressed, a general analysis of the participant characteristics and the session characteristics is made to understand the conditions of the collected data. Secondly, the utterance statistics are analyzed to determine what types of conversations emerged during the 20 different sessions, in accordance with **RQ1**. Thirdly, an analysis of the learning opportunities is made where five of the 20 sessions have been chosen to look further into, in accordance with **RQ2**. Lastly, all results are analyzed together in relation to the intentions of the designers to determine to what extent they align, in accordance with **RQ3**.

6.1 Participant and session characteristics

The compilation of the participant characteristics shows that the predominant part of the participants (80,7%) either belonged to the age intervals of younger than 13 or older than 35. These groups typically consisted of children with their parents or children with their grandparents. As the expressed main target group of the exhibition *Mathrix* is children between the ages of 13 and 18, this means that the results show that only 10,6% of the participants actually belonged to that target group (the remaining 8,7% were older than 18 but younger than 36). However, families of various ages are also identified as a secondary target group. During the two days that the data collection was executed, the distribution of the ages of the studied groups were representative of the total amount of visitors. In general, there were not many children between the ages of 13 and 18 visiting the exhibition at all.

The session characteristics reveal that during most of the sessions, 15 sessions, the participants played three game rounds or more. Most groups evidently enjoyed or at least were intrigued by the game and the exhibit and wanted to try again. Playing several game rounds and staying at the exhibit for a while did not always mean that the participants made a lot of conversation though, as only 9 sessions consisted of 40 utterances or more. These 9 sessions also did not completely corelate to those where three or more game rounds were played. It is clear that some of the studied groups talked to each other much more and some did not, without a correlation to whether they played many game rounds or not. It is probably not unusual that some of the visitors interacting with the exhibit do not talk to each other so much, regardless of how interested they are and how long they choose to stay at the exhibit. The fact that these kinds of sessions are

represented in the data collection rather support the claim that authentic visitor experiences were reached. There can be many possible explanations to why there seems to be no clear pattern between playing many game rounds, staying at the exhibit for a longer while and having a lot of conversation. The most crucial factor is probably related to the extent of support and scaffolding from the older participants, as they were typically the ones initiating and keeping up the conversations. No general analysis will be made of how the older participants supported the conversations during the sessions as that is not the main focus of this study, however, this is one of several aspects that are further acknowledged in chapter 6.3 where the learning opportunities in relation to five chosen sessions are analyzed.

6.2 Conversation characteristics

Analyzing the total utterance statistics from a general view reveals the observation that the predominant part, 73,0%, belongs to the main category of Learning talk. This means that the very majority of the conversations revolved around the mathematical knowledge and skills that the game was designed to foster. The part of the main category Learning talk connected to game mechanics was slightly larger than the part connected to mathematical talk. 34,4% of the utterances belonged to the category of mathematical talk and 38,6% to the category of game mechanics. This means that the participants were slightly more prone to using a language that acted as a representation of the underlying mathematical content rather than using the explicit mathematical language. For example, the participants discussed the rules and the meaning of the game and commented on where they chose to place their dots, but by using utterances that could not be categorized as strictly mathematical. A large part of the game mechanics (24,9% of all utterances) contained the utterances related to the organization of the game, where participants were taking turns, choosing and allocating colours and beginning and ending the game rounds. This can probably be explained by the game of the Voronoi exhibit being designed to be played by several players which naturally resulted in most sessions (95%) having several participants playing. In 65% of the sessions there were three or four participants playing and in 30% of the sessions there were two. Also, as each game round was relatively short, the mean time being about 1 minute and 22 seconds, this allowed these procedures to be repeated many times during each session. These utterances can be seen as a part of the learning process of how to play the game and how to interact with and use the integrated mathematical content, which makes them equally important in relation to learning talk.

The utterances belonging to the different subcategories of the main category of Learning talk connected to mathematical talk have been

analyzed in relation to the SOLO taxonomy as defined by Briggs and Collis (1982) and described in chapter 4.5.1.2. The five stages of the taxonomy are illustrated in figure 6.2 on page 58, onto which the different subcategories are also mapped.

The most common utterances belonged to the subcategories of observation and prediction. These were utterances where the participants made statements about what they saw on the display and what they thought would happen as the pattern was formed and after it was completed. As the formation of the pattern is the 'main event' of the game, the most interesting and exciting part of the game, this result is not very surprising. Even though the placements of the dots occupied much more time, the formation of the pattern generated more discussion between the participants as more observable things happened during this time. The utterances of observation can be seen as the least complex utterances belonging to the unistructural stage as they simply involved one aspect of the game: watching the pattern. The utterances of prediction are placed in the relational stage as a prediction usually involved connecting the placements of the dots with the formation of the pattern.

The second most common utterances belonged to the subcategories of strategy management and strategy types. These utterances had a more complex character where the participants reflected on what strategies to use, the fact that a strategy might be needed in the first place and whether or not tested strategies were successful. These utterances also included statements where the participants identified specific strategies in relation to where they were placing their dots. These utterance subcategories are placed in the relational stage as they involved making connections between different aspects of the game. In the presented results, utterances were only added to these strategy type subordinate groups when a participant specifically talked about using a strategy. If a participant used a strategy, or seemed to use a strategy, but did not talk about it, no utterance was added to any of the strategy type subordinate groups. This means that many of the participants probably used the different strategy types, without it showing in the utterance statistics. During the data analysis, six different strategy types emerged among the transcripts. These were quickly identified as three different pairs of strategies, according to:

The strategies of placing one's own dots...

in the middle of the display ––– on the edges of the display
far from each other ––– close to each other
far from other participants' dots ––– close to other participants' dots

These pairs of strategies seemed to present different levels of relating one's owns dots to something (see figure 6.1 on page 58). The first level that the participants reached was usually placing the dots in relation to the appearance of the display. The participants talked about whether they would place their dots in the middle or on the edges of the display. Further on, after trying a game round or two of placing the dots in the middle or on the edges and looking at the result, many of the conversations continued to relate to each participant's own dots. Here the participants realized that they could place their dots close to each other or far from each other and that these choices would create different outcomes. Some of the participants also reached the last level, where they realized that they needed to take into account what the others did and where they had placed their dots. The overall strategy that seemed to be the most successful in the end was when the participants related the placements of their dots to *all three* different levels. That is, when they understood that all three levels of relating their dots to something had an impact on the outcome of the game, and that all three levels had to work together. A few participants also realized that the order in which the dots were placed made a difference, where the last player usually had a better starting position as they could make their choice after the others had already made theirs. In total there were 12 utterances that were connected to these realizations (1,3% of the utterances).

The least common utterances in relation to the Learning talk of mathematical talk belonged to the subcategories of inquiry, interpretation and connection. The utterances in relation to inquiries were mostly related to wonders of who would win. A few utterances were in relation to why the winner won. These utterances are placed in the multistructural and the relational stage, where the inquiries in relation to who will win are seen as understanding several but unconnected aspects of the game and the inquiries in relation to why the winner won are seen as connecting different aspects of the task. The utterances where the participants interpreted the outcomes of the games and discussed connections between the seating positions and the outcomes, as already mentioned above, can be seen as much more complex. Here the participants were reflecting on the effects of their chosen strategies, trying to create an understanding of what happened to be able to recreate similar or dissimilar results in the upcoming game rounds. The utterances belonging to the subcategories of interpretation and connection are placed in the final stage of the SOLO taxonomy, the extended abstract stage, as the participants making these utterances are able to make strong connections beyond the original task of the game.

Figure 6.1: *The three levels of strategically placing the dots in the game in relation to something.*

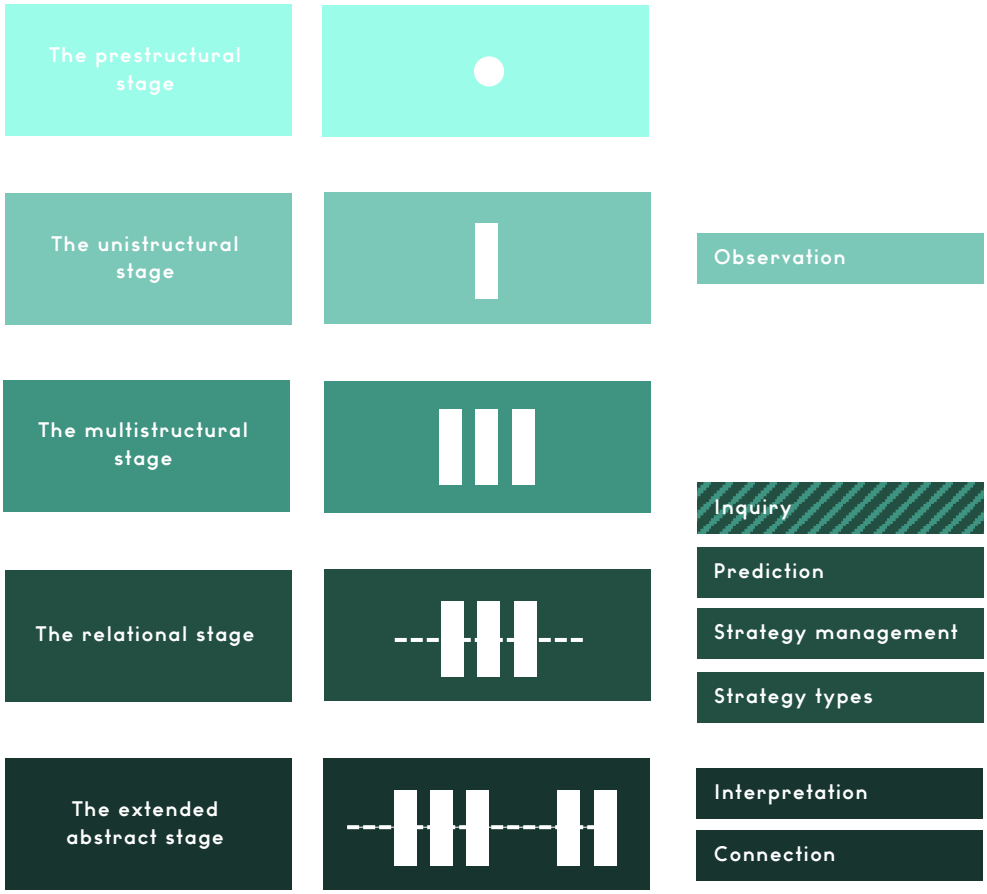

Figure 6.2: *The different subcategories of Learning talk (mathematical talk) mapped onto the five stages of the SOLO taxonomy. The second column shows a visual representation of the different stages.*

The remaining two main categories of System talk and Affective talk occupied 11,8% and 15,2% of the total amount of utterances. During the data collection, the impression of the researcher was that the utterances that belonged to the main category of System talk occupied a very large part of the conversations. Most of the participants had problems with the display and the physical buttons at some point which always became a discussion that involved all participants when they were trying to solve the problem. In hindsight, these utterances were not that many compared to those related to learning. Evidently, the game and the thought behind the game functioned better than the first impression that it gave. The fact that the main category of Affective talk occupied 15,2% of the utterances is not surprising nor unwanted. In a game where several participants compete against each other, emotional expression will naturally appear. The affect that emerges and is shared, also functions as a motor to the gameplay and the conversations. As Michael and Chen (2005) states, if possible the learners should have fun while learning and preferably that fun should be collaboratively shared, as it encourages the will to play and hence the opportunities to learn.

Another interesting observation is that in some of the sessions, all main categories are not represented. Session 9 did not include any utterances belonging to Learning talk (mathematical talk) or Affective talk, session 4 did not include any utterances belonging to System talk or Affective talk and session 11 did not include any utterances belonging to System talk. Worth noting though is that all of these three sessions consisted of 9 or less utterances, which is far below the mean of 45,1 utterances per session. When the participants make such few utterances, it is not surprising that they do not manage to cover all main categories.

To summarize, the types of conversations that emerge during a typical gameplay and exploration of the interactive exhibit, usually involves a lot of game mechanics but also a lot of mathematical talk. Usually some affective talk and system talk also occurs, but to a lesser degree. The talk related to game mechanics is present to a relatively large degree in most sessions, as all participants organize and comment the game at some point. The degree of mathematical talk is also relatively large in most sessions, but slightly less than the game mechanics. The most common mathematical talk includes observing and predicting the pattern, however, many of the participants discuss and use different game strategies and try to interpret and understand the outcomes of the different games. A usual gameplay also includes some individual or collective problems with the interface when trying to place the dots on the display and some reactions to the outcomes of the game. The expressions of displeasure are more common than those of pleasure, however, interestingly no participants express a displeasure of playing

the game. Displeasure is rather expressed in relation to the actions of the other players.

6.3 Learning opportunities

Session 2, 7, 15, 19 and 20 are chosen to be analyzed further based on the criteria presented in chapter 4.5.2. The analyses are done by using the method of interaction analysis. Two of the chosen sessions are the top ones in figure 5.6 on page 49. The remaining three are spread out in the figure, indicating that a large total percentage of Learning talk (mathematical talk) might not always corelate with a large percentage of more complex utterances. The most relevant extracts from the transcriptions have been chosen and they are presented in Appendix XIII, XIV, XV, XVI and XVII. Table 6.1 on page 61 summarizes the characteristics of each of the five participant groups and their respective sessions.

In chapter 6.3.6.1, common patterns of the learning trajectories of the five following sessions are identified, described and discussed, which results in four different stages of the learning trajectories. The names of these stages, defined as **Engaging**, **Observing**, **Interpreting** and **Testing**, are used in the analyses of the five chosen sessions.

6.3.1 Session 2

Session 2 is by far the longest of all sessions, in relation to session time, number of game rounds and the number of utterances. Also, it contained 51,8% Learning talk (mathematical talk), which makes it the session with the most mathematical talk of all.

The session begins with **P4** and **P5** being uncertain of how to play the game, where the first game rounds are characterized by quick and random placements of the dots on the screen and looking at what happens. Utterances like "*I don't understand anything*", "*it was just a test*" and "*let's see what happens*" are made. When observing the outcome of the first games, **P4** quickly notices that the colours of the other participants are '*stealing*' his space, while **P5** ponders if there is a '*smart way*' of playing the game. After another game round, the third and youngest participant **P6** who has been mainly silent so far, is also starting to interpret the game with the help of the comments from **P4** and **P5**. **P5** states that "*he ruined the game for me, I had taken several areas here and he went and stole them*" whereas **P6** answers and says that "*otherwise you would have gotten all of that area*".

During the following game rounds, the actions revolve around stealing space from each other by placing the dots close to one another and the

ID	Participants (gender)	Age interval	Game rounds	Utterances	Session duration	Learning talk (mathematical talk)
2	P4 (M)	<13	12	195	16:12	51,8%
	P5 (F)	>35				
	P6 (M)	<13				
7	P19 (F)	>35	4	85	07:37	20,8%
	P20 (M)	<13				
	P21 (F)	<13				
15	P38 (F)	<13	8	87	08:16	24,2%
	P39 (F)	>35				
	P40 (M)	<13				
	P41 (M)	<13				
19	P51 (M)	22-25	5	51	08:13	50,9%
	P52 (M)	22-25				
	P53 (M)	19-21				
20	P54 (F)	>35	6	59	09:22	35,6%
	P55 (M)	13-15				
	P56 (M)	<13				
	P57 (M)	>35				

Table 6.1: *A summary of the characteristics of each of the five participant groups and their respective sessions.*

conversations revolve around discussions of what parts of the display are offering less space to conquer (in particular placing the dots on the edges are considered to be less strategic). Many utterances also include **P5** asking **P4** what it is that makes him win almost every single game round. Apart from stating that he is always the last player, **P4** does not have any particular answer to why he wins as he answers *"I just think big"*. The main character of the conversations seems to be that **P5**, the older participant, initiates and fuels the conversation by asking **P4** and **P6** how they are thinking and why and by observing and interpreting what happens on the display in relation to the choices they have all made when placing the dots. **P4** and **P6**, with **P4** being the most active talker of the two, are intrigued both by the game and the scaffolding from **P5**, exploring and searching for the answers of her questions and the answers of why the pattern is created in the way that it is.

During the course of the gameplay, the participants repeatedly enter all the stages of **Engaging**, **Observing**, **Interpreting** and **Testing**. In general, it is after the formation of the different patterns, during the **Interpreting** stage, where the conversations blossom the most, as the participants interpret the outcomes together and have vivid discussions of who has stolen the space from who and what it means for each participant respectively. The fact that they successfully enter these stages so many times, together with the continual scaffolding from **P5**, are probably two of the reasons as to why the session and its conversations turn out so fruitful in terms of learning talk.

6.3.2 Session 7

Session 7 only involved 20,8% Learning talk (mathematical talk) but has been chosen based on the relatively high percentage of strategic talk and interpretation, as well as the large number of utterances.

The session begins with **P19** dominating the conversation by being pretty much the only participant talking. She instructs **P20** and **P21** in relation to how to use the buttons and the display and also observes and comments the formation of the pattern. Utterances like *"you won then... yes, you got the largest area"* and *"should we try again then, now that we know what it is about?"* are made. After the first game round, **P19** has explained to **P20** and **P21** what the game is about and that they should strategically consider where to place their dots. As soon as **P20** starts to understand the setup of the game, he jumps right in and states that *"I knew it, because you took that one and I was trying to place mine on top of that so that I would get... and here too"*. **P19** answers by saying that *"yes, that's what you thought, and you would get all of that, yes, but I stole some from you there I think"*. They quickly realize the fun in the game as they start to understand the consequences of their own choices.

As session 7 only contains 4 game rounds, the learning progression is very steep and the participants quickly start talking about placing their dots where no one else has placed their dots, and, similar to session 2, they talk about '*stealing*' space from one another. In the end, **P20**, similar to **P4**, quite trivially states that the reason to him winning is that "*I knew it, I just pressed*".

During the first game round, **P19** seems to be the only participant passing the stages of **Engaging** and **Observing**. She seems to be entering the stage of **Interpreting** and shares her insights so that all participants can reach the **Testing** stage right away. After this initial game round, the group quickly reaches more complex levels of understanding as they repeatedly enter the stages several times. In general, the conversations are mainly driven by the scaffolding from **P19**, and she makes the most statements about what happens and why. She interprets what the other participants say, and repackages their statements into more clearly expressed thoughts. This is probably one of the reasons why **P20** is not himself really able to identify the reasons behind his wins.

6.3.3 Session 15

Session 15 was chosen based on the same criteria as session 7. Only 24,2% of the conversations involved Learning talk (mathematical talk), but there was a relatively high percentage of strategic talk and interpretation, as well as a large number of utterances.

This session is slightly different from the other sessions being analyzed as the participants enter the game during different phases of the session. The session begins with **P38** being the only actively playing participant and **P39** simply watching. This game round is characterized by utterances of organizing the game and observing the outcome. During the second game round, **P39** enters the playing of the game and **P38** and **P39** continues with a similar game round of exploring the game and observing the pattern that is formed. Utterances such as "*and then it's my turn again*" and "*blue won*" are made. At the end of the following game round, **P39** expresses an understanding of the game, without really stating what she means, and as a result of this the remaining participants **P40** and **P41** are invited to the game. During the remaining game rounds, the conversation quickly starts to revolve around '*stealing*' each other's space and '*ruining*' each other's strategies. **P39** states that she is going to "*go and bother you because you are all being so crowded over there*" as she is placing her dot just next to the dots of the other participants. **P38** also states that **P41** ruins his strategy, as she says that **P41** is "*destroying where I had placed my dot*".

During the first game rounds, the participants seem to only enter the stages of **Engaging** and **Observing**. It is not until after a few game rounds have been played, that the older participant expresses an understanding of the game. This quickly results in the remaining game rounds changing focus, reaching the stage of **Interpreting** and staying there for a longer time. As the participants are collaboratively reaching a more complex understanding of the consequences of their actions, they are naturally entering the stage of **Testing** as they want to test their new insights. In this session, the older participant **P39** does not initiate any complex mathematical conversations but rather focus on the organizing of the gameplay and the turn taking of each participant. This is clearly reflected by the Learning talk of game mechanics occupying 62,1% of the total amount of utterances, being the second largest percentage of game mechanics of all sessions.

6.3.4 Session 19

Session 19 contained 50,9% Learning talk (mathematical talk), which makes it the session with the second largest percentage of mathematical talk of all.

The conversation of this session is already from the beginning focused on trying to create an understanding of the game and the most successful strategies, which is probably a result of the participants being slightly older. However, similar to the other sessions, the session begins with a game round where the participants work out the purpose of the game, making utterances such as *"just the goal in the end is to get as much space as possible with your dots"*, *"you want as much space as possible, okay"* and *"now I understand the game better"*. During the second game round **P53** states that *"I think we tried to go against each other too much here"*, indicating that the third level of relating one's owns dots to something might have already been reached.

The following game rounds are characterized by an exploration and a collaborative construction of knowledge where the participants work together to understand and verbalize what they believe are the best strategies of the game. **P53** states that he thinks that *"being in your spot is the best, I think the last dot is the best, you have... like in the last place you can just look at which dots give you the most area"*, when talking to **P51** which sits to the very right of the display meaning that he is the last one to place his dot each round. Further on during the discussions, they also bring up the subject of the appearance of the display, making utterances such as *"probably the edges, because the center is going to be divided by this [points at the middle of the display with numerous different colours]"*, *"you want to go as far as possible"* and *"you want to be as close to the center but no one goes further than you"*. Together they seem to make sense of their own

individual thoughts, ending up with more and more clear statements as they build on each other's thoughts. Towards the end of the session, **P51** states that visualizing the screen helps him think, and **P53** says that *"with the last turn you can really be a little vigilant, probably I put it here, I would have claimed more space than you"*. He continues with *"I guess you have to consider who has got the most space currently"* whereas **P51** answers with *"yeah, and then optimize"*.

The participants in this session clearly reaches all stages of **Engaging**, **Observing**, **Interpreting** and **Testing** right away. They immediately start interpreting the outcomes of the game, instead of simply observing the outcomes. They collaboratively reach a complex level of understanding, and stays there for a long period of time during each game round. In general, the conversation is dominated by the thoughts and utterances of **P51** and **P53**. **P52** only makes a few (9 out of 51) utterances mainly related to observing the formation of the pattern and affective utterances. There clearly is no participant being much older than the others, initiating the conversation and fueling the discussion. The conversation is equally driven by both **P51** and **P53**, and the session is characterized by a collaborative sense-making between the two. The participants also make several statements about the connection between the seating positions and the winner of the game, even to the point that they decide to change positions during the game to try and see if it affects the outcome of the game.

6.3.5 Session 20

Session 20 contained 35,6% Learning talk (mathematical talk), which makes it the session with the sixth most mathematical talk of all, however, session 20 has mainly been chosen based on the relatively high percentage of strategic talk and interpretation.

The session begins with two game rounds where the participants explore the purpose of the game and make comments about the playing of the game. **P54** quickly states that *"one probably has to play a few times to understand how it [works]..."*. She also comments on the fact that she is always the one to start and the placements of the dots in relation to the appearance of the display by saying *"I wonder if it's good, we always place the dots a little like this, in the corners"*. **P55** quickly answers that he does not place the dots on the edges, he focuses on the middle. During the next game round, **P55** starts to realize that the other participants are surrounding his dots, whereas **P54** answers that *"I have to know if it's a good strategy to surround someone"*. **P55** wins the game round which leads to **P54** questioning her own choices and still wondering what is the best strategy. The following game round is characterized by **P54** and **P55** being overly focused on repeatedly competing about *'stealing'* each

other's space, resulting in **P56** being the winner this time. As **P54** and **P55** was occupied trying to *'ruin'* the game for each other, **P56** was left with a lot of space to conquer. **P56** states that he was aiming to *"keep myself as far away as possible from the others"*. After this game round, all four participants have a common discussion making statements such as *"it's good to stay at the edges of the display"*, *"yeah, but now I tried it like this, and it's not smart to stay close to your own dots in the middle because then you are crowded by the others"*, *"I placed them where no one else was because then I get a bigger area"* and *"sometimes when you sabotage for others, you sabotage for yourself"*.

This session is similar to some of the others based on the fact that the first half of the session, where several game rounds are played, is very focused on the stages of **Engaging** and **Observing**. However, in the middle of the session, the participants stop at the **Interpreting** stage for a very long time, having a long discussion where they compare and interpret different game strategies. This never results in all participants entering the **Testing** stage, as only **P55** and **P56** plays the last few rounds. In general, the conversation is mainly driven by the utterances of **P54**, wondering about the best strategies. However, it is the youngest participants **P55** and **P56** that seem to reach the most complex levels of understanding. One explanation for this could be that **P54** deliberately takes a step back, to give space to the thoughts and reasonings of the younger participants.

6.3.6 General conclusions

In this chapter, the analyses of the five chosen sessions are generalized by identifying common patterns in the learning trajectories of the participants and by identifying possible success factors.

6.3.6.1 Learning trajectories

Looking at the five chosen sessions from a holistic view, four different stages of the learning trajectories seem to naturally appear in a repeated process. In the beginning phases of the analysis, these stages were defined as **Engaging**, **Observing**, **Interpreting** and **Testing**. Interestingly, these stages more or less coincide with the experiential learning process defined by Kolb (Kolb, Boyatzis & Mainemelis, 1999) and these connections will be explained further below. In the description of the four stages, the defined names are used. However, in figure 6.3 on page 68, the experiential learning process defined by Kolb is used as a foundation onto which the identified stages are mapped. The analyses of the five chosen sessions shows that all participants do not enter all stages during all game rounds, but typically, during a session as a whole all stages are reached by at least one of the

participants at some point. It seems that the most successful sessions are those that manage to reach all of these four stages at some point.

Stage one: Engaging

This stage is characterized by fascination and exploration but also by uncertainty. The participants are intrigued by the setup of the game and work together to figure out its purpose. Typically, they jump right in and start playing without fully understanding what they are doing and why. This step somewhat coincides with what Kolb defines as the first stage of the experiential learning process. He calls it the stage of concrete experience, where the learner uses its senses and perceptions to engage in the situation (Kolb, Boyatzis & Mainemelis, 1999). In the five analyzed sessions, and in all 20 sessions as a whole, the learning situations are characterized by the participants being put in a position they have no previous knowledge about as Voronoi diagrams seem to be unknown to all participants of the case study. This aspect is not clearly expressed in the theory of Kolb, which makes the first stage in the learning trajectories of this interactive exhibit slightly different from the first stage of the experiential learning process defined by Kolb.

Stage two: Observing

This stage is characterized by observation, prediction, inquiry and many times surprise. As the participants are watching the pattern as it takes form on the display, they try and predict what will happen and comment on the outcomes. They typically express wonders of who will win and are sometimes surprised by the result. This step coincides very well with what Kolb defines as the second stage of the experiential learning process. He calls it the stage of reflective observation, where the learner uses the experiences as the basis for observations and reflections (Kolb, Boyatzis & Mainemelis, 1999).

Stage three: Interpreting

This stage is what occupied most of the conversations of the five chosen sessions. This stage is characterized by interpretation, strategy management and connections. The participants do not only observe the pattern that has been formed on the display in front of them, they also collaboratively try to make sense of what just happened. They discuss different outcomes and transform their experiences into knowledge and hypotheses in the form of game theories and game strategies. This step coincides very well with what Kolb defines as the third stage of the experiential learning process. He calls it the stage of abstract conceptualization, where the observations and reflections from step two are assimilated and distilled into abstract concepts and theories that can be tested and acted on (Kolb, Boyatzis & Mainemelis, 1999).

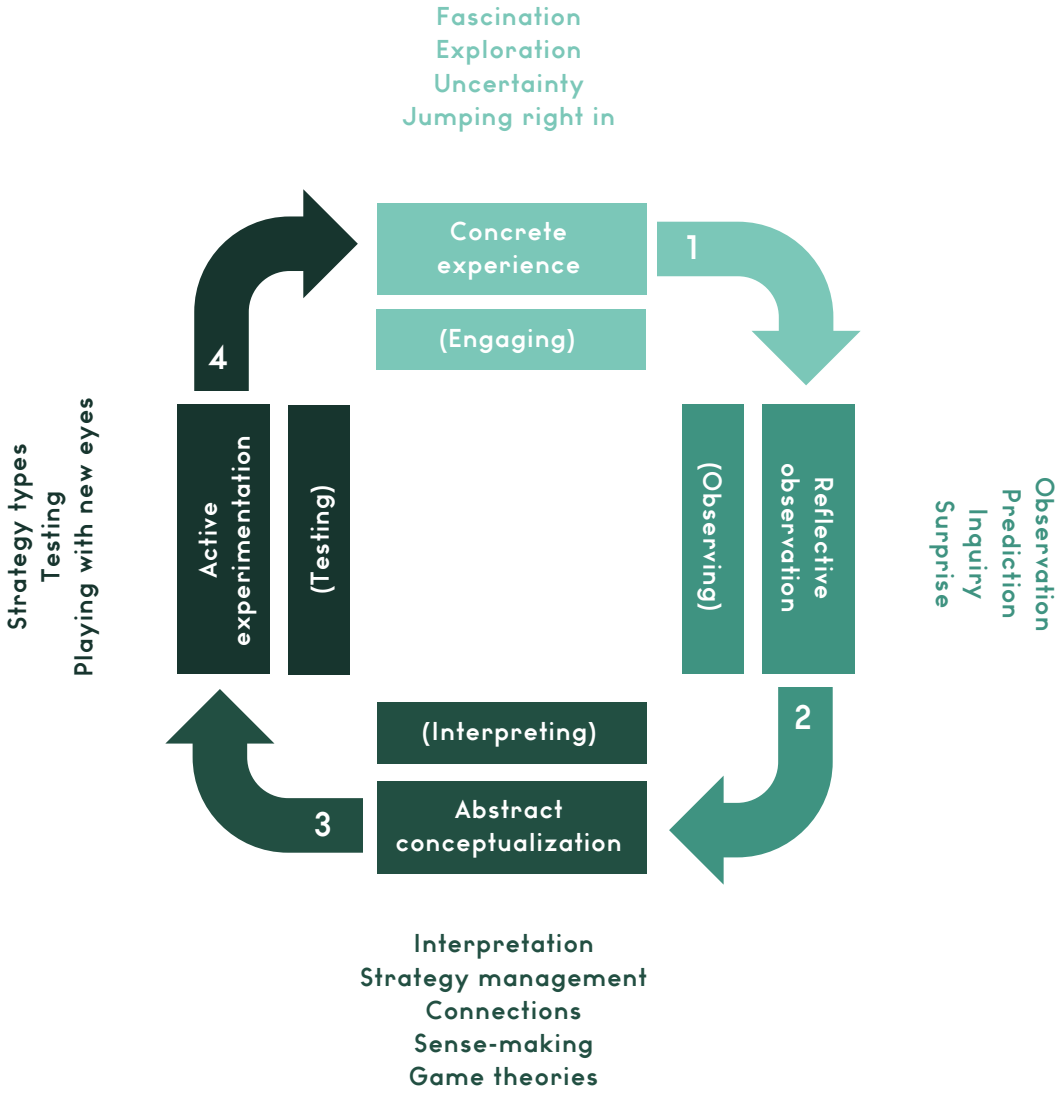

Figure 6.3: *The four identified stages of the learning trajectories mapped onto the experiential learning process defined by Kolb.*

Stage four: Testing

The last stage is characterized mainly by the participants testing different strategy types. They start playing the game once again but with slightly new eyes, trying out their newly created hypotheses. At this stage, they are more certain of what to do and can focus on the game strategies they want to test. Once again, the game offers feedback in terms of new patterns being formed, and new experiences are created. This step coincides very well with what Kolb defines as the fourth stage of the experiential learning process. He calls it the stage of active experimentation, where the concepts and theories from step three can be actively tested, the learners can get feedback and hence create new experiences (Kolb, Boyatzis & Mainemelis, 1999).

6.3.6.2 Success factors

In this chapter, five possible success factors are identified in relation to the exhibit as a whole. These factors all appeared to some extent when looking at the five chosen sessions. The first three factors are present to some extent in almost all of the sessions as they are appearing somewhat automatically based on the design and preconditions of the exhibit. The last two seem to be reached only in the most 'successful' sessions. The five success factors are illustrated in figure 6.4 on page 72, where the purpose of the stair is to demonstrate that the first step is the most frequently achieved and the fifth step is the least frequently achieved.

The power of the game element. This factor is the most apparent success factor identified during the analyses of the chosen sessions. Every single one of the actively playing participants in the chosen sessions were captivated by the game element of the exhibit. It generated interaction, both with the mathematical content of the game and with the other opponents, as the learners collaboratively tried to solve the two main 'problems' of the game. The first problem being the conquering of the largest area, and the second problem being figuring out *how* the conquering of the largest area actually took place. All these aspects, the captivation, the generated interaction and the collaborative problem-solving, go in line with what is stated as the main strengths of game-based learning (Corti, 2006; Felicia, 2014; McCall, 2009).

The learners as active explorers. What was first identified as a possible obstacle during the overall data collection, later turned out to be one of the most successful factors of the exhibit: the absence of clear instructions. As the digital interface of the game offered no guidelines or hints as to how the game actually functioned, and as none of the participants read the information board next to the exhibit, all

participants were put in a position where they were exploring something they knew nothing about. During the course of the game, no information, knowledge or answers were given from the exhibit except who was the winner of each round. This forced the participants to try and answer their own questions and to actively take part in their own learning processes. One can argue that this aspect of the game is where the learners actively construct their own knowledge, in accordance with the ideas of constructivism, as they are not passively receiving any information at all. Each new insight created throughout the gameplay is built upon old knowledge. Either upon the knowledge already existing in the mind of the learner, or upon the knowledge created in the previously played game rounds (Amineh & Asl, 2015; Olusegun, 2015). The importance of the learner being an active explorer is also emphasized in the theories of experiential learning by Kolb as well as in the theories of social constructivism developed by Dewey. Kolb stated that knowledge is created through the transformation of experience where the environment around the learner is crucial in the constructing of the new knowledge. The learner needs to be actively engaged in something that creates these experiences, experiences that the learner can relate his or her observations and theories to (Kolb, Boyatzis & Mainemelis, 1999). Dewey stated that learners need to be involved in activities where they explore common interests (Phillips & Soltis, 2020), which is exactly what happens when the participants interact with the game presented in the exhibit.

The collaborative learning. The game and the exhibit is not only designed to support active exploring, it is also designed to be used by several visitors at once. When several visitors engage in the same content, and when their respective choices affect each other, conversation between the visitors tend to appear. This goes in line with what Dewey stated is the best way of learning a new concept: through normal communication with others (Phillips & Soltis, 2020). By creating the apparent social context of the learning situation, as emphasized as one of the most important aspects of learning by psychologist and social constructivist Vygotskij, learners naturally learn from each other and help each other build their own knowledge (Phillips & Soltis, 2020). In many of the sessions, a type of collaborative sense-making process took place, where the learners reached conclusions and realizations with the help of each other. In other sessions, one participant acted as a so-called more knowledgeable other, typically one of the older participants, providing the younger learners with the suitable support to be able to reach the current conclusions and realizations. It seems that, as the learning situation is created so that the learning takes place in a collaborative manner, the learning in this context does not have to be related to the learners' stages of cognitive development as stated by Piaget (Phillips & Soltis, 2020). As long as the learners are within the

so-called zone of proximal development, defined by Vygotskij (Phillips & Soltis, 2020), and at least one more knowledgeable other is present, learning opportunities seem to arise. Even though not noted, the ages of the youngest participants were most likely from 8 years old and up (based on the memory of the researcher), meaning that according to the theories of Piaget, they were able to or almost able to understand and solve abstract problems and to think conceptually (Phillips & Soltis, 2020).

The fueling of the conversation. In the five chosen sessions, there was one common factor that was impossible to overlook. All conversations were clearly fueled by one or several participants, taking almost all initiatives and asking all questions. However, it seemed as this participant or these participants did not necessarily have to be the one or the ones reaching the conclusions and realizations, acting as a more knowledgeable other as defined by Vygotskij (Phillips & Soltis, 2020). As long as someone was taking the lead and triggering a conversation, the conversations seemed to have a higher chance of reaching complexity in terms of the learning talk. This seems to indicate that the sole presence of an active and collaborative learning situation is many times not enough to create learning opportunities.

The focus on reflective discussions. The fifth and final identified success factor deals with whether the main part of the conversations were focused on what happened before the creation of the Voronoi diagram or after. Many sessions tended to get stuck on the more simple utterances of observation, inquiry and prediction. In these sessions, the participants were so eager to start a new game round that they rarely discussed the outcomes for more than a short while, if they even did so. Only a few sessions tended to place their main focus on the reflections in the interpretations of the patterns during the presented outcome of each game round. In these sessions, where the five chosen sessions are included, it seemed as these reflective discussions gave the learning processes time to take form which fostered more complex understanding and learning opportunities.

Figure 6.4: *The five identified success factors, where the purpose of the stair is to demonstrate that the first step is the most frequently achieved and the fifth step is the least frequently achieved.*

6.4 Alignment with the intentions of the exhibit designers

In this chapter, the participant, session and utterance statistics together with the learning opportunities are analyzed in relation to the intentions of the designers to determine to what extent they align. Each learning and design objective is presented in table 6.2 and 6.3 on page 74.

An overall analysis of the compiled results of the case study shows that the majority of the conversations that emerged during the gameplays were focused on strategical and mathematical thinking. The game fostered learning trajectories where many of the participants were clearly and repeatedly entering the stages defined as engaging, observing, interpreting and testing. This lead to fruitful conversations and learning processes where the participants were given the opportunities to practice and assimilate the knowledge and the skills that the game was designed to foster. Regarding the first three learning objectives, the analyses that have been made have not been focused on investigating to what extent these objectives were fulfilled, which makes it hard to make any confident conclusions. Based on the overall findings, however, the Voronoi exhibit can be considered a successful educational game where the mathematical content is well integrated into the game without interrupting the fun and enjoyable aspects. This means that the first three learning objectives were also, most likely, fulfilled to some extent.

The majority of the participants were attracted by the exhibit and engaged in its content. As the game was designed to be completely exploratory, with no mathematical instructions or explanations added to the interface, one can argue that the informative aspect was slightly lower. However, with the help of each other, many of the participants managed to come to their own, mostly correct, conclusions. During the data collection, almost all groups of participants that approached the exhibit chose to stay, and most of them stayed for a longer while. This indicates that the exhibit caught the visitors' interest and managed to keep it. What could be concluded as the very main strength of the exhibit, however, is the fact that it attracted and stimulated participants of all ages and at all levels. All participants, from a few years old to over 70 years old, seemed to be intrigued by and interested in the mathematical content of the game. Participants of all ages entered the different stages of the learning trajectories, which indicates that any type of visitor is given the opportunity to learn.

Learning objectives
To provide the visitors with a different view of mathematics as a mind-set and an integrated part of humanity
To create awareness of patterns in nature in general and particularly Voronoi diagrams, including how the patterns can be explained with mathematical models and that they can be used in other contexts
To create awareness of how powerful mathematical models are in general
To foster strategical thinking, creating discussions on how to strategically conquer as much area as possible
To foster mathematical thinking, creating discussions on how the model works and the properties of a Voronoi cell

Table 6.2: *The five identified learning objectives.*

Design objectives
An appealing and attractive activity
An engaging and interactive activity
An exploratory activity
An activity that quickly catches the visitors' interest
A stimulating activity that retains the visitors' interest
A visually striking activity
An activity that is adequately fun and informative
An activity catching people at different knowledge levels, from the most simple to the most qualified level

Table 6.3: *The eight identified design objectives.*

7. Discussion

7.1 Validity of the study

To be able to draw any conclusions in relation to the validity of the study, the presented findings need to be related to other similar research studies. As all nine previous studies presented in chapter 3 have developed their own coding systems, in the same way that a specific coding system was developed in this study, a simple comparison cannot be done. The aim is therefore to try and find some previous studies where the main categories of Learning talk, System talk and Affective can be somewhat identified and compared to the results of this study. The findings that are mainly compared are the fact that, in this study, the Learning talk occupied 73,0% of the utterances, the System talk 11,8% of the utterances and the Affective talk 15,2%.

The first previous study that might give comparable insights is the study conducted by Allen (2003), looking for learning in visitor talk at an Exploratorium in San Francisco. The study of Allen (2003) is focusing on a frog exhibition including different elements where she has recorded the conversations of the visitors and coded the presence or absence of different types of talk for each of these elements. This means that the counting of the different types of talk is made in a completely different way than in this study. Allen (2003) concludes that at 83% of the elements visited, the visitors are engaging in some kind of Learning talk. Looking at the remaining 17% of the elements visited where no Learning talk was expressed, the visitors were either not talking at all (14%) or only talking about navigation and organization (3%). Her idea of Learning talk includes the main categories of Perceptual talk, Conceptual talk, Connecting talk, Strategic talk and Affective talk. Comparing this to the coding system of our study, all her main categories would be included in our main category of Learning talk except her main category of Affective talk. As the results of the study of Allen (2003) only presents presence or absence, where the results of all subcategories are viewed separately without connections to each other, there is no way of knowing how many percent of the elements visited included only Perceptual talk, Conceptual talk, Connecting talk and Strategic talk. The results of Allen (2003) only show that at 57% of the elements visited, the visitors engaged in Affective talk. Even though impossible to really compare, the findings from this study seem to corelate with the findings from the study of Allen, where both studies show a large degree of Learning talk present.

The second previous study having a similar focus is the study conducted by DeWitt and Hohenstein (2010) where they are investigating and comparing student discussions on different scientific topics being

presented to the students in museums and then in classrooms. The most relevant categorization that they make, is the differentiation between Content-related talk and Procedural talk. They also divide the Content-related talk into the categories of Explanation, Fit, Description, Read, Description (visual), Content-superficial, Affective, Attention and Other. In relation to this study, all types of Content-related talk except the Affective talk, can be considered similar to the main category of Learning talk. The Procedural talk is more closely related to the System talk, as it is defined as all types of talk not including any deeper engagement with the educational material (DeWitt & Hohenstein, 2010). Looking at the results from the study of DeWitt and Hohenstein (2010), the similarities are striking. The Content-related talk, where the category of Affective talk has been excluded, corresponded to 66,8% of the total student talk. The Affective talk alone corresponded to 6,0% of the total student talk, and the Procedural talk to 27,2%. One explanation as to why the types of talk interpreted as Learning talk are slightly lower and the types of talk interpreted as System talk are higher than this study could be explained by the fact that the design of the educational game of Voronoi managed to incorporate many of those procedural utterances into the Learning talk, as the procedures of the game were well integrated with the mathematical content of the game.

The third and final previous study that the results are compared to is the study also executed at the science center of Universeum but focusing on another exhibition called the OpenSpace exhibit were visitors can explore open research data from space that requires visualizations to be accessible (Pareto et al., 2023). Similar to this study, they categorize the visitor talk in terms of talk related to Learning, The visualization system and Experience. These main categories can be seen as another way of expressing the main categories of Learning talk, System talk and Affective talk. However, they only categorize utterances as Affective talk when the expressions of the visitors are affective and unrelated to the exhibit content. The results from their study show that 72,0% of the visitor talk is related to Learning, 23,0% is related to The visualization system and 5% is related to Experience (Pareto et al., 2023). The fact that the talk corresponding to System talk is more present is probably not surprising, as the interface of the OpenSpace exhibit seems to be much more elaborate and complicated. The percentage of Learning talk, however, is almost exactly the same.

It seems that, when comparing this study to three of the previous studies, the overall results are fairly similar. This strengthens the validity of the identified learning talk of this study. A more detailed study of the separate subcategories compared would have to be done to further make sure that the type of learning talk identified in this study is reflected in the studies mentioned above. However, as these previous

studies were used when developing the coding system of this study, this would indicate that they are similar enough to be able to make these comparisons.

Additionally, in relation to the validity of the study, the characteristics of the participants have to be considered. Looking at the age intervals of the 57 participants that were taking part in the case study, the results show that only 6 out of 57 participants (10,5%) belonged to the main target group of children between the ages of 13 and 18. This indicates that the case study did not succeed at investigating the embodied conversations and learning opportunities of the intended main target group. However, as mentioned in chapter 6.1, almost all visitors of the exhibition during the two busy days of the data collection were children younger than 13 years old or adults older than 35 years old, which means that the chosen participants clearly represents the typical visitor group. Also, as mentioned in chapter 1.1, families of various ages are identified as the secondary target group. Therefore, the participants of the case study are seen as relevant.

7.2 Limitations to the study

The first limitation to the study is related to the influence of the researcher. Quite early during the data collection of the audio-visual recordings, it felt evident that some instruction was needed in order for the visitors to start interacting with the exhibit. But as seen in the summary of the misconceptions of the gameplay in Appendix XII, these instructions tended to decrease during the course of the data collection. This seem to indicate that the amount of instructions given were not always needed. When the researcher became more confident of letting the participants act on their own, it turned out that instructions were not always needed to such a large degree. In retrospect, this is something that affected the completely authentic learning situation.

Another aspect that probably affected the data collection, and that is related to the mentioned influence of the researcher, is the fact that most people tend to make greater efforts when they know they are being watched and analyzed, also known as ‘the Hawthorne effect’ (Adair, 1984). This is an aspect that is difficult to completely eliminate as the participants need to be made aware of their participation in the case study. However, during the course of the data collection, the participants never gave the impression of being affected by the fact that they were being watched. No participants, except three girls belonging to the same session, mentioned or talked about the audio-visual recordings during the sessions. It seemed as if the game was so intriguing that they forgot about the case study.

The third aspect that can be seen as a limitation to the study is the fact that the interaction analyses were mainly focused on the verbal rather than the nonverbal activities of the participants. The visual components of the collected data was mainly used as a support to understand what the conversations were referring to. This means that this case study does not completely investigate what people do, but rather what people talk about doing. This does not include all actions and interactions. Also, the interaction analysis was only done by one person, the researcher, which lowers the validity of the results.

7.3 Future research

Because of the limitations to the execution of the interaction analyses, it would be desirable to further develop the analyses of the collected data. These analyses could be more focused on the nonverbal actions of the participants to further understand the learning opportunities arisen. Also, it would be desirable to collect additional data from new sessions, where the participants are given no instructions at all. This would lead to even more authentic learning situations where the methods and findings from this study could hopefully be useful and further validated.

Another interesting aspect in relation to future research is whether or not the developed coding system can be believed to be useful in other contexts. The 37 identified subordinate groups are most likely too detailed to be applicable if one were to study other exhibits or educational artefacts, but the 16 subcategories and the three main categories are most probably applicable. The subcategories connected to game mechanics is most likely not applicable in contexts where a game is not played, however, they can probably be altered to fit the context. Even if a game is not played, these types of utterances are probably still made. For example, commenting on the actions that are made or on the organization of the activity.

8. Conclusions

The fundamental idea behind this study was based on a will to investigate and try to understand what types of conversations emerged during a typical gameplay and exploration of the Voronoi exhibit by the general public. The aim of investigating these conversations was then to develop the analyses of the conversations into deeper analyses of the learning opportunities arisen. These findings were then compared to the intentions of the exhibit designers to be able to determine the so called 'success rate' of the exhibit.

It can be concluded that the majority of the conversations that emerged during the gameplays were focused on strategical and mathematical thinking. A total of 73,0% of the utterances from the participants were related to learning, either explicitly through mathematical talk or represented as talk of game mechanics. The most common subcategories were gameplay organization, gameplay comments, observation, prediction and strategy management. The game fostered learning trajectories where many of the participants were clearly and repeatedly entering the stages defined as engaging, observing, interpreting and testing. This lead to fruitful conversations and learning processes where the participants were given the opportunities to practice and assimilate the knowledge and the skills that the game was designed to foster. Also, five success factors were identified that seemed to be crucial in terms of the success of the exhibit. These were: the power of the game element, the learners acting as active explorers, the collaborative learning, the fueling of the conversations from one or several participants and the creation of reflective discussions.

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Appendix I: The full text of the information board

What do the wings of a dragonfly and the coat of a giraffe have in common? Both display something known as a Voronoi pattern. Such patterns can be seen in a wide variety of contexts, e. g. in the cloves of a garlic bulb, in growing colonies of cells, when forests spread, and when diseases spread.

Voronoi patterns occur naturally when something develops at the same rate of growth from separate points. Each small area (cell) in the pattern has a point from which it grows. The lines between the areas are always midway between adjacent points.

This gives us a mathematical model of a Voronoi pattern. By marking out a set of points and drawing the boundary lines, you can create your own Voronoi diagram.

A Voronoi diagram is an example of how we can describe natural phenomena using mathematics and thus better understand how they arise, and we can also use the resulting models in other contexts.

Voronoi diagram pinpointed source of infection

The Voronoi diagram is probably best known for having been used to pinpoint the water source that caused the major cholera outbreak in London in the 1850s. By comparing a map showing the distribution areas of water sources and the advance of the cholera epidemic, it became clear which source the infection came from.

Voronoi diagrams as an aid to urban planning

Voronoi diagrams can be used to plan key social infrastructure. The points in a diagram can be used, for example, to represent hospitals or schools and the associated Voronoi cells the catchment areas.

Other applications for Voronoi diagrams

Voronoi diagrams have many interesting application within a diverse number of fields, including biology, computer science, statistics, archeology – and not least art.

Figure A.1: *The full text of the information board.*

Appendix II: Participation Form

Study of Mathematics Talk at the Voronoi interactive exhibit

You are being invited to take part in a master's thesis research project from the department of Communication and Learning in Science at Chalmers University of Technology. Before you decide, it is important for you to understand why the research is being done and what it will involve. Please take time to read the following information carefully and discuss it with others if you wish. Ask me if there is anything that is not clear or if you would like more information. If you decide to take part, you will be given a copy of this participation form.

Thank you for considering participating in this study. Your contribution to our understanding of mathematical learning is greatly appreciated!

What is the study about?

The study aims at investigating what kinds of conversations are induced during a typical gameplay and exploration of the Voronoi interactive exhibit. The study thereby examines the exhibition station and not you. Nothing you do is either wrong or right so you are more than welcome to interact with the exhibition in whatever way you please.

Why study this?

The aim of the Voronoi interactive exhibit is to offer learning opportunities for visitors. I want to investigate what mathematical understanding, reasoning and problem-solving skills arise among visitors while interacting with the content of this exhibit and which learning opportunities can be associated to these.

How is the study conducted?

If you wish to participate, you will have to fill in a short questionnaire stating your age, gender and nationality. After answering these questions, you are free to explore the Voronoi interactive exhibit together with your party for as long as you like. Your interaction will be recorded in audio and video where only the display and your hands will be visible. These recordings will then be transcribed, analyzed and compared to other visitors' and will be shared only with my supervisor and examiner.

Data protection and confidentiality

Any personal identifying information will be removed from the data. The data production process will comply with the General Data Protection Regulation (GDPR). Once the data is gathered, it will be stored in a password-protected computer. The data will be kept until the thesis is submitted and presented. After that, it will be deleted.

Withdrawal of consent

You are free to withdraw your consent at any point by contacting me and stating your unique ID-number.

Your ID-number is: _____

Contact details:

Student: Teresia Thilén
teresia.thilen@gmail.com
Supervisor: Lena Pareto
lena.pareto@gu.se

Appendix III: Informationsformulär för deltagande

Studie av matematikprat vid den interaktiva utställningsstationen Voronoi

Du inbjuds att delta i en forskningsstudie som en del av ett masterexamensarbete vid institutionen för Vetenskapens kommunikation och lärande på Chalmers tekniska högskola. Innan du bestämmer dig är det viktigt att du förstår varför forskningsstudien görs och vad den kommer involvera. Ta dig tid att läsa följande information noggrant och diskutera med andra om du vill. Fråga mig om det är något som är oklart eller om du vill ha mer information. Om du väljer att delta, kommer du att få en kopia av detta informationsformulär.

Tack för att du överväger att delta i den här studien. Ditt bidrag till vår förståelse av matematiskt lärande uppskattas stort!

Vad handlar studien om?

Studiens mål är att undersöka vilka typer av konversationer som skapas vid interaktion och utforskande av utställningsstationen vid namn Voronoi. Studien utvärderar alltså stationen och inte dig. Ingenting du gör är antingen rätt eller fel och du är välkommen att interagera med stationen på vilket sätt du vill.

Varför studera detta?

Syftet med utställningsstationen Voronoi är att erbjuda besökare möjligheter för lärande. Jag vill undersöka vilka typer av matematisk förståelse, matematiska resonemang och problemlösningsförmågor som uppkommer bland och används av besökare medan de interagerar med stationen och vilka lärandemöjligheter som kan associeras med dessa.

Hur genomförs studien?

Om du vill delta behöver du fylla i ett frågeformulär som anger din ålder, ditt kön och din nationalitet. Efter dessa frågor är du fri att undersöka stationen så länge du vill tillsammans med ditt sällskap. Din interaktion kommer att spelas in med ljud och video från ovan och bara fånga dina händer och skärmen. Dessa inspelningar kommer sedan att transkriberas, analyseras och jämföras med andra besökares och kommer endast att delas med min handledare och min examinator.

Dataskydd och sekretess

All personlig information som kan identifiera dig kommer att tas bort från datan. All hantering av data kommer att följa dataskyddsförordningen (GDPR). När uppgifterna väl har samlats in kommer de att lagras i en lösenords-skyddad dator. Datan kommer att sparas tills dess att masterexamensarbetet är inlämnat och presenterat. Därefter kommer den att raderas.

Ånger av samtycke

Du kan ångra ditt samtycke när som helst genom att kontakta mig och ange ditt unika ID-nummer.

Ditt ID-nummer är: _____

Kontaktuppgifter:

Student: Teresia Thilén
teresia.thilen@gmail.com
Handledare: Lena Pareto
lena.pareto@gu.se

Appendix IV: Consent Form & Demographic Information

Consent Form

I, _____, agree to participate or agree the participation of my child, _____, in the research study titled "Study of Mathematics Talk at the Voronoi interactive exhibit", conducted by Teresia Thilén, who has discussed the research study with me.

I have received, read and kept a copy of the participation form. I have had the opportunity to ask questions about the study and I have received satisfactory answers. I understand the general purposes, risks, and methods of this study.

I consent to participate in the research study and the following has been explained to me:

- The research may not be of direct benefit to me
- My participation is completely voluntary
- My right to withdraw from the study at any time without any implications to me
- The risks including any possible inconvenience, discomfort or harm as a consequence of my participation in the research project
- The steps that have been taken to minimize any possible risks
- What I am expected and required to do
- Who I should contact for any complaints with the research or the conduct of the research
- I am able to request a copy of the research findings and the report
- Security and confidentiality of my personal information

In addition, I consent to:

- Audio-visual recordings of any part or of all research activities
- Publication of results from this study on the condition that my identity will not be revealed

Name: _____

Signature: _____

Date: _____

Demographic Information

How old are you?

- ☐ Younger than 13
- ☐ 13 to 15
- ☐ 16 to 18
- ☐ 19 to 21
- ☐ 22 to 25
- ☐ 26 to 35
- ☐ 36 or older

What gender do you identify as?

- ☐ Female
- ☐ Male
- ☐ Other (please specify): _____
- ☐ Wish not to answer

What is your nationality?

_____

Appendix V: Samtyckesformulär & demografisk information

Samtyckesformulär

Jag, _____, samtycker till att delta eller samtycker till att mitt barn, _____, deltar i forskningsprojektet med namnet "Studie av matematikprat vid den interaktiva utställningsstationen Voronoi", som genomförs av Teresia Thilén, som har diskuterat forskningsprojektet med mig.

Jag har mottagit, läst och behållit en kopia av informationsformuläret för deltagande. Jag har haft möjlighet att ställa frågor om forskningsprojektet och jag har fått tillfredsställande svar. Jag förstår de allmänna syftena, riskerna och metoderna för detta forskningsprojekt.

Jag samtycker till att delta i forskningsprojektet och följande har förklarats för mig:

- Forskningen kan inte vara av direkt nytta för mig
- Mitt deltagande är helt frivilligt
- Min rätt att dra mig ur studien när som helst utan några konsekvenser för mig
- Riskerna, inklusive eventuella besvär, obehag eller skada som en följd av mitt deltagande i forskningsprojektet
- Åtgärder som har vidtagits för att minimera eventuella risker
- Vad jag förväntas och krävs göra
- Vem jag ska kontakta vid klagomål om forskningen eller genomförandet av forskningen
- Att jag kan begära en kopia av forskningsresultaten och rapporten
- Information om säkerhet och sekretess för min personliga information

Dessutom samtycker jag till:

- Ljud- och bildinspelning av alla eller delar av ovan beskrivna forskningsaktiviteter
- Publicering av resultaten från denna studie under förutsättning att min identitet inte avslöjas

Namn: _____

Signatur: _____

Datum: _____

Demografisk information

Hur gammal är du?

- ☐ Yngre än 13
- ☐ 13-15
- ☐ 16-18
- ☐ 19-21
- ☐ 22-25
- ☐ 26-35
- ☐ 36 eller äldre

Vilket kön identifierar du dig som?

- ☐ Kvinna
- ☐ Man
- ☐ Annat (vänligen ange)
- ☐ Avböjer från att svara

Vilken är din nationalitet?

_____

Appendix VI: Interview guide

The interviews were based on three sections:

1. The role of the interviewee
2. The intended learning objectives associated with the exhibit
3. The intentions with the physical and digital design of the exhibit to support the intended learning objectives

1. The role of the interviewee

What is your line of work?
What was your specific role when it came to the creation of Mathrix?
Could you shortly give an account of the process of the design of the exhibit, and explain which parts you have been a part of?
Were you involved in the overall design of the whole exhibition?
Were you only involved in the Voronoi exhibit?

2. The intended learning objectives

Where did the idea for the theme of this exhibit come from?
Why did you create this specific exhibit?
Why did you choose this specific theme?
What would you say are the main learning objectives?
What are the visitors supposed to learn and explore?
What kind of mathematical talk did you want the visitors to use?

3. The intentions with the physical and digital design

Where did the idea for the physical and digital design of this exhibit come from?
What was the idea behind the setup of the gameplay?
What was the idea behind the text on the information board?
When was the overall physical and digital design created, before or after the completion of the learning objectives? Did that affect the final design?

Extra

Is there anything you would have wanted to do differently? Why?
Is there anything you were not satisfied with? Why?

Appendix VII: Interview transcripts

Table A.1 contains the collection of the most relevant quotes (in English and Swedish, where the original quotes were all in Swedish) from the separate interviews. The quotes have been categorized according to 'the intended learning objectives', 'the intentions with the physical and digital design' and 'the identified elements of improvement'. They are marked with the letters from A to D, representing the different interviewees according to:

A: Håkan Sigurdsson
B: Philip Gerlee
C: Mats Blysing
D: Lena Pareto

The intended learning objectives	
"Det var lite det som det hela gick ut på; att ge besökarna en helt annan bild av matematiken. Att matematik inte är ett skolämne utan ett tankesätt; att hela vår mänsklighet bygger på ett matematiskt tänk. Det är inte bara något vi gör i skolan och sedan tittar i facit om det är rätt svar". **(A)**	*"That was a part of the whole idea; to give the visitors a different view of mathematics. That mathematics isn't just a school subject but a mind-set; that our whole humanity is built upon a mind-set of mathematics. It's not just something we do in school and then check the solution for the correct answer".* **(A)**
"Det är det som är lite vitsen, just det här 'det här har jag inte tänkt på innan' [att mönstret finns och hur det skapas]. Man vill tända en lampa". **(A)**	*"That's kind of the idea, this 'I haven't thought of this before' [that the pattern exists and how it's formed]. You want to enlighten a spark".* **(A)**
"Syftet är att upplysa om begreppet Voronoi och om mönster i naturen. Att man kan lägga på en matematisk modell som också kan användas i andra sammanhang". **(A)**	*"The purpose is to make people aware of the notion of Voronoi and of patterns in nature. That it's possible to apply a mathematical model that can also be used in other contexts".* **(A)**
"Man behöver inte förstå hur Voronoi fungerar, men man ska veta att det finns sådana här matematiska mönster i naturen och att de återkommer på många ställen". **(C)**	*"You don't have understand how Voronoi works, but you should know that these mathematical patterns exist in nature and that they occur in many places".* **(C)**
"Jag tror att vi tänkte att det här blir ett bra sätt att illustrera vad Voronoidiagram är för någonting. Om man tänker sig att man vill lära ut det här, att då ha en definition som är väldigt teknisk, det är inget som man kan förvänta sig att besökarna tar till sig. Men att göra det till ett spel på det sättet, tanken	*"I think that our idea was that this is a good way of illustrating what a Voronoi diagram is. If you imagine that you want to teach this, using a definition that is very technical is not anything you can expect the visitors to understand. But to make it into a game like this, the idea is that it becomes much more*

är att det blir mer lätt tillgängligt. Tillsammans med utställningstexten blir det något slags lärande som sker. Att man förklarar var de här sakerna dyker upp i naturen och hur man kan använda dem. Då tänker jag att spelet blir ett sätt att illustrera vad det faktiskt är för någonting". **(B)**	*easily available. Together with the information text, some kind of learning takes place. You explain where these things appear in nature and how you can use them. Then I imagine that the game is a way of illustrating what it really is".* **(B)**
"Det finns ju inget svar för det beror på hur de andra spelar, det är det som är det intressanta. Det finns ingen bra strategi mer än att du måste förstå Voronoiprincipen. Ju mer du förstår hur den kommer att reagera på dina prickar och de andras prickar, desto mer yta får du". **(C)**	*"There is no clear answer because it depends on what the others do, that's what's so interesting. There is no successful strategi more than the fact that you have to understand the principle of Voronoi. The more you understand how the pattern will react on your dots and the others' dots, the more area you will conquer".* **(C)**
"Det är spännande att se hur mycket taktik det ändå går att lägga in i det. Det blir aldrig samma mönster". **(C)**	*"It's exciting to see how much strategy you can put into it. The pattern is never the same".* **(C)**
"Första gången man spelar kanske man inte tänker på något annat än att jag vann. 'Vi gör en gång till' säger den som förlorar och till slut så måste man börja tänka till. Då får man börja titta och analysera och se, hur kommer det sig att det blir så mycket yta här? Då måste man börja tänka Voronoi för att vinna helt enkelt. Så den som förstår Voronoi bäst vinner". **(C)**	*"The first time you play, you might not think of anything else than the fact that you won. 'Let's try another time' says the one that lost and eventually you have to start to think. Then you start to look and analyze, how come the area gets so big here? Then you simply have to start thinking Voronoi to win. The one who understands Voronoi the best, wins".* **(C)**
"Jag försökte tänka ut en bra strategi för hur man borde placera sina punkter men kunde inte komma på någon självklar så tyckte att det var en spännande utmaning utan självklar lösning. Något som förhoppningsvis ger upphov till diskussioner om hur modellen fungerar och vilka egenskaper en Voronoicell har, och dels hur man kan tänka strategiskt för att få till så stora ytor som möjligt. Båda bra aktiviteter som uppmuntrar och tränar ett matematiskt tänkande". **(D)**	*"I tried to think of a good strategy on how to place your dots but couldn't think of any obvious strategy so I thought it was an exciting challenge without a clear solution. Something that will hopefully create discussion on how the model works and the properties of a Voronoi cell, and also on how to think strategically to conquer as much area as possible. Both are great activities that foster and practice a mathematical way of thinking".* **(D)**

Table A.1: *The most relevant quotes from the separate semi-structured interviews in relation to the intended learning objectives.*

The intentions with the physical and digital design	
”Först skulle det bara vara ett fint bildspel som snurrade där inne där man såg olika naturfenomen där naturen jobbar efter Voronoiprincipen. Då var det svårt att få in matematiken i det på ett bra sätt och det var inte så engagerande. Man vill gärna att folk ska bli delaktiga i det och göra någonting. Man tänker mycket djupare om man måste agera på något sätt. Då tyckte vi det var roligt att använda Voronoi till att tävla lite. En tävling är något som drar igång folk och gör att man vill prova flera gånger”. **(C)**	*“At first it was just supposed to be a nice spinning slide show where you could see different phenomena where nature works based on the principle of Voronoi. At that point, it was difficult to include the mathematics in a good way and it was not that engaging. Preferably you want people to take part and to actively do something. You think much more deeply if you have to act in any way. We thought it was fun to use Voronoi to compete. A competition is something that triggers people and makes them want to try several times in a row”.* **(C)**
”När jag såg de animerade exemplen som man får om man googlar på modellen på hur ett antal punkter sprider sig likformigt ut från punkterna tills de stöter på något och bildar ett Voronoi diagram, fick jag genast idén om att göra ett spel av detta. Att försöka välja punkter så att dessa bildade så stor yta som möjligt. Det är ju visuellt effektfullt när de färgade cellerna breder ut sig så det var också en viktig komponent”. **(D)**	*”When I saw the animated examples you find if you Google the model, showing a number of points spreading from the dots until the hit something and form a Voronoi diagram, I immediately got the idea of making a game out of it. To try and place dots so that these became as large as possible. It’s visually striking when these coloured cells spread which was also an important component”.* **(D)**
”Tanken var bara ett roligt, tilltalande spel där man själv är delaktig i någonting. Där är ju spel oftast väldigt bra, de lockar folk till att göra saker”. **(A)**	*“The idea was just a fun, appealing game where you are taking part in something. Games are usually very successful at that, they attract people to start doing things”.* **(A)**
”De [Universeum] var rätt mycket ute efter interaktivitet i utställningen och det här var ett naturligt sätt att göra det på. Att göra ett spel av det är ett väldigt enkelt sätt att göra det interaktivt och medryckande”. **(B)**	*“They [Universeum] were looking for interactivity in the exhibition and this was a natural way of doing it. To make a game out of it is a very easy way of making it interactive and entrancing".* **(B)**
”När de [Universeum] berättar om sina pedagogiska mål så kan jag översätta hur det skulle vara roligt som upplevelser. Vad kan man göra för upplevelser för att beskriva det här? En av de stora utmaningarna i det är att det ska vara lagom skojigt och lagom lärorikt”. **(C)**	*”When they [Universeum] tell me about their pedagogic goals, I am able to translate that into a fun experience. What activities can be done to describe this? One of the big challenges is to make it adequately fun and informative”.* **(C)**

”Oavsett om de förstår det eller inte, är tanken att de ska se något skoj att göra och så börjar de jobba med det och så börjar de fundera ’vad är det här?’. Man kan gå lite olika vägar. En del i publiken, de går dit och så börjar de läsa informationen och så tänker de, ’jaha, det här var intressant, det här måste jag prova’. En del går direkt fram och i bästa fall så läser de informationen. En del av dem läser inte alls informationen och är där och trycker, men de får någon slags uppfattning av det för nästa gång de hör Voronoi eller ser de här mönstren så kan de relatera tillbaka till det här och se att det fanns en matematisk anknytning i alla fall. I det bästa fallet går de in och faktiskt lär sig någonting. Det är att försöka fånga folk på väldigt många olika stadier, från det allra enklaste till det lite mer kvalificerade stadiet”. **(A)**	*“Regardless if they understand or not, the thought is that they should see something fun to do and then start working with it and start thinking ‘what is this?’. There can be different routes. Some go there and start reading the information and think ‘oh, this was interesting, I have do try this’. Others approach the exhibit directly and, at best, they read the information. Some of them do not read the information at all and are there pressing the buttons, but they do get some idea of it because the next time they hear Voronoi or see these patterns they can relate back to this and see that at least there was a mathematical connection. At best, they actually learn something. It’s all about catching people at different stages, from the most simple to the most qualified stage”.* **(A)**
”Vi pratar mycket om upplevelsebaserat lärande, vi vill locka in folk att arbeta med montern”. **(A)**	*“We talk a lot about experiential learning, we want to attract people to work with the exhibit”.* **(A)**
”Det här mönstret kom upp och vi tyckte det var väldigt intressant eftersom vi har mycket natur på Universeum, då blev det som en naturlig koppling”. **(A)**	*“This pattern came up and we thought it was very interesting since we have a lot of nature at Universeum, it became a natural connection”.* **(A)**
”Att man placerar ut prickar och att man har olika färger, det var klart ganska tidigt. Det som vi diskuterade en del var i vilken ordning man skulle få göra sakerna. Man skulle kunna tänka sig att den ena personen placerar ut alla sina prickar och sedan så gör den andra personen det. Jag tror att vi till slut valde det som det blev eftersom det kändes mest rättvist”. **(B)**	*”The fact that you place dots and that you have different colours, that was done quite early. Then we discussed quite a lot in what order one should be allowed to do these things. One could imagine that one person places all their dots first and then the other person does it. I think we ended up with it being as it is since it felt the most fair”.* **(B)**
”Vi har haft mycket diskussioner kring hur mycket text man ska ha genom åren på olika sätt. Motargumentet mot mycket text har varit att folk läser i alla fall inte. Men även om det är en av tusen som läser så blir man väldigt irriterad om den inte skulle finnas där”. **(A)**	*“We’ve had many discussions over the years on how much text to include. The counterargument has been that people don’t read anyway. But even if it’s one in a thousand that reads, one would be very annoyed if it wasn’t there”.* **(A)**

”Vad gäller texten på väggen så var det några saker vi ville förmedla. Modellens användbarhet: att modellen finns i naturen men att den också används till vitt skilda användningsområden. Budskap: hur kraftfullt det är med modeller. Den matematiska principen för modellen som ju är ovanligt enkel för att vara en matematisk modell”. **(D)**	*”In terms of the information on the wall there were a few things that we wanted to convey. The usability of the model: that the model exists in nature but can also be used for a variety of applications. Message: how powerful models are. The mathematical principle of the model which is unusually simple as a mathematical model”.* **(D)**
”Vi pratar inte om matematiken i stationen. Hade vi gjort det hade risken varit att man fastnade i matematiken och inte det roliga. Vi ville att de skulle fortsätta söka sanningen på något sätt. Skulle vi förklara hur man räknar hade det blivit en ganska komplicerad station. Vi höll den på den absolut mest stimulerande nivån”. **(C)**	*“We don’t talk about the mathematics in the exhibit. If we had done that, we would have risked getting stuck in the mathematics and not the fun. We wanted them to continue searching for the truth in a way. If we would have explained how to calculate, it would have been a much more complicated exhibit. We kept it at the most stimulating level”.* **(C)**
”Det har ju egentligen inte med naturen att göra, det är ju inte så att Voronoimönsterna som skapas i naturen tävlar mot varandra, de bara uppstår så att säga. Men vi skapar ett tävlingsmoment i det som gör att det får en extra dimension och som gör att det kanske blir roligt att jobba med det” **(A)**	*“It reality it has nothing to do with nature, it’s not like the Voronoi patterns that are created in nature are competing against each other, they just appear so to speak. But we create an element of competition which gives it an extra dimension, that might make it fun to work with”.* **(A)**
”Vi har haft många brainstorms om vad man skulle kunna tänkas göra. Just det här att man ska ta yta genom att placera punkter gör att man får tänka till lite. Det blir ju strategiskt hur man ska placera sig och vinna så mycket yta som möjligt”. **(A)**	*“We’ve had many brainstorms on what to possibly do. This thing about conquering space by placing dots forces you to think a little extra. The placements of the dots becomes strategical so that you can conquer as much area as possible”.* **(A)**
”Man får inte känna sig dum. När man står där måste man kunna göra någonting, man måste förstå gaska snabbt vad det går ut på. Känner jag mig dum så vill jag inte fortsätta och det är ett misslyckande”. **(A)**	*“You are not supposed to feel dumb. When you stand there you have to be able to do something, you have to understand quite quickly what the purpose is. If you feel dumb you don’t want to continue and that is a failure”.* **(A)**
”Det handlar också om att saker rör sig på en gång. Man kan omedelbart få saker att hända”. **(A)**	*“It’s also about things moving right away. You can get things to happen immediately”.* **(A)**
”Tanken är att man börjar med att prova och ser vad som händer och sedan börjar man förstå vad det är. Istället för att skriva det på näsan på folk vad som händer. Det ligger lite i utforskandets natur, att du ser och testar	*“The idea is that you start by trying and looking what happens and then you start to understand what it is. Instead of writing what is happening on peoples’ faces. It’s in the nature of exploring, that you see and try*

och sedan så börjar du undra. Istället för att du får hela förklaringen och sedan gör du". **(A)**	*and then you start to wonder. Instead of getting the explanation and then start doing".* **(A)**
"Det är inte uppenbart vad som är den optimala strategin. Det kanske gör det mer intressant än vad jag hade tänkt. Det är någon slags added value". **(B)**	*"The optimal strategy is not obvious. Maybe that makes it more interesting than what I had thought. It's a kind of added value".* **(B)**

Table A.2: *The most relevant quotes from the separate semi-structured interviews in relation to the intentions with the physical and digital design.*

The identified elements of improvement	
"Tanken från början var väl lite mer att ha ett bord som man stod runtomkring, men det var av utrymmesskäl". **(A)**	*"The thought from the beginning was to have a table that you could gather around, but it was because there was not enough space".* **(A)**
"Vi är definitivt inte nöjda med att man inte bara kan vara två. Det fanns inte utrymme att ändra det. Vi tänker oss att vi ska göra en förbättring där". **(A)**	*"We are definitely not satisfied with the fact that you can't play just two. There was no space to change it. We are thinking that we will make an improvement there".* **(A)**
"Man behöver vara fyra för att det ska börja bli ett mönster. Med två blir det inte så många punkter och då blir det inte så många färger. Helt enkelt är det så att det var lägsta nivån för då blev det ett tillräckligt tydligt mönster". **(C)**	*"You need to be four in order for it to start to become a pattern. If you are two you don't get as many dots and not as many colours. It was simply the lowest level to form a pattern that was clear enough".* **(C)**
"Det blev ungefär så som jag hade tänkt mig. Det som stör mig lite grann är att om man inte spelar alla spelare, att de där punkterna hamnar slumpmässigt på skärmen. Man fattar inte riktigt vad som händer är känslan. Det hade varit bättre att man bara är så många som man är i spelet". **(B)**	*"It turned out just about what I imagined. What bothers me a bit is that if you don't play with all colours, those dots are placed on the display randomly. You get the feeling that you don't really know what is happening. It would have been better if you were only as many players as you were people".* **(B)**
"Det hade kunnat vara tydligare i interfacet att det är datorn som spelar. De [punkterna] kommer väldigt snabbt bara". **(B)**	*"The interface could have been more clear concerning when it is the computer playing. They [the dots] just appear very fast".* **(B)**
"Alla montrar är ju prototyper. Innan man lägger ner jättemycket krut [på montern], så måste man testa den". **(A)**	*"All exhibits are prototypes. Before you put a lot of effort [into it], you have to try it".* **(A)**
"Jag tror att Lena och jag kanske hade velat ha lite mer formler inblandade, men någonstans måste man sätta en gräns. Så	*"I think Lena and I might have wanted some more formulas involved, but you have to draw the limit somewhere. As soon as you*

fort du visar en formel så måste du förklara alla ingående termer. I allmänhet har vi inte den kapaciteten att gå så djupt in i saker och ting". **(A)**	*present a formula you have to explain all terms. Generally, we do not have the capacity to go that deep into things".* **(A)**
"De saker jag hade önskat ändra är, i rangordning: Möjligheten att se allas andel av ytan. Möjligheten att vara färre spelare med var sin färg. Möjligheten att ha matematiska beskrivningar direkt på skärmen som visade på modellens egenskaper, de som nu är i stillbilder på väggen, för att fler skulle få den matematiska biten och inte bara den spelstrategiska. Möjligheten att lägga till punkter under spelets gång [formationen av mönstret] vilket hade gjort det spelmässigt mer intressant. Möjligheten att ha turneringar, kanske att man hade spelat bäst av fem?". **(D)**	*"The things I would have wanted to change, presented in order, are: The ability to see everyone's share of the display. The ability to play with fewer players each having their own colour. The ability to have mathematical explanations directly on the screen showing the properties of the model, which is now presented in stills on the wall, so that more people would get the mathematical part and not just the strategical parts related to the game. The ability to add dots during the game [the formation of the pattern] which would have made the game more interesting. The ability to play tournaments, maybe that you could have played who is the best out of five rounds?".* **(D)**
"Jag hade nog kanske hellre sett lite mer information. Man hade kunnat haft någon sådan animering som visar hur sådana mönster växer. Jag hade kanske önskat att informationen lästes mer. Hade man kunnat få till det på något sätt så hade det klart varit bättre". **(B)**	*"I might have wanted some more information. There could have been an animation showing how these patterns grow. I would have wished that the information was more frequently read. If that could have been reached in any way it would have been clearly better".* **(B)**
"Det är klart att man alltid kommer undra om det hade varit bättre att lägga in en liten animation i slutet som beskriver matematiken, men man vill hålla uppe lekfullheten och tempot i att man ska prova igen. De som har svårt för matematik, de har lätt för att känna att 'oj, nu blir det massa matematik här', och då går de. Jag ville verkligen att de skulle spela flera gånger". **(C)**	*"Of course you will always wonder if it would have been better to attach a small animation at the end, describing the mathematics, but you rather want to preserve the playfulness and the pace so that they try again. Those who find mathematics difficult can easily feel that 'wow, now there is a lot of mathematics here', and then they leave. I really wanted them to play several times in a row".* **(C)**
"Jag ville att den skulle stå fritt så att man stod i var sitt hörn eller två på var sin sida. Vi flyttade runt montrarna för att utnyttja ytan vi hade och det var en jätteutmaning att få plats med allt. Sen vet jag inte om det är så fel att man står bredvid varandra. Det bildar ju någon slags grupptillhörighet, att det är vi mot skärmen. Men jag hade hellre	*"I wanted it to stand freely so that you would stand in your own corner or two people on each side. We moved around the exhibits to take advantage of the space that we had and it was a huge challenge to fit everything. However, I'm not so sure that it's so wrong that you stand next to each other. It creates a kind of group attribute, that it's us against*

velat att det var personligare, att man stod runt ett bord och tittade på varandra. Då hade det nog uppmuntrat mer att man försökte ta sin egna planhalva, man hade nog lagt prickarna runt sig själv mer. Då kanske man inte hade uppnått bästa resultatet till en början, innan man förstod att man kunde sätta prickar i de andras hörnor. Det tror jag hade varit en bra tröghet i inlärningen". (C)	*the display. But I would rather have wanted it to be more personal, that you were standing around a table looking at each other. Then it would have encouraged people to try and conquer their own corner first, placing your own dots around yourself. Maybe then you wouldn't have reached the best result in the beginning, before you understood that you can place your dots in the corner of the others. I think that would have allowed the learning process to successfully slow down".* (C)
"Jag hade inte velat ha de fysiska knapparna alls. 'Nu trycker jag på gul och så sätter jag min', det blev ett krångligt interface. Man hade bara velat gå runt bordet och så sätter du din färg när det är din tur. Vi kom inte på det förrän en bit in i utvecklingen att det faktiskt var ett problem. Då hade det varit bättre att vara runt bordet för då hade man inte kunnat trycka på varandras knappar". (C)	*"I wouldn't have wanted the physical buttons at all. 'Now I press yellow and then I place mine', it became a complicated interface. You would have just wanted to go around the table and then you place your colour when it's your turn. We didn't realize it until further on in the process, that it actually was a problem. then it would have been better to stand around the table because they you wouldn't have been able to press each other's buttons".* (C)
"Det gjordes ingen prototyp på Voronoi och jag kan konstatera att det hade varit bra om vi gjort det då det visade sig att önskemål i spelupplägg och interaktion inte kunde realiseras. Det var för svårt och dyrt enligt utvecklarna". (D)	*"There was never a prototype of the exhibit Voronoi and I can state that it would have been a good thing to do as it turned out that desires in relation to the setup of the game and the interaction couldn't be realized. It was too difficult and expensive according to the developers".* (D)

Table A.3: *The most relevant quotes from the separate semi-structured interviews in relation to the identified elements of improvement.*

Appendix VIII: Learning talk (mathematical talk), utterance statistics

	Frequency	%	%	%
Learning talk (mathematical talk)	**310**	**34,4**		
Observation	**105**		**11,7**	
Observing the formation of the pattern	100			11,1
Observing the placements of the dots	5			0,6
Inquiry	**29**		**3,2**	
Wondering how the pattern will be formed	26			2,9
Wondering why the winner wins	3			0,3
Prediction	**64**		**7,1**	
Predicting how the pattern will be formed	64			7,1
Interpretation	**20**		**2,2**	
Interpreting the formation of the pattern to understand the outcome of the game	20			2,2
Connection	**12**		**1,3**	
Making connections between the winner and the seating positions during the game	12			1,3
Strategy management	**46**		**5,1**	
Realizing the need for a strategy	6			0,7
Reflecting on what strategy to use	21			2,3
Realizing a strategy is successful	14			1,6
Realizing a strategy is not successful	5			0,6
Strategy types	**34**		**3,8**	
Using the strategy of placing the dots far from other participants' dots	5			0,6
Using the strategy of placing the dots close to other participants' dots	10			1,1
Using the strategy of placing the dots far from each other	6			0,7
Using the strategy of placing the dots close to each other	2			0,2
Using the strategy of placing the dots in the middle of the display	7			0,8
Using the strategy of placing the dots on the edges of the display	4			0,4

Table A.4: *Overview of the frequencies and percentages of the utterances belonging to the main category of Learning talk (mathematical talk). The frequencies and percentages are each presented per main category, per subcategory and per subordinate group.*

Appendix IX: Learning talk (game mechanics), utterance statistics

	Frequency	%	%	%
Learning talk (game mechanics)	**348**	**38,6**		
Gameplay comments	**124**		**13,8**	
Making comments about the rules of the game	41			4,6
Making comments about the playing of the game	69			7,7
Making comments about the outcomes of previous games	14			1,6
Gameplay organization	**224**		**24,9**	
Organizing the start and the end of the gameplay	60			6,7
Choosing and allocating colours	89			9,9
Organizing taking turns	75			8,3

Table A.5: *Overview of the frequencies and percentages of the utterances belonging to the main category of Learning talk (game mechanics). The frequencies and percentages are each presented per main category, per subcategory and per subordinate group.*

Appendix X: System talk, utterance statistics

	Frequency	%	%	%
System talk	**106**	**11,8**		
Game interface issues	**43**		**4,8**	
Talking about issues with the use of the physical buttons	11			1,2
Talking about issues with the use of the display	32			3,6
Game interface instructions	**63**		**7,0**	
Instructing how to use the game interface	63			7,0

Table A.6: *Overview of the frequencies and percentages of the utterances belonging to the main category of System talk. The frequencies and percentages are each presented per main category, per subcategory and per subordinate group.*

Appendix XI: Affective talk, utterance statistics

	Frequency	%	%	%
Affective talk	**137**	**15,2**		
Pleasure	**33**		**3,7**	
Expressing pleasure of the outcome of the game	6			0,7
Expressing pleasure of the actions of the computer or the other participants	4			0,4
Expressing a will to play the game	6			0,7
Expressing pleasure of playing the game	17			1,9
Displeasure	**38**		**4,2**	
Expressing displeasure of the outcome of the game	9			1,0
Expressing displeasure of the actions of the computer or the other participants	26			2,9
Expressing a reluctance to play the game	3			0,3
Surprise	**25**		**2,8**	
Expressing surprise of the actions of the computer or the other participants	11			1,2
Expressing surprise of how the pattern is formed	14			1,6
Uncertainty	**21**		**2,3**	
Expressing uncertainty of how to play the game	21			2,3
Praise	**20**		**2,2**	
Expressing praise to oneself or to another participant	20			2,2

Table A.7: *Overview of the frequencies and percentages of the utterances belonging to the main category of Affective talk. The frequencies and percentages are each presented per main category, per subcategory and per subordinate group.*

Appendix XII: Misconceptions of the gameplay

In the beginning of most of the sessions, the participants had to be instructed to be able to understand the interactive exhibit. The misconceptions of the gameplay can be divided into the four following themes:

1. **Understanding how to use the physical buttons and the display**. This misconception revolved around the fact that the coloured buttons had to be pressed before any dot could be placed on the display, the fact that no dot would be placed on the display unless there was only one finger touching the screen at once and the fact that the participants had to press not too hard but also not too loose on the display to be able to place a dot.

2. **The main purpose of the game and how it works**. This misconception mainly revolved around the formation of the pattern, that is, the details of how the pattern of different colours was developed. It also revolved around the goal of the game, to conquer as much display area as possible, and the fact that the game could only play with the same order of the colours (if the participants wanted to change order in the game they had to change seats).

3. **Understanding the fact that the game will always have four players, no matter how many colours are chosen by the participants of the game**. This misconception revolved around the fact that if there were less than four colours chosen by the participants of the game, the game itself would play the remaining colours. These dots would then be placed randomly on the display during the game.

4. **Understanding what it means to conquer the largest area of the display**. This misconception revolved around the fact that the conquered areas did not have to be connected as one unit. The sizes of each area was summarized, regardless of them being next to each other or not.

Table A.6 summarizes the number of sessions in which the different themes had to be brought up by the researcher. It also shows the total number of times that the different themes were brought up as many themes had to be brought up several times during each session. Theme 1 was the most common theme (15 sessions, 36 mentions) closely followed by both theme 2 (15 sessions, 23 mentions) and theme 3 (15 sessions, 24 mentions). Theme 4 was the least highlighted (2 sessions, 3 mentions).

ID	Theme 1	Theme 2	Theme 3	Theme 4
1	5	3	2	0
2	3	3	3	0
3	3	2	1	0
4	3	1	1	0
5	6	1	1	0
6	1	1	2	0
7	2	1	2	0
8	4	3	2	0
9	2	1	1	0
10	0	0	1	0
11	1	1	1	0
12	1	1	0	0
13	1	2	1	0
14	1	1	1	0
15	2	0	2	0
16	1	1	0	0
17	0	1	2	0
18	0	0	0	2
19	0	0	0	0
20	0	0	0	1
Number of sessions	15	15	15	2
Number of mentions	36	23	24	3

Table A.8: *Overview of the frequencies of the different themes that had to be brought up by the researcher, in relation to the different misconceptions of the gameplay.*

Appendix XIII: Extracts from session 2

Session 2 involved three participants, where **P4** and **P6** were male under the ages of 13 and **P5** was female over the age of 35. The participants were all Swedish and the utterances are presented in their original form.

P4	*Jag förstår ingenting*	Expressing uncertainty of how to play the game
	-	
P5	*Nu kommer nog mönstret, va?*	Making comments about the playing of the game
P5	*Nu ska vi se vad som händer*	Wondering how the pattern will be formed
P5	*Jag tog alla mina väldigt tätt*	Placing the dots close to each other
P5	*Det var ingen smart lösning*	Realizing a strategy is not successful
P5	*Jag tänkte jag skulle pröva göra det mycket tätt, det var bara ett test*	Placing the dots close to each other
	-	
P4	*Nej, du klämmer ut mig [P4 pekar på en av de andra färgerna som tar upp P4:s plats]*	Observing the formation of the pattern
	-	
P5	*Jag vet ju inte om det finns något smart sätt*	Realizing the need for a strategy
P5	*Nu ska vi se vad som händer med hans*	Wondering how the pattern will be formed
P5	*Nu ska jag också sätta uppe i hörnet*	Making comments about the playing of the game
P4	*Nej, jag måste förstöra*	Placing the dots close to other participants' dots
P5	*Nej, går du in och förstör för mig nu! [P4 placerar sin prick mellan P5:s utplacerade prickar]*	Expressing displeasure of the actions of the computer or the other participants
P4	*Men du tar ju allt [P4 kommenterar mönstret som börjar skapas]*	Predicting how the pattern will be formed
P5	*Det är säkert att du gjorde det*	Predicting how the pattern will be formed
P6	*Kolla, nu får jag mycket*	Observing the formation of the pattern
P4	*Hoppas jag*	Observing the formation of the pattern
P5	*Ja, du tar allt*	Observing the formation of the pattern
P5	*Det är så man kan göra också*	Realizing a strategy is successful
	-	

P5	*Såg du vad han gjorde nu? Han gick in och förstörde för mig för jag hade tagit flera områden här. Gick han in och förstörde för mig [P5 pekar på P4:s gröna område som ligger mellan P5:s gula områden]*	Interpreting the formation of the pattern to understand the outcome of the game
P6	*För annars skulle du fått hela den ytan*	Interpreting the formation of the pattern to understand the outcome of the game
P4	*Du skulle ha vunnit annars*	Interpreting the formation of the pattern to understand the outcome of the game
	-	
P5	*Nu ska jag in och förstöra för [P4:s namn]*	Placing the dots close to other participants' dots
P4	*Nej, det kan du inte, för du är före mig, jag är efter dig*	Making connections between the gameplay and the seating positions during the game
P5	*Ja, just det, det var ju dumt*	Making connections between the gameplay and the seating positions during the game
P5	*Det är inte värt att vara för lång ute på kanten då kanske*	Reflecting on what strategy to use
	-	
P5	*Men vad är det som gör att man vinner då, [P4:s namn]? Eftersom du vinner så mycket. Vad är det, hur tänker du?*	Wondering why the winner wins
P4	*Jag tänker bara stora*	Reflecting on what strategy to use
P5	*Hur tänker du? Du måste ju ha någon strategi tänker jag. Hur du sätter dem*	Wondering why the winner wins
P4	*Jag sätter dem, jag bara sätter dem*	Reflecting on what strategy to use
P5	*Du har ju fattat en grej, att du ska förstöra för andra*	Expressing praise to oneself or to another participant
P5	*Förstöra för andra är du väldigt bra på*	Expressing praise to oneself or to another participant
P5	*Jag undrar bara hur du får till det att du vinner*	Wondering why the winner wins
P5	*Det är bara genom att du förstör*	Realizing a strategy is successful
P4	*Mm*	Realizing a strategy is successful
	-	
P4	*Nu har jag en väldigt bra strategi*	Realizing a strategy is successful
P5	*Nu ska vi se vad han har för strategi, nu går han in och förstör*	Placing the dots close to other participants' dots
P5	*Oj!*	Expressing surprise of the actions of the computer or the other participants

P4	*Bra! In där! [P4 kommenterar att P6 placerar sin prick ganska nära P5:s prickar]*	Expressing pleasure of the actions of the computer or the other participants
P4	*Där sitter den bra*	Expressing pleasure of the actions of the computer or the other participants
P6	*Min, kolla nu, den växer [P4: namn] [P6 pekar på en stor blå yta i hörnet av displayen]*	Observing the formation of the pattern
P5	*Ja, det är ju en bra strategi*	Realizing a strategy is successful
P5	*Nu är det blå igen, ja [syftar på att blå vann]*	Observing the formation of the pattern
	-	
P4	*Det kan vara för att jag är sist, det kan vara för att jag är sist*	Making connections between the gameplay and the seating position during the game
P5	*Ja, Kanske, vi ska se*	Making connections between the gameplay and the seating positions during the game
P6	*Men jag var inte sist förut, jag var…*	Making connections between the gameplay and the seating positions during the game
P5	*Vinner du nu igen så är det att du är så himla smart, fattar spelet*	Expressing praise to oneself or to another participant
	-	
P4	*Hur vann jag! Va!*	Expressing surprise of how the pattern is formed
P5	*Du fick det området för dig själv, förstörde här borta*	Interpreting the formation of the pattern to understand the outcome of the game
	-	
P5	*Du har tagit ganska långt upp, kanske om du hade haft en prick där nere också*	Interpreting the formation of the pattern to understand the outcome of the game
P4	*Kolla här, om jag inte hade satt den där, då skulle du komma där och ta allt det här [P4 pekar på sina blåa ytor bredvid P5:s gula ytor]*	Interpreting the formation of the pattern to understand the outcome of the game
P5	*Kolla, jag förstörde ju där borta, jag tog ju de där två blåa prickarna där, så där satte jag min första*	Interpreting the formation of the pattern to understand the outcome of the game
P6	*Fast den är väl större? Röd är väl större? [P6 tycker att den röda färgen borde ha vunnit]*	Expressing surprise of how the pattern is formed
P5	*Den räknar ut här på procenten, så det är nog rätt*	Making comments about the rules of the game
P5	*Det kanske bara var en procents skillnad*	Making comments about the rules of the game
	-	
P5	*Nu vann jag, en vinst*	Observing the formation of the pattern

P4	*Jag vann mest*	Making comments about the outcomes of previous games
P4	*Du snodde min, jag skulle sätta min där för att blocka dig [P4 pekar på P5:s gula ytor]*	Interpreting the formation of the pattern to understand the outcome of the game
P6	*Du skulle ha tagit där*	Interpreting the formation of the pattern to understand the outcome of the game
P5	*Titta nu när du satte din lite längre ned här nu, nu fick du ganska mycket där på den [P5 pekar på P6:s röda yta]*	Interpreting the formation of the pattern to understand the outcome of the game
P6	*Ja, faktiskt*	Interpreting the formation of the pattern to understand the outcome of the game
P5	*Men så förstörde [P4:s namn] den*	Interpreting the formation of the pattern to understand the outcome of the game
P4	*Grön låg väldigt bra till, om inte jag skulle ha satt den*	Interpreting the formation of the pattern to understand the outcome of the game

Appendix XIV: Extracts from session 7

Session 7 involved three participants, where **P20** was male under the age of 13, **P21** was female under the age of 13 and **P19** was female over the age of 35. The participants were all Swedish and the utterances are presented in their original form.

P19	*Aha, just det, så då väljer man en färg då, till exempel att du har den gröna*	Instructing how to use the game interface
P19	*Då vann du kanske då, ja, då fick du mest yta*	Observing the formation of the pattern
P19	*Ska vi prova igen? Nu vet vi vad det går ut på*	Organizing the start and the end of the gameplay
	-	
P19	*Då ska du tänka då hur du placerar dina prickar*	Realizing the need for a strategy
	-	
P19	*Nu var det, nu ska vi se, vänta… det kanske är grön? Röd? Det är ganska jämnt*	Predicting how the pattern will be formed
P20	*Ja, röd!*	Observing the formation of the pattern
P19	*Ja, faktiskt*	Observing the formation of the pattern
P20	*Jag visste det, för att du tog den och jag skulle försöka att lägga över den så att jag skulle få, och här också [P20 pekar på P19:s röda yta och sin egen gröna yta precis bredvid varandra]*	Interpreting the formation of the pattern to understand the outcome of the game
P19	*Ja, du tänkte så ja, att du skulle få hela där, ja, men jag tjuvade lite av dig där tror jag*	Interpreting the formation of the pattern to understand the outcome of the game
P19	*Det är lite roligt när man förstår hur det fungerar*	Expressing pleasure of playing the game
P19	*Sen är det svårt att beräkna*	Expressing uncertainty of how to play the game
	-	
P19	*Ja, det där kan ha varit en bra idé [P20 placerar sin prick där ingen annan prick är]*	Realizing a strategy is successful
P19	*Blå, sista blåa [P21:s namn]*	Organizing taking turns
P19	*Nu, ganska bra för den gröna tror jag*	Predicting how the pattern will be formed
P19	*Ja!*	Expressing pleasure of the outcome of the game
P20	*Yes!*	Expressing pleasure of the outcome of the game
P20	*Jag kunde, jag bara tryckte jag*	Expressing praise to oneself or to another participant

Appendix XV: Extracts from session 15

Session 15 involved four participants, where **P38** was female under the age of 13, **P40** and **P41** were male under the age of 13 and **P39** was female over the age of 35. The participants were all Swedish and the utterances are presented in their original form.

P39	*Jaha, nu ska vi se om jag…*	Wondering how the pattern will be formed
P38	*Vi får se om jag vinner en gång till*	Wondering how the pattern will be formed
P39	*Ja, gult, då börjar jag, där*	Organizing taking turns
P39	*Så, du ska få se här*	Making comments about the playing of the game
P38	*Man trycker på någon av de här knapparna. Se nu kommer de, färgerna*	Instructing how to use the game interface
P39	*Nu tror jag att jag listade ut där*	Realizing a strategy is successful
P39	*Ja, så är det ser du*	Realizing a strategy is successful
	-	
P39	*Nu ska jag bort och störa där för ni knör ju så mycket [P39 placerar sin tredje prick nära de andras prickar]*	Placing the dots close to other participants' dots
P39	*Och så [P38:s namn]*	Organizing taking turns
P40	*Jag kommer ta bort mycket*	Predicting how the pattern will be formed
P38	*[P41:s namn], han förstör! Du förstör där jag hade min [P41 placerar sin prick mitt emellan P38:s prickar]*	Expressing displeasure of the actions of the computer or the other participants
P39	*Titta!*	Observing the formation of the pattern
P40	*Hur mycket tar gul ens!*	Observing the formation of the pattern
	-	
P38	*Jag vill vinna!*	Expressing a will to play the game
P39	*Det gäller att sätta prickarna rätt, nu så*	Realizing the need for a strategy

Appendix XVI: Extracts from session 19

Session 19 involved three participants, where **P53** was male between the ages of 19 and 21 and **P51** and **P52** were male between the ages of 22 and 25. The participants were all Dutch and the utterances are presented in their original form.

P51	*So now I am not supposed to do anything yet?*	Expressing uncertainty of how to play the game
P51	*Just the goal in the end is to get as much space as possible with your dots*	Making comments about the rules of the game
P51	*You want as much space as possible, okay*	Making comments about the rules of the game
P52	*Now I understand the game better*	Making comments about the playing of the game
	-	
P52	*Looks like I just won*	Observing the formation of the pattern
P53	*I think we tried to go against each other too much here*	Interpreting the formation of the pattern to understand the outcome of the game
	-	
P51	*Let's do one more at least*	Organizing the start and the end of the gameplay
P51	*Then we can adapt more*	Reflecting on what strategy to use
P52	*You want the computer to win?*	Expressing surprise of the actions of the computer or the other participants
P51	*To like if the good spots, that's a really good strategy, good spots go together*	Reflecting on what strategy to use
P51	*So what's the, I need to think about this a bit more*	Reflecting on what strategy to use
P53	*The red is in a bit weird place [the red dots placed by the computer are both far out on the edge of the display]*	Expressing surprise of the actions of the computer or the other participants
P51	*I question this computer*	Expressing surprise of the actions of the computer or the other participants
P53	*I would have gotten all of this [P53 points at a big green area with a narrow blue area in the middle]*	Interpreting the formation of the pattern to understand the outcome of the game
P51	*I think blue... yes*	Predicting how the pattern will be formed
P52	*Oh, what?*	Expressing surprise of how the pattern is formed
P51	*Okay, what is the best strategy? It's like the, putting dots together?*	Reflecting on what strategy to use

P53	*I think being in your spot is the best, I think the last dot is the best, you have... like in the last place you can just look at which dots give you the most area*	Making connections between the gameplay and the seating positions during the game
P51	*Like in general, when you start...*	Reflecting on what strategy to use
P53	*Like probably the edges, because the center is going to be divided by this [P53 points at the middle of the display with numerous different colours]*	Placing the dots on the edges of the display
P51	*You want to go as far as possible*	Placing the dots far from other participants' dots
P51	*If you just start in the center*	Placing the dots in the middle of the display
P53	*You want to be as close to the center but no one goes further than you*	Placing the dots in the middle of the display
P51	*Yeah, and it's, should we do it one more time?*	Organizing the start and the end of the gameplay
P53	*I want to be in your place*	Choosing and allocating colours
P51	*Okay [P53 and P51 changes seats]*	Choosing and allocating colours
P51	*It's kind of a nice game, it makes for some nice conversation*	Expressing pleasure of playing the game
P51	*If you think about, yeah, maybe if I put... so I have, yeah, green*	Reflecting on what strategy to use
P51	*I just have to visualize the screen*	Reflecting on what strategy to use
P51	*Damnit*	Expressing displeasure of the outcome of the game
P53	*Like with the last turn you can really be a little vigilant, probably I put it here, I would have claimed more space than you*	Making connections between the gameplay and the seating positions during the game
P51	*Just like put it here*	Interpreting the formation of the pattern to understand the outcome of the game
P53	*I guess you have to consider who has got the most space currently*	Reflecting on what strategy to use
P51	*Yeah yeah, and then optimize*	Reflecting on what strategy to use
P53	*Like I was trying to go in the middle to put off red again*	Placing the dots close to other participants' dots

Appendix XVII: Extracts from session 20

Session 20 involved four participants, where **P56** was male under the age of 13, **P55** was male between the ages of 13 and 15, **P54** was female over the age of 35 and **P57** was male over the age of 35. The participants were all Swedish and the utterances are presented in their original form.

P54	*Man behöver nog spela några gånger för att förstå hur den...*	Expressing uncertainty of how to play the game
	-	
P54	*Det är alltid jag som får börja*	Making comments about the playing of the game
P54	*Undrar om det är bra, vi sätter ju alltid lite så här i hörnet [prickarna är placerade i hörnet men inte jättenära kanten]*	Reflecting on what strategy to use
P55	*Inte jag! [P55 placerar sin prick mitt på displayen]*	Placing the dots in the middle of the display
P54	*Nej, du sätter dina lite på mitten, [P55:s namn]*	Placing the dots in the middle of the display
P55	*Nej!*	Expressing displeasure of the actions of the computer or the other participants
P55	*Ska ni omringa mig nu? [P54 placerar sin prick i närheten av P55:s prick]*	Expressing displeasure of the actions of the computer or the other participants
P54	*Jag måste veta om det ä ren bra strategi att omringa någon*	Reflecting on what strategy to use
P55	*[P54:s namn]! Ta där! [P55 pekar långt ut på kanten av displayen]*	Making comments about the playing of the game
P54	*Nej, jag får vara här borta [P54 placerar sin prick på andra sidan av displayen men fortfarande ganska nära kanten]*	Making comments about the playing of the game
P55	*Åh. Jag vann! [P55 pekar på en stor grön yta]*	Observing the formation of the pattern
P55	*Men en gång till*	Organizing the start and the end of the gameplay
P54	*Det måste ju finnas någon bra strategi*	Reflecting on what strategy to use
P56	*Jag har inte vunnit en enda gång*	Making comments about the outcomes of previous games
P56	*[P54:s namn], du måste trycka på den gula*	Instructing how to use the game interface
P55	*Men varför ska du ta i mitten?*	Expressing displeasure of the actions of the computer or the other participants
P54	*Nej! [P55:s namn]! [P55 placerar sin prick precis bredvid P54:s prick]*	Expressing displeasure of the actions of the computer or the other participants

P55	*Nej! [P54 placerar sin prick precis bredvid P55:s prick som är precis bredvid P54:s förra prick]*	Expressing displeasure of the actions of the computer or the other participants
P54	*Nej, nu måste vi ju hjälpa, nu måste vi ju röra om i grytan lite för de här borta med*	Making comments about the playing of the game
P54	*Nej, [P55:s namn]! [P55 placerar sin prick in närheten av P54:s tredje prick]*	Expressing displeasure of the actions of the computer or the other participants
P57	*Så grön den här gången med*	Observing the formation of the pattern
P57	*Du har en bra strategi [P56:s namn]*	Expressing praise to oneself or to another participant
P57	*Tre av fyra*	Making comments about the outcomes of previous games
P56	*Hålla sig så långt ifrån någon annan*	Placing the dots far from other participants' dots
P54	*Men det är ju bra att hålla sig ute i kanterna*	Placing the dots on the edges of the display
P54	*Man ser ju att det är smart att hålla sig...*	Placing the dots on the edges of the display
P57	*...i ytterkant*	Placing the dots on the edges of the display
P54	*Ja, men nu testade jag ju så här, det är ju inte smart att hålla sig jämte sina prickar i mitten för då blir man ju intryckt*	Realizing a strategy is not successful
P56	*Jag satte dem där ingen annan var för då får jag en större yta*	Placing the dots far from other participants' dots
P56	*[P55:s namn] hade ju lättast för han var ju sist, du hade ju svårast [syftar på P54 som var först]*	Making connections between the gameplay and the seating positions during the game
P54	*Jag tänkte att det var konstigt att jag alltid fick börja*	Making comments about the playing of the game
P55	*Ibland när man saboterar för andra så saboterar man för sig själv*	Realizing a strategy is not successful
P55	*Jag försökte vara så långt ifrån alla andra som möjligt för då får jag ju en större yta*	Placing the dots far from other participants' dots

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