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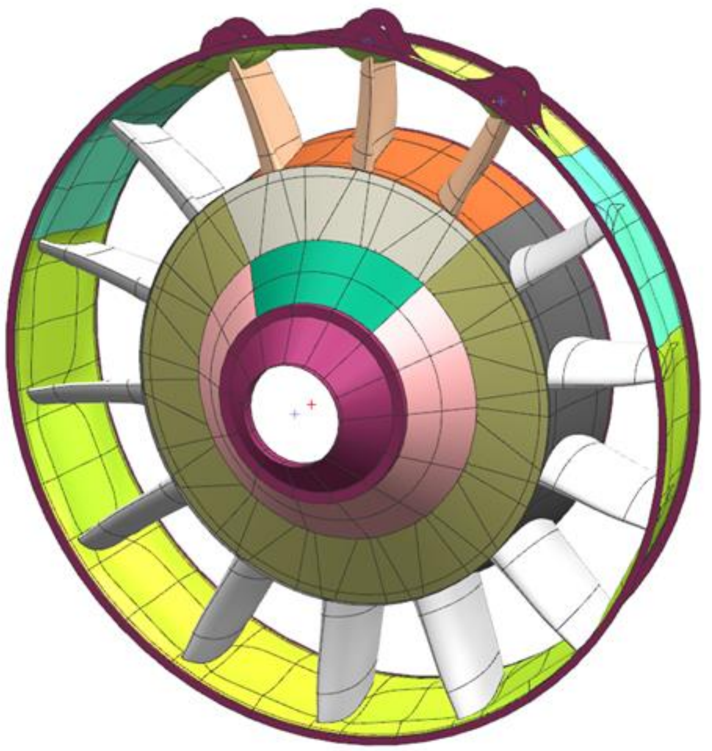

# Increased Thermal Robustness of Turbine Structure

*Master´s thesis in Product Development*

MATTIAS LARSSON
SEBASTIAN MARKLUND

Department of Product and Production Development
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2014

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Cover:
The Turbine Structure (Aerospace, 2014)

Chalmers Reproservice
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## Abstract

This report is a result of the master´s thesis work carried out in corporation with GKN Aerospace Engine Systems in Trollhättan, Sweden and the department program at Chalmers University of Technology in Gothenburg, Sweden, during the spring of 2014.

During the development of a jet engine, changes on the system level can have a great impact on components manufactured by GKN. In previously developed jet engines it has been found that increased temperatures have shortened the life span of the Turbine Structure, this due to a design that is sensitive to thermal variation.

In this thesis a method for identification and reduction of sensitivity to thermal variation is developed. The methods investigated are based on Robust Design theory and statistical models. Design of Experiments is used for investigating how variance in the thermal zones is affecting the stress levels in the TS.

To be able to withstand changes in thermal loads, statistical methods have been investigated and implemented in order to design aerospace engine components towards an increased thermal robustness.

The TS was divided into four thermal zones that were varied according to different experimental design plans. A Taguchi L9 experimental plan was carried out and all combinations of the varied temperature inputs were simulated in ANSYS to achieve the corresponding temperatures in the thermal zones. These temperatures were then used as input to the structural simulation that shows corresponding stresses in the TS.

The Design of Experiment results showed that main effects derive from the thermal zones 2 and 3. Geometrical variations together with variation in thermal boundary conditions were also studied to determine what the most sensitive geometrical parameters were. With the correct set up of experimental design and statistical methods, further investigation on the geometry impact of thermal variation can be studied.

A methodology regarding analysis of Design of Experiment data was established throughout this thesis work. Procedures for automated investigation of areas of interest were evaluated as support for more efficient analysis of thermal robustness.

**Acknowledgements**

This report is a result of the master thesis work made by Mattias Larsson and Sebastian Marklund at the Department of Product and Production Development at Chalmers University of Technology. We wish to thank our supervisors, Dr. Johan Lööf at GKN Aerospace and Associate Professor Lars Lindkvist at Chalmers University of Technology. Their assistance in form of planning and revising supported this project to stay on track to reach the set up goals.

Without the expertise and dedication from GKN employees this thesis had not been able to reach the highly valued results. We especially want to thank Carlos Arroyo for his excellence in aerothermodynamics engineering that helped and supported the work throughout the whole project. Carlos constructed the foundation that made this work possible.

We also want to thank Richard Sävenå and Visakha Raja for their brilliance in solid mechanics engineering. This thesis work was made possible only with Richard's and Visakha's knowledge, help in programming and help with simulation setup.

Finally we want to thank Petter Andersson for his guidance throughout this work, with the support in interpreting the data and set up of statistical aids.

Gothenburg, 2014

Mattias Larsson & Sebastian Marklund

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## Explanation to abbreviations

EWB – Engineering Workbench

NTL – Newton’s Third Law of motion

LPC – Low Pressure Compressor

HPC – High Pressure Compressor

LPT – Low Pressure Turbine

HPT – High Pressure Turbine

TS – Turbine Structure

TBH – Tail Bearing Housing

TRF – Turbine Rear Frame

BC – Boundary Condition

DoE – Design of Experiments

DoF – Degrees of Freedom

GKN – GKN Aerospace Engine Systems

ANSYS – Fluid dynamics and solid mechanics simulation software

MATLAB – Mathematical and statistical software

MINITAB – Statistical software

LE – Leading Edge

TE – Trailing Edge

ModeFrontier – Multi-objective optimization software

CUMFAT – Analysis for determination of component life and fatigue

# 1. Introduction

This introductory chapter presents GKN together with the description of the background and current situation to the problem area. The introduction also explains the purpose of this thesis and its goal definitions. A description of the limitations and structure of the thesis report are also presented.

## 1.2 Company Description

GKN Aerospace Sweden AB is the parent company for the division GKN Aerospace Engine Systems within the GKN Group. GKN Aerospace serves a global customer base and operates in Europe and North America. The company is one of the world´s independent tier supplier to the global aviation industry, with GKN as a global leader in the aero structures and engine components manufacturing for both the military and civil market. With over 100 years of aerospace experience, extensive knowledge utilization and advanced manufacturing technologies, GKN delivers high-valuable integrated assemblies in both metallic and composite materials. (Aerospace, 2014)

The company has approximately 12 000 employees distributed in more than 35 facilities across four continents. The GKN Aerospace AB headquarters is located in Trollhättan, where manufacturing of engine components and the development of the Turbine Structure is taking place.

GKNs vision and goals are to get an even broader range of product families within the core structures, engine assemblies, transparencies and niche technology markets. The design and manufacture of high level integrated aircraft assemblies and sub-assemblies for OEM´s and Tier One customers. Finally GKN strives for an expansion into adjacent markets with similar product technologies and manufacturing capabilities. (Aerospace, 2014)

"*GKN Aerospace is committed to being the best value solution for our customers worldwide to meet aerospace and defense needs*" (Aerospace, 2014)

## 1.3 Background

In the modern world with an increasingly extensive global competitive market, shortened lead times in product developments is not only a recommendation, it is a vital key for success. The development time reflects how responsive the company can be to competitive forces and technological developments. It also tells how quickly the company receives the economic returns from the project´s resource and time efforts (Ulrich, 2011).

During the development of a jet engine, changes on the system level can have a great impact on components manufactured by GKN. Due to aim of shortened lead times the need of early decisions in the development phase are of great importance. The resulting costs of changes in the development phase are heavily dependent on in which part of the progress stage the changes are made. Changes made at an early stage in the development phase are substantially more affordable than major changes made late in the process, which often result in an exponentially increased total cost for implementation of the new procedures.

In previously developed jet engines it has been found that increased temperatures have shortened the life span of the TS and this due to a design that is sensitive to thermal variation. To be able to withstand changes in thermal loads, statistical methods need to be incorporated in order to design aerospace engine components towards an increased thermal robustness. To gain thermal robustness it is important to explore alternative design configurations with respect to thermal uncertainties. In negotiation with GKN customers there is a need for engineering know how, trade off curves etc. as support, when providing powerful arguments in new engine programs. To account for these challenges there is a demand to develop robust methods and simulation support in early phases of the product development cycle.

As a part of decreasing lead times in product development at GKN and to make the product development processes more effective the company has started a work team called Engineering Workbench, hereafter called (EWB). EWBs´ main function is to create a cross functional team from different engineering disciplines. EWD also supports a more effective exchange of important information across the engineering disciplines at GKN.

## 1.4 Purpose and Aim

As described in the previous chapter the urge for early decision making in the development process is a key factor for success in the modern global competitive market. This quest for early decision making requires that the decisions are based on facts, which in turn has to be based on qualitative available information. To gain this important information at an early stage in the innovation process reliable and effective methods are invaluable. One important part of this thesis is to explore the possibilities for implementation of these methodologies to gain valuable knowledge at an early stage in the development process. This research is closely connected to the improvement capabilities of the EWB group at GKN.

The purpose of this thesis is to explore and understand what impact a variation in boundary conditions has on important design properties in the development phase. A methodology is needed for identifying sensitive parameters and how variation in boundary conditions is treated. The task is to define possible methods and tools that should be used to improve engineering efficiency and utilization of statistical tools to decrease sensitivity on GKN developed components when changes are made on the system level on the TS.

Definition of a set-based approach is needed in order to get knowledge about how robust the TS are to changes made on the system level. The thesis should result in a more engineering efficient way to handle variations in boundary conditions and parameters in early development phases.

## 1.5 Research questions

This thesis will regard the following three research questions. The outline of the thesis is based on these questions.

1. **How to identify key design parameters that are coupled to thermal variation?**
   This research question will address how key design parameters can be identified and how these are treated during the ongoing development project.
2. **How is it possible to in an efficient way handle variations in key design parameters during ongoing development projects?**
   This question will address how a methodological approach can be implemented to obtain a robust design with respect to thermal and geometric attributes.
3. **How can Design of Experiments (DoE) be implemented to improve thermal robustness in an early phase of the development project?**
   This question will address how design of experiment can be implemented in multidisciplinary simulations in order to improve thermal robustness in early stages of product development.

## 1.6 Delimitations

Limited resources in terms of time, budget and knowledge makes it necessary to establish boundaries of the thesis project performed at GKN. Research will be done on one single component in the engine, the TS, and not additional parts of the engine. Examination and analysis will be made on already existing models at GKN. Tests and implementations will not be evaluated and examined through an economical point of view. Delimitations will also be done to only include those parameters that affect the investigated component. Also the investigation should be on a low level of detail in the component, with only the most important parameters included for observation. Some geometric features are also fixed and should not be taken into consideration when investigating variation of geometric parameters. The mentioned features will be described at a later stage. The tools and methods used are limited to the software and tools available at GKN.

## 1.7 Secrecy

Due to secrecy reasons the authors of this report are bound to follow laws and regulations under confidentiality agreement with GKN. For this reason, results and conclusions have been removed in the public version of this thesis. Data has been normalized, figures are modified, reduced in quantity and names of variables have been changed. Valid results are given in the internal report available at GKN.

## 1.8 Structure of the report

The structure of this thesis report is outlined in a scientific format referred to Chalmers standard form for master thesis reports. The report will be subdivided into the following sections.

**Chapter 1.** Provides a presentation of GKN Aerospace Engine systems and the description of the relevant problem which is the basis for this master thesis. The goal and scopes for this project are presented in terms of research questions, together with delimitations and assumptions.

**Chapter 2.** Contains the introduction of jet engine theory, a description of the examined component and the related thermal issues. This chapter also describes the tools and methods used to succeed with the ongoing work.

**Chapter 3.** Explains the method of how this thesis work was carried out. The method describes how the theory and methodologies earlier presented are meant to be implemented in the ongoing thesis work.

**Chapter 4.** Presents what methodologies that were used in this master thesis and what tools that were finally implemented. The chapter also presents the results derived from the thesis work.

**Chapter 5.** Reflects on how well the results derived in this work answer the research questions. The chapter deals with the discussion about the generated results, and debates about how relevant the findings are in an engineering manner.

**Chapter 6.** Presents the sources of error that was found throughout the thesis work.

**Chapter 7.** Presents the conclusions drawn from the derived results of this thesis work.

**Chapter 8.** Presents the developed methodology that is to be used when investigating thermal robustness.

**Chapter 9.** Presents future work that can be followed by this thesis work.

**Chapter 10.** Presents the list of references

**Chapter 11.** List of all appendices.

## 2. Theory

This chapter contains the introduction of jet engine theory, a description of the examined component and the related thermal issues. It also describes the tools and methods used to succeed with the ongoing work.

### 2.1 Jet engine functionality

The imminent majority of today's commercial airplanes use jet engine propulsion. "Jet engine" is a broad definition of a variety of engines using Newton's laws of motion saying that for every action of force there is an equal and opposite reaction force. Jet engine, in common parlance, may also be referring to as "internal combustion air breathing jet engine".
In a jet engine air is accelerated through the engine and gives the air a change in momentum. From NTL this translates to the thrust equation

$$F = \dot{m}(V_0 - V_1) \tag{2.1}$$

Where $\dot{m}$ is the mass flow rate of air and $V_{0,1}$ is the relative speed before and after the jet engine. From this it is also clear that the purpose of the engine is to produce large volumes of exhaust gasses that move at high velocity. Figure 2.1 below shows a basic view over a civil bypass jet engine. Air first enters the front facing fan that sucks in air to feed the compressors. The airstream enters the low pressure compressor (LPC). In this stage the larger quantity of air enters the bypass canal for direct thrust. Small volumes of air, typically around 10% depending on engine set-up, enter the jet engine core for later ignition. After the LPC the air is further compressed in the high pressure compressor (HPC) to a stage where the pressure ratio is between 20:1 and 80:1, much depending on how many stages of blades that is present in the first two compressor stages.

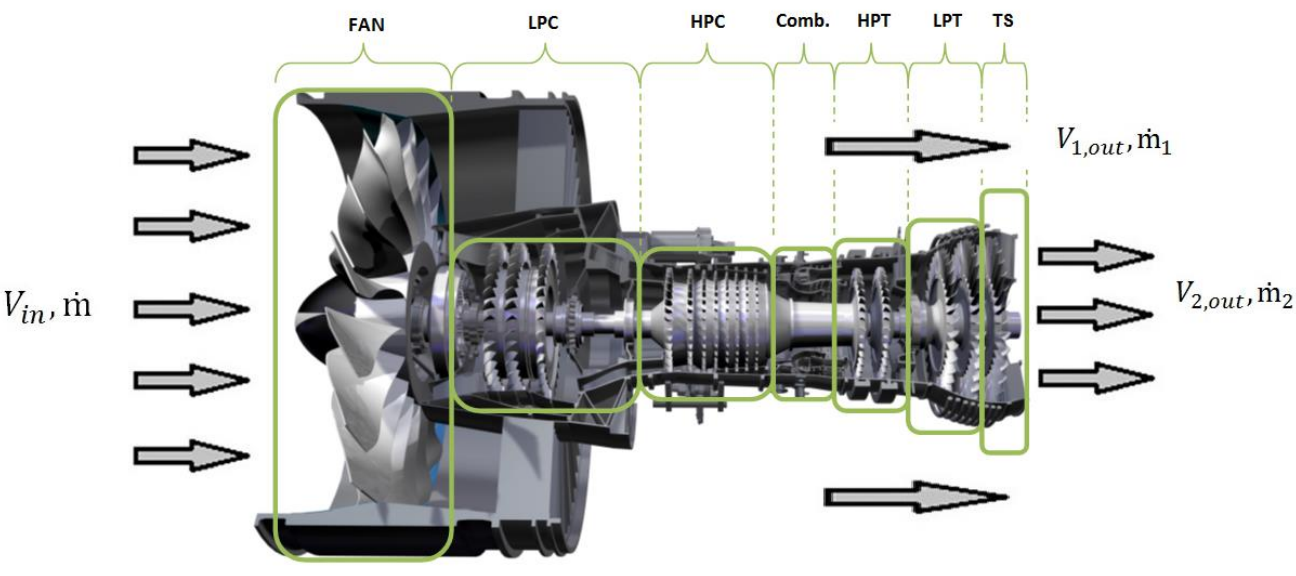



*Figure 2.1. PW1000G jet engine sectors.*

To overcome the aerodynamic drag of an airplane the thrust needs to be larger than the aerodynamic drag. The compressor raises the pressure of the incoming air and forces the air towards the combustion chamber. To create a higher velocity of the exhaust gasses fuel is mixed with the compressed air in the combustion system and a light spark ignites the air-fuel mixture. The ignited fuel mixture then expands with high pressure and accelerates through the turbines of the engine. A fraction of this energy is used to produce thrust. The hot gasses from the combustor enter the high pressure turbine (HPT), and the HPT extracts the energy produced in the combustor by transforming the energy to kinetic energy. In the same way as the compressor, the turbine consists of rotating discs of blades and static vanes. Turbine blades transform the pressurized stream of gasses from the combustion into kinetic energy that is used to produce thrust and compression. In a two-axial bypass engine the kinetic energy from the HPT is used for compression in the HPC. This since the HPC is mounted on the same shaft as the HPT, see figure 2.1. After the HPT the hot gasses enter the low pressure turbine (LPT). The LPT is mounted on the same shaft as the fan and the LPC. The extracted energy from this stage is used to produce thrust at the fan and for compression of air in the LPC. All of the remaining energy blows out as exhaust gasses at the exit nozzle at back of the jet engine, but between these stages the jet stream passes the Turbine Structure (TS). The TS is also often called tail bearing housing (TBH) or turbine rear frame (TRF) and this component is the topic of this thesis and will explained further in the following chapters.

In a commercial jet engine the exhaust gasses only produce a small amount of thrust where the larger proportions of thrust is produced from the fan that pushes large volumes of air around the core of the engine into the bypass canal. This air flow is also called “by pass air”. This configuration is optimized for the most fuel efficient flight cycle and is therefore more fuel efficient than for example a military jet engine that demands rapid changes in altitude and speed. In military engines all of the air from the fan enters the engine core for combustion and all of the thrust is gained from the exhaust of these gasses. Figure 2.2 shows a common turbojet for military purposes. This configuration on the other hand requires large volumes of fuel, but produces larger thrust.

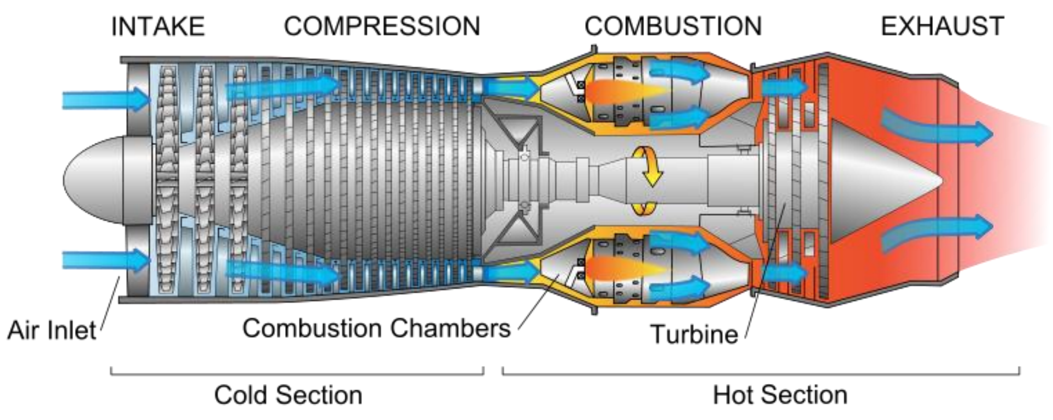



*Figure 2.2. Cross-section view of a military jet engine.*

## 2.2 Turbine Structure

This report will consider some of the problems that occur in the TS during normal operating conditions. The TS can most easily be seen as an exhaust to the jet engine. It is mounted on the airplane wing and carries a large proportion of the weight of the jet engine, see figure 2.3. The part has to withstand high temperatures that arise due to exposure of hot exhaust gasses, but also the forces from aerodynamic drag that is created when the TS redirect the jet stream that exits the LPT.  The TS also functions as a tail bearing house that holds the bearing for the jet engine shafts and has numerous interfaces to different parts of the jet engine. Some of the interfaces that can easily be seen are the interfaces to the LPT, the wing, the tail cone and the bearings, but the TS also contains important hydraulic and electrical tubing as well as sensors for the engine control system.

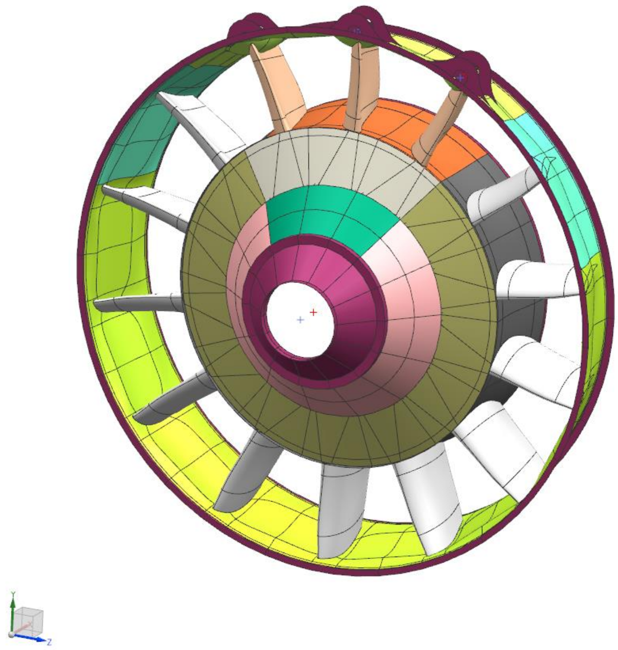


*Figure 2.3 The Turbine Structure (TS)*

In figure 2.3 a TS can be seen with a configuration of 14 struts (also called vanes) that are used to redirect the jet stream exiting the LPT. The struts are often hollow and contain hydraulics and electrical tubing. In some engine configurations the struts are also internally cooled in the same way as for the turbine blades in order to reduce the temperature and thermal expansion of the struts. The center structure consists of the hub that also provides structural support for the rotor bearings and the hub cone. On top of the TS there are for this engine three mount lugs that holds the engine to the airplane’s wings. The mount lugs is placed on the outer case of the TS and the whole structure is welded together by industrial robots.

## 2.3 Thermal management

Thermal management will in this report regard thermal zones bordering the Turbine Structure (TS) and the thermal boundary conditions that act on the TS during operation. GKN Aerospace has found that the TS suffer from a lowered life due to high sensitivity of low cycle fatigue during the flight cycle. This occurs mainly when the airplane is at idle phase between flights. During idle the temperatures exiting from the LPT increases. A simple, but ill-considered, conclusion would be that this is the cause of the problem, but when looking closer the problem becomes more complex. Many different cavities can found within the TS. All of these cavities have different temperatures during the flight cycle and the temperatures varies with changes in a range of variables such as altitude, airspeed, humidity, air temperature, air density and thrust.

Computerized simulations can be performed in order to evaluate how a jet engine performs during operation. Disciplines in fluid dynamics and heat transfer are used to calculate how variations in surrounding temperatures affect stress distributions in the TS. In order to understand the underlying processes of this phenomenon fundamental principles of heat transfer are needed.

Heat transfer focuses on the energy transfer that occurs due to temperature gradients between areas of different temperature levels.  Heat transfer occurs as a result of three different mechanisms, conduction, convection and radiation (Kutz, 2009).

### 2.3.1 Conduction

This mechanism focuses on the transfer of energy through direct contact between molecules and therefore the transfer of energy between areas of high temperatures to the areas of lower temperatures. A substance ability to transfer energy through conduction is represented by the constant of thermal conductivity κ. The thermal conductivity varies for different materials and is often denoted in material specifications. The fundamental relationship for heat transfer is termed Fourier's law of heat conduction. For a three-dimensional expression of the heat transfer over time, the following heat diffusion equation is used (Kutz, 2009):

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+q=\rho c_p\frac{\partial}{\partial x}\frac{\partial T}{\partial t} \quad (2.2)$$

Where $c_p$ is the specific heat capacity and  ρ the density of the material. q denotes the internal heat generation. $c_p$ and ρ are tabulated data given by material specifications.

### 2.3.2 Convection

This mechanism focuses on the transfer of energy that is being transferred through a motion of a fluid. The heat transfer rate can be described by Newton's law of cooling and is denoted:

$$q = hA(T_1 - T_2) \quad (2.3)$$

Where h is the heat transfer coefficient and A is the surface area (Kutz, 2009).

### 2.3.3 Radiation

This mechanism focuses on the transfer of energy that occurs through electromagnetic waves or through photons. The radiation mechanism is not contributory to rising temperature levels and is not considered further.

### 2.3.4 Thermal expansion

When a material experiences a change in temperature, the material wants to expand due to increased atomic vibration. The linear thermal expansion equation is written:

$$\Delta\mathcal{E} = \mathcal{E}_0\alpha(\mathrm{T}_1 - \mathrm{T}_0) \tag{2.4}$$

Where $\mathcal{E}$ is thermal expansion and $\alpha$ is the thermal expansion coefficient (Lundh, 2000). $\mathcal{E}$ and $\alpha$ are tabulated data given by material specifications. Hookes Law states that the stress is equal to the thermal expansion multiplied with the elastic modulus E

$$\sigma = \mathrm{E}\varepsilon \tag{2.5}$$

This implies that the change in temperature raises the thermal expansion, which leads to an increased stress level in the component.

## 2.4 Robust Design Methods

This chapter describes the fundamentals of robust design and its principles. Different tools implemented in the philosophy of robust design are described.

*"Validation requires documented evidence that a process consistently conforms to requirements. It requires that you first obtain a process that can consistently conform to requirements and then that you run studies demonstrating that this is the case. Statistical tools can aid in both tasks."* (Taylor, 1991)

### 2.4.1 Robust Design

One definition of a robust process or product is the ability to perform as intended even during non-ideal conditions. The term *noise* is used to describe uncontrolled variation in the process that may affect the outcome or performance. The activity in engineering development processes that aims at minimizing this sensitivity to uncontrolled variations that may affect performance is called "*Robust design".*

The goal is to find the combination of which parameters and what numerical values they should range between, that generates the least sensitive to uncontrolled variation. An experimental approach is used to find these robust points.

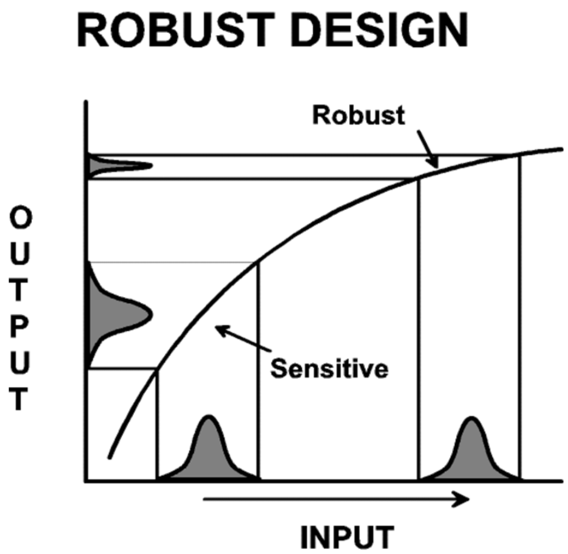



*Figure 2.4. Well selected inputs make the output less sensitive to the variation of the input.* (Taylor, 1991)

As shown in figure 2.4, a more robust design results in less variation and higher quality, but without additional costs. There exists a variety of tools for identifying key inputs and the sources of variation. One of those is Analysis of variance (ANOVA), which is a statistical study including statistical methods to determining if significant differences exist between populations. (Taylor, 1991). ANOVA is further described in chapter 2.12.

Robust design methods refer to the different methods of selecting the optimal values for inputs. As shown in figure 2.4, when nonlinear relationships exist between inputs and outputs, a careful selection of inputs makes the outputs less sensitive to a variation in these inputs. This means that the distribution of variation can stay the same, but with less impact on the outputs.

### 2.4.2 Taguchi method

The modern robust design methodologies has its origin from the 1950s and the Japanese engineer Genichi Taguchis ideas regarding improving quality by minimizing the negative effects of variation, rather than trying to eliminate the variation itself.

Taguchi divides the inputs in three different categories; noise factors, signal factors and control factors. These three inputs are all affecting the output or response. The noise factors are representing the type of variation that is uncontrollable in the process. The relation is shown in figure 2.5. (Forslund, 2012)

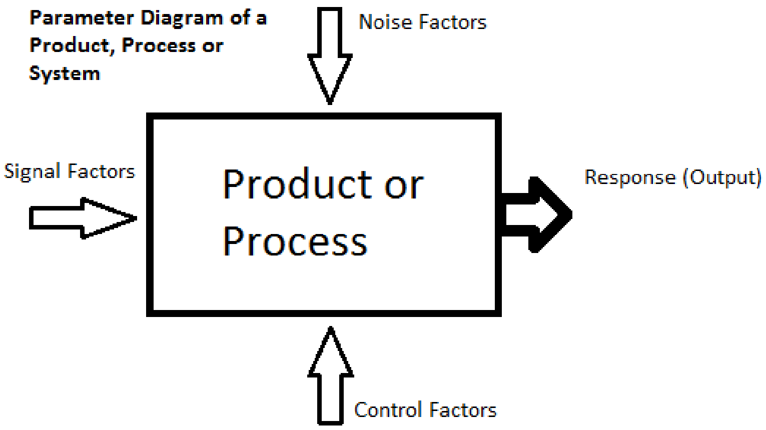



*Figure 2.5. Visualizes the parameter diagram of a product, process or a system.*

The utilization of noise and control factors are very suitable when it comes to investigation of real processes. In computer simulations though the experiments are deterministic, which means that all parameters are controllable, even the noise factors. (Forslund, 2012)

## 2.5 Experimental design

Experimental design is used to discover and examine the relationship between inputs and output. The definition of an experimental approach is the treatment of a group of parameters with the interest in observing the impact of the response. The construction and execution of the experiments has of course a major impact on the credibility in the results, which makes the validation of the experiment design extremely important.

To show the relationship between inputs and output a response surface can be used to graphically display an estimated equation from the simulated response and thereafter be able to predict the response from a change in input. (Ulrich, 2011)

### 2.5.1 Design of Experiment

This chapter will explain the methodology of Design of Experiments, hereafter also called DoE, and its use.

**Introduction to DoE**

In the product development phase early decisions usually have a significant impact on later project results. A cornerstone in a successful product development is to base decisions on facts and making the right choices early in the concept phase. Therefore knowledge accumulation has to be done at an early stage in the development phase but also in an effective and rapid way. A method for gathering this early knowledge is set up of experiments to provide information regarding important design and process parameters. The experiments needs to be planned and executed in a structured way to achieve best possible products and processes at the lowest cost. This form of controlled experiments gives an empirical base for decision making when examining parameters and their effect on output results. It is also an important tool to significantly reduce the time required for the experimental investigation. The procedure is efficient for finding multiple parameters effect on the investigated performance, as well to study each parameters individual to see which factor that has more or less influence on the performance. (Ulrich, 2011)

*"Design of experiments (DOE) is a systematic, rigorous approach to engineering problem-solving that applies principles and techniques at the data collection stage so as to ensure the generation of valid, defensible, and supportable engineering conclusions. In addition, all of this is carried out under the constraint of a minimal expenditure of engineering runs, time, and money."* (NIST, 2013a)

Design of Experiments is a method for assessing and quantifying the robustness in a design. The utilization of DoE as a method for analyzing variation in design parameters is an effective way to gain knowledge about how these parameters affect the robustness of the product. (Aerospace, u.d.)

Statistical Design of Experiments is a method for managing variation while learning the most from limited resources in what factors influence performance. Performing the correct DoE is an efficient utilization of available resource to gain required knowledge. (Simpson, 2013)

The initial step of DoE is to determine what the objectives of the experiment plan are, and selecting the correct parameters or *factors* to study. A well designed experiment plan extracts maximum information obtained from the required experimental effort.

To better understand the fundamental properties of DoE, the different steps when developing the design of experiment plan and the interpretation of results will be explained hereafter.

The basic structure for all DoE's follows these seven steps:

1. Set objectives
2. Select process variables
3. Set up of experimental plan
4. Execution of the experiment design
5. Screening to find important factors
6. Analyze and interpret the results
7. Use results or repeat the process

**Set objectives**
This initial step defines what are the objectives are for the experiment. This question answers what should be examined, and what kind of result that are needed to make relevant conclusions. This is a very critical step and therefore a great understanding is required before initiating an analysis.

**Select process variables**
Examine which input and output parameters that are important. This is made from a cause and effect analysis and engineering experience. This step is very crucial for the outcome of the DoE, where a bad choice of parameters will affect the total credibility of the results.

**Set up of experimental plan**
The way the experimental plan is set up depends on what the objectives is for the experiments and what number and type of factors that will be investigated. At this step number of factors (parameters) and how they will be varied is set up in an experimental plan matrix. Different methods are used for varying the factors in different combinations. This will be further explained later.

**Execution of the experiment design**
Run the experiments according to the set up experiment plan. Depending on the experiments this step may be very time consuming and require a lot of resources. A well-defined experiment plan can therefore save substantially recourses as time and costs.

**Screening to find important factors**
With help of the results obtained from the experiments contrasts and the effect of each factor and its interactions will be calculated. This effect shows how strongly a variance in a certain input parameter is affecting the output response.

**Analyze and interpret the results**

Statistical analysis needs to be done on the results to find out how likely it is that a certain factor really influence an output. This will be further described in chapter 3.7.

**Use results or repeat the process**

When the results are validated and proved one can draw conclusions from the analyzed results. Are the effects reliable and the right factors chosen, or should the experiments be repeated with another configuration? Maybe the first set up of experiments was to discover main effects in a screening design, and now a more in depth understanding is needed with a design for response surface. (NIST, 2013b)

### 2.5.2 Design of Experiments methodologies

This chapter describes relevant DoE design setups and their aim.

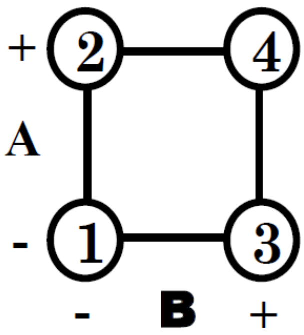
**Factorial Design**

In Design of Experiments the full factorial experiment plan is method for examine two or more parameters or "*factors*", each with different set of values or "*levels"*. According to the setup of the experiment plan then all possible combinations of these factors and their interactions are examined.

The number of experiments run for a full factorial defines by the number of levels of the factors squared by number of factors chosen, for example a two level – 4 factor design need $2^4$ = 16 runs. All fractional designs can be expressed as the notation$I^{k-p}$, where $I$ is the amount of levels for each factors, $k$ represents the amount of factors and $p$ the fraction of the full factorial that is used. The term $p$ defines the amount of generators, in other words the number of effects and interactions that can't be estimated entirely independent of each other.

Often a full factorial design is too comprehensive to be feasible in an economically or time consuming perspective. Through "engineering experience" the designer can carefully choose to remove some (often the majority) of the combinations, to make the experimental plan more compact. (Ulrich, 2011)

When making statistical analysis of generated results from factorial experiments, the *Sparsity-Of-Effects Principle* states the model often is dominated by main effects and low order interactions. This gives that main effects and two-factor interactions will result in the most significant responses in a factorial experiment, and the sparsity-of-effects principle actually refers to the thought that only a few effects in a factorial experiment plan will be significant. (Wu. C.F. Jeff, 2000)

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| 3 | - | + |
| 4 | + | + |

*Figure 2.6. The design space and experiment plan for a simple* $2^2$ *factorial design.* (Assarlind, 2014)

**Factor effects in DoE**

The main effect is the average effect of increasing the level of one factor from lower to higher value in the design matrix. The effect corresponds to the influence that a change in level for a certain factor has on the response. The effects are calculated as the difference between the averages of responses when the factors are, respectively, set at higher and lower levels. The interaction effects can be described as the effect of one factor influenced by the levels of another factor.

The corresponding calculation of effects for both factors and their interaction for the example in figure 2.6, are illustrated in figure 2.7;

$$l_A = \frac{y_2 + y_4}{2} - \frac{y_1 + y_3}{2} \qquad l_B = \frac{y_3 + y_4}{2} - \frac{y_1 + y_2}{2} \qquad l_{AB} = \frac{y_1 + y_4}{2} - \frac{y_2 + y_3}{2} \qquad (2.6)$$

| | A | B | AB | Experiment result |
|---|---|---|---|---|
| 1 | - | - | + | $y_1$ |
| 2 | + | - | - | $y_2$ |
| 3 | - | + | - | $y_3$ |
| 4 | + | + | + | $y_4$ |
| | $l_A$ | $l_B$ | $l_{AB}$ | |

*Figure 2.7. The corresponding effects for the two factors and their interaction.*

In practice higher order levels than two is rarely used, since response surface methodology is a more efficient way to investigate the correlation between factors and the corresponding response. It is also hard and very inefficient to use a design for more than two levels, compared to the response surface designs. (Assarlind, 2014)

A full saturated fractional design can investigate, at most, $2^n$ -1 factors; for example, seven factors needs 8 experiments.

**Resolution**

In fractional factorial design the ability to separate the low order interactions and main effects from each other is presented in the term of resolution. The resolution corresponds to the minimum length of the defining relation minus 1. In practice, a resolution of at least three and not over five is appropriate depending on the objective of the observation. A resolution of three can estimate main effects, but the factors may be confounded with two factor interaction effects, that is for example, $2^{3-1}$ where the defining relation is I= ABC ( factors A, B and C). Resolution IV estimates main effects and two-factor interactions, even if these interactions could be confounded by other factor interactions, $2^{5-1}$ (I=ABCD). Resolution V can estimate main effects and independent two-factor interactions. The two factor interactions may be confounded by three factor interactions though, $2^{5-1}$ (I = ABCDE). (Assarlind, 2014) (NIST, 2013c)

| | Factors | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Run | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 | FULL | III | | | | | | | | | |
| 8 | | FULL | IV | III | III | III | | | | | |
| 16 | | | FULL | V | IV | IV | IV | III | III | III | III |
| 32 | | | | FULL | VI | IV | IV | IV | IV | IV | IV |
| 64 | | | | | FULL | VII | V | IV | IV | IV | IV |
| 128 | | | | | | FULL | VIII | VI | V | V | IV |

*Figure 2.8. Different designs available dependent on number of factors and how well they can estimate interactions indicated by the resolution. (Assarlind, 2014)*

### Orthogonal Arrays

A correct set up factorial designs allow unbiased estimates of effects of factors and interactions because these are *orthogonal* to each other. An Orthogonal design gives that the effect of one factor is cancelled out by averaging for other factors. The cross product of the design matrix with itself is diagonal. An example of an orthogonal design is shown in figure 4.5, in the appearance of a L9 Taguchi design.

The full factorial can be split into two saturated designs, which are exemplified in figure 2.9. They both represent a fractional factorial design that corresponds to a full factorial when added together.

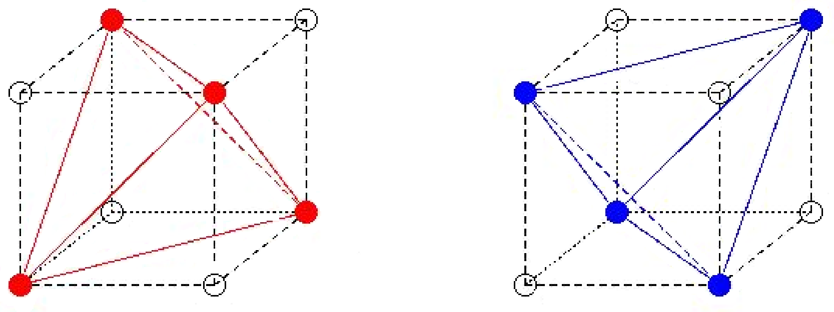


*Figure 2.9. The saturated design with three factors and four experiments is the half of a full factorial design. Both subsets above are orthogonal.* (Buydens, u.d.)

Experiments shown in red

| | A | B | C |
|---|---|---|---|
| Experiment 1 | - | - | - |
| Experiment 2 | + | + | - |
| Experiment 3 | + | - | + |
| Experiment 4 | - | + | + |

Experiments shown in blue

| | A | B | C |
|---|---|---|---|
| Experiment 5 | + | + | + |
| Experiment 6 | - | + | - |
| Experiment 7 | - | - | + |
| Experiment 8 | + | - | - |

*Figure 2.10. The two corresponding experiment plans for the subsets from figure 2.9.*

Another rule when dealing with statistical experiments is that the factorial experiments should be in a randomized order to eliminate the impact of bias on the experimental results. (Buydens, u.d.)

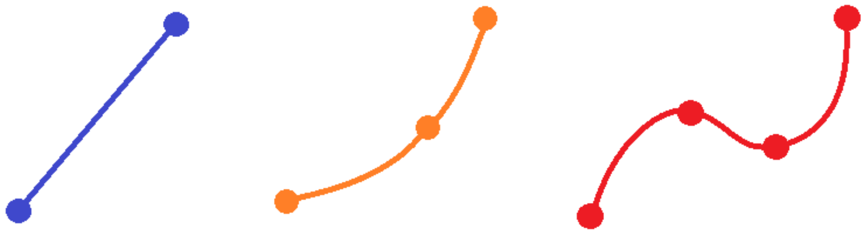


*Figure 2.11. The representation of linear, quadratic function and cubic function.*

A response that has a linear function only needs a design matrix that is set up by two levels on each factor. If the response is behaving as in figure 2.11, with a quadratic function, the number of levels on the factors has to be at least three. The addition of center point to a two level factor setup can't estimate the quadratic effects, but detect them in an efficient way.

### 2.5.3 Taguchi Methods

In Japan during the 50s and 60s, Dr. Genichi Taguchi developed techniques for DoE and implemented several key ideas for experimental design. Taguchi´s parameter design offers an efficient and systematical approach to optimize design for performance.

The Taguchi designs are similar to the fractional factorial designs, with a standardized set of orthogonal arrays, but with the implementation of two array matrices for each designed experiment. The Taguchi design methods are popular when it comes to screening objectives. (NIST, 2013d)

Taguchi´s design for experiments uses two major tools;

1. Signal to Noise ratio, where control factors and noise factors are separated. The control factors represent the parameters that can be controlled and varied by the engineer. Noise factors represent the variation that emerges in for example manufacturing, and are uncontrolled.

2. The use of orthogonal arrays, which accommodates many design factors simultaneously. The purpose of the orthogonal array is to investigate as many factors as possible, with as minimum effort as possible.

A common tool stressed in Taguchi methods is the S/N ratio or, Signal-to-Noise ratio. The desired values are named signal while undesired values are called noise. The noise factors are manipulated to create variation, and from the results the control factors can be chosen to minimize the effect from the generated disturbance, i.e. a more robust design.
There are three categories of the S/N ratio:

1. Nominal the best: $\frac{S}{N} = 10\log\frac{\tilde{y}}{s_y^2}$
   In this case a nominal value provides the best characteristics. A specific value is most desired, and both smaller and larger values are worse.

2. Smaller the better: $\mathrm{S/N} = -10\log 1/\eta(\sum y^2)$
   In this case a smaller S/N ratio provides the best characteristics. The ideal value is zero.

3. Larger the better: $\mathrm{S/N} = -10\log 1/\eta(\sum(1/y^2))$
   In this case a smaller S/N ratio provides the best characteristics. The ideal value is zero.

*Table 2.1. Choice of Taguchi "L-designs" dependent on number of factors and levels*

| Number of Levels | Number of Parameters (P) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| 2 | L4 | L4 | L8 | L8 | L8 | L8 | L12 | L12 | L12 | L12 | L16 | L16 | L16 | L16 | L32 |
| 3 | L9 | L9 | L9 | L18 | L18 | L18 | L18 | L27 | L27 | L27 | L27 | L27 | L36 | L36 | L36 |
| 4 | L'16 | L'16 | L'16 | L'16 | L'32 | L'32 | L'32 | L'32 | L'32 | | | | | | |
| 5 | L25 | L25 | L25 | L25 | L25 | L50 | L50 | L50 | L50 | L50 | L50 | | | | |

| Number of Levels | Number of Parameters (P) | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 |
| 2 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 | L32 |
| 3 | L36 | L36 | L36 | L36 | L36 | L36 | L36 | | | | | | | | |
| 4 | | | | | | | | | | | | | | | |
| 5 | | | | | | | | | | | | | | | |

The well-known Taguchi orthogonal arrays are the "L´s". For example, with four factors and three levels the orthogonal array Taguchi L9 is chosen. See table 2.1 for different combinations.

*Table 2.2. The orthogonal arrays are shown for the yellow, blue and red combinations.*

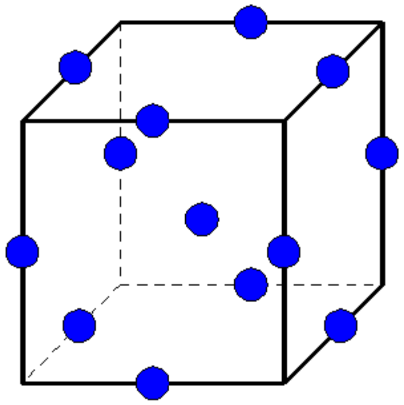
| Experiment | A | B | C | D |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 2 | 2 | 2 |
| 3 | 1 | 3 | 3 | 3 |
| 4 | 2 | 1 | 2 | 3 |
| 5 | 2 | 2 | 3 | 1 |
| 6 | 2 | 3 | 1 | 2 |
| 7 | 3 | 1 | 3 | 2 |
| 8 | 3 | 2 | 1 | 3 |
| 9 | 3 | 3 | 2 | 1 |

### 2.5.4 Box Behnken

Box Behnken is an efficient design for estimation of first and second order interactions and is suitable for response surface methodology, invented by Georg E.P Box and Donald Behnken in the 1960.The setup of a Box Behnken design require some guidelines stated below.

At least three levels are required for Box Behnken, and the factors are leveled as “+1”, “0” and “-1”. The design appropriate for a quadratic model, as stated, consisting the product of two factors.

The experiment plan can be considered as a combination of a two level fractional or full factorial designs, but with an incomplete block design. Each block is designed with a certain number of factors are varied with all combinations, while the remaining factors are kept at the nominal value.



*Figur 2.13. The Box-Behnken design with spherical design space.*

As can be seen in figure 2.13, the design space in Box-Behnken consists of sphere that protrudes throug the original design space box. Each midpoint of the box is tangential to the surface of the sphere. This design leads to fewer run than a original center composite design with the same amount of factors, which makes Box Behnken less expensive. (NIST, 2013e)

*Table 2.2 The setup of different Box Behnken designs.*

| Number of factors | Number of factors varied in each block | Number of blocks | Factorial points in each block | Total runs with one center point | Number of coefficients in quadratic model |
|---|---|---|---|---|---|
| 3 | 2 | 3 | 4 | 13 | 10 |
| 4 | 2 | 6 | 4 | 25 | 15 |
| 5 | 2 | 10 | 4 | 41 | 21 |
| 6 | 3 | 6 | 8 | 49 | 28 |
| 7 | 4 | 7 | 8 | 57 | 36 |

### 2.5.5 Placket-Burman Design

When interactions between factors are negligible, the idea of Placket-Burman Design is to find a sequence of experiments where all combinations of levels for any couple of factors or parameters, are presented the same number of times throughout the experiment plan.

Placket-Burman is very efficient way of screening between a large set of controlled factors, but only when the main effects are of interest. The reason why Placket-Burman is suitable for screening is because main factors are, in general, confounded with two factor interactions. This makes Placket-Burman as a design method very economically for detecting large main effects, but with the assumptions that interactions are negligible in comparison with the major main effects. When applying the Placket Burman Design the Pareto principle is present, which means that the assumption is made that only a few of the factors are considered contributing with major main effects. This makes Placket Burman the ultimately screening design, when a few major factors need to be identified. (NIST, 2013f)

Placket- Burmans are so called cyclic designs, where the matrix is generated by one line of "+" and "-", with the next line including same sequence, but shifted by one position with the last line is "+" only.

| Run | A | B | C | D | E | F | G | H | I | J | K |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 |
| 2 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 |
| 3 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 |
| 4 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 | -1 |
| 5 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 | -1 |
| 6 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 | 1 |
| 7 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | 1 |
| 8 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 |
| 9 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 |
| 10 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 | 1 |
| 11 | 1 | -1 | 1 | 1 | 1 | -1 | -1 | -1 | 1 | -1 | 1 |
| 12 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |

*Figure 2.14. Example of Placket Burman where 11 factors and their main effects are examined with 12 runs.*

## 2.6 Response surface methodology

Response surface methodology is a collection of mathematical and statistical methods that are used to develop an empirical model of a response. Usually response surface methods are used when the objective is to optimize a response or to predict the outcome of a certain input setting of the process. (Mukhopadhyay, 2010).

$$y = f(x_1, x_2, x_3, x_4) + \mathcal{E} \qquad (2.7)$$

Where y is the response and $x_1, x_2, x_3, x_4$ are temperatures at different points. $\mathcal{E}$ represents the noise or error in the response. Response functions are usually approximated with a low-order polynomial with a first order model. A multiple linear approximation model are written as

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \mathcal{E} \qquad (2.8)$$

The lack of fit of any model can be calculated from statistics or by graphical analysis of the results. There is often curvature in a response and in those cases a higher degree of polynomial are applied to the approximation model. A second-order model takes curvature into account by taking interaction effects between the factors into consideration

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum\sum_{i<j} \beta_{ij} x_i x_j + \mathcal{E} \qquad (2.9)$$

For a complete description of the process an even more detailed approximation model can be used, but this is highly unusual since a response surface of second-degree are often accurate enough for small regions of the response surface. A full cubic model with all possible terms can be seen down below

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum\sum_{i<j} \beta_{ij} x_i x_j + \sum\sum\sum_{i<j<k} \beta_{ijk} x_i x_j x_k + \mathcal{E} \qquad (2.10)$$

Where $\beta_{ijk}$ are regression coefficients and $x_{ijk}$are factor inputs. In order to estimate curvature in the system a three-level design is needed and in order to get the most efficient results a proper design matrix with appropriate experimental runs are needed. (NIST, 2013d). Figure 2.15 illustrates how fitted lines change with increasing polynomial order of the approximation model.

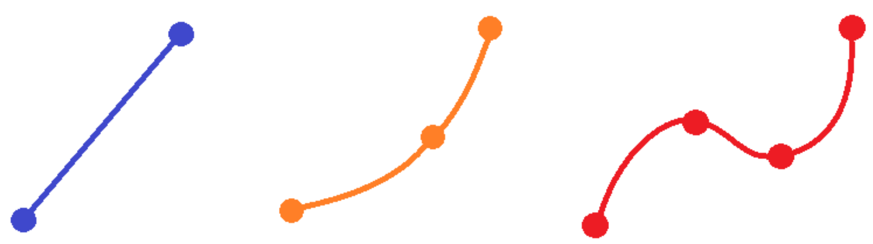


*Figure 2.15. Linear-, quadratic- and cubic line fittings.*

### 2.6.1 Neural Network Function

Sometimes it is desirable to combine several individual sets of simulation data to create a response function. A method that makes this possible is the Neural Network Function that maps numeric input to a set of output targets. The method is feasible when a need of combining different sets of data, and the data is established for different purposes originally.

MATLAB and ModeFrontier has a Neural Network toolbox which can be used for data fittings, but also other applications than such as clustering, pattern recognition, dynamic system modeling and control. In MATLAB, the application assists in selecting data, create the network and evaluate the performance with mean square error and regression analysis.

A so called two-layer feed-forward network with sigmoid hidden interconnections and linear output neurons is created. This network can fit multi-dimensional mapping problems given consistent data and enough neurons in its hidden layer. The network in MATLAB is trained with a background propagation algorithm, and in ModeFrontier a genetic algorithm is used to create the neural network response function shown in figure 2.16 (MathWorks, 2014).

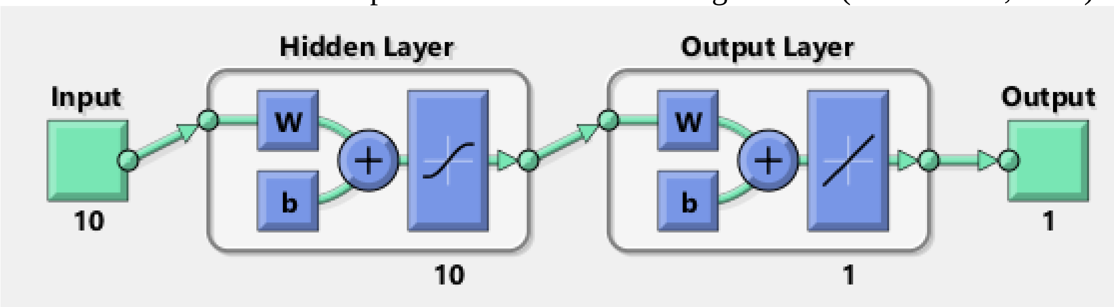



*Figure 2.16. The two-layer feed forward network used in MATLAB*

## 2.7 Regression theory

The approximation models mentioned in previous chapter are often regarded as regression models. Regression is a statistical method used to estimate the relationship between input variables and the output response. Regression can be applied in both linear, nonlinear and multiple manners depending on the amount of input factors. When model assumptions have been made, the coefficients of the model need to be estimated. The most common method is the least-square regression model since this method effectively captures factor effects as well as interaction effects while being insensitive to model error such as coefficient variance. (Simpson, 2013). The least square regression method seeks to minimize the squared error calculated from the residuals and the total variance

$$R^2 = 1 - S_r/S_t \qquad (2.11)$$

The $S_t$ value represents the total variance of the response;

$$S_t = \sum_{i=1}^{n}(\tilde{y}_i - \bar{y}_i)^2 \qquad (2.12)$$

where $\bar{y}$ is the mean value of the response vector. A model with low $R^2$ value does not guarantee that the model fits the response data well, which also means that the model doesn't represent the real process in a satisfying way. Another value often used in close relation to $R^2$ is the $R^2_{adj}$ which is the adjusted value. The $R^2_{adj}$ value increases when significant terms are added to the model and decreases when they are removed. Thus if insignificant terms are added to the regression model, the increase of $R^2_{adj}$ is small. $R^2_{adj}$ should be close to $R^2$ to ensure a good approximation model.

$$R^2_{adj} = 1 - \left(\frac{n-1}{n-DOF}\right)(1 - R^2) \qquad (2.13)$$

Where n is the number of experiment and DOF is the degrees of freedom in the approximation model. The least-square regression method is calculated differently depending on the approximation model used in the scientific investigation. A linear regression model seeks to minimize the following

$$S_r = \sum_{i=1}^{n}(\tilde{y}_i - y_i)^2 = \sum_{i=1}^{n}(\tilde{y}_i - \beta_0 - \beta_1 x_1 - \beta_2 x_2 - \beta_3 x_3 - \beta_4 x_4)^2 \qquad (2.14)$$

Where $\tilde{y}_i$ is the measured response from the experiment and $y_i$ are the response given by the linear approximation model. Often when there are four independent factors there are interaction effects between these that contribute to the response. A quadratic model with interactions instead seeks to minimize

$$S_r = \sum_{i=1}^{n}(\tilde{y}_i - y_i)^2 = \sum_{i=1}^{n}(\tilde{y}_i - \beta_0 + \beta_i x_i + \beta_{ii} x_i^2 + \sum\sum_{i<j}\beta_{ij} x_i x_j)^2 \qquad (2.15)$$

The same correlation exists for the cubic approximation model, but will not be mentioned further here, since higher degree polynomials would result in the danger of over-fitting the model.

Results of the least-square regression analysis for different approximation models can be seen in table 4.12. It is theoretically possible to create a model that fits the data exactly. However this will result in a highly oscillating function that cannot be used to predict how the response is affected by a change in input variables in an effective way (Simpson, 2013).

Determination of test points in matrices depends on the assumed polynomial order of the assumed mathematical model that is to be used for the response function. The polynomial order can often be determined from historical testing or by consulting engineers with first-hand knowledge and experience within the area of focus.

## 2.8 Statistical correlation

Correlation is a statistical technique often used when the purpose is to identify the degree of relationship between two variables i.e. it tells the degree to which two variables tend to move together. The correlation coefficient r is calculated by

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{n\sigma_x\sigma_y} \tag{2.16}$$

where n is the sample size i.e. the size of the response vector, x is the factor and the respective factor mean value, y is the response vector and the respective response mean value. $\sigma_x$, $\sigma_y$ is the square root of the factor and response variance (Benjamin S. Blanchard, 1990)

$$\sigma_x = \sqrt{\sum x^2 - (\sum x)^2} \tag{2.17}$$

$$\sigma_y = \sqrt{\sum y^2 - (\sum y)^2} \tag{2.18}$$

## 2.9 Analysis of variance (ANOVA)

Together with the equations mentioned in section 2.7 and 2.8 it is possible to analyze the relationship between response and input. This sort of analysis is called an ANOVA analysis. The purpose of analysis of variance, or ANOVA, is to test the difference between means when there are several populations in an experiment. It is a statistically based decision tool that is helpful to derive the significance of all main factors and their respective interactions. (Rama Rao. S, 2012).

The ANOVA table for a simple linear regression model looks as follows:

*Table 2.3. Typical ANOVA table*

| **Source** | **DOF** | **SS** | **MS** | **F** | **P-value** |
|---|---|---|---|---|---|
| **Model** | k | SS(model) | MS(model) | F(model) | |
| **Factor** | k-1 | SS(factor) | MS(factor) | F(factor) | |
| **Error** | n-k | SS(error) | MS(error) | | |
| **Total** | n-1 | SS(total) | | | |

The second column in ANOVA tables shows the degrees of freedom for a factor i.e. the amount of other possible combinations of each factor. Where k is the number of levels used in the experimental plan and n are the number of experiments. An ANOVA analysis can also be applied to evaluate a regression model to determine the accuracy of the model. In such cases the DOF for the complete model is represented by the amount of factors included within the model. For any regression model with k factors and for any experiment with n observations, the degree of freedom is calculated as shown in table 2.4.

*Table 2.4 Degrees of freedom in ANOVA analysis*

| **Source** | **DOF** |
|---|---|
| **Model** | k |
| **Error** | n-(k+1) |
| **Total** | n-1 |

The SS column shows the sum of squares and is calculated in the same manner as in the least square regression method presented in eq. (2.12) and (2.13). The total sum of squares become

$$SS_{Total} = SS_{Model} + SS_{Error} \tag{2.19}$$

The third column MS shows the mean sum of squares and is calculated in the same way as the sum of squares, but divided with the number of experiments and thus shows the mean sum of squares for the theoretical regression model and for the residuals. (Meier, 2013)

### 2.9.1 Verification and Validation

The goal of a simulation model is to represent the reality as accurate as possible. Simulation models have an increasing importance in modern product development, and works as tool for decisions-making. In especially the aerospace industry, simulation and other computer aided tools plays a significant role in the development of the product. (Forslund, 2012)

When using the models and the results established from them, is it important that the information generated is “correct” and really explains the reality in an expected way. The term *validation* is here used to express how well the simulation model, and the results based on it, really represents the reality (Forslund, 2012). In this thesis no validation of the actual simulation data will be considered, but the awareness must be there when interpreting the results. Nevertheless, validation in form of statistical methods and tools has to be carried out on data that are underlying the decisions made through this project.

Verification refers to the evaluation if an internal process complies with specifications or requirements, in contrast to validation that refers to the assurance that the process meets stakeholder’s interests.

### 2.9.2 Fischer F-test

The F-test provides useful statistics that shows the statistical significance of the regression coefficients. The F-test is a test function for the null hypothesis that shows if there is no linear relationship between the factors. (Norrby, 2012). The F statistics is calculated in the following manner

$$F = \frac{MS_{model}}{MS_{error}} = \frac{SS_{model}/DOF_{model}}{SS_{error}/DOF_{error}} \quad (2.20)$$

A model or a factor with high F-value that exceeds the critical value will show that there is a significant effect that is unlikely due to chance. (Winter, 2014). The critical value can be found in tabulated data when given the DOF values of the numerator and denominator of the F ratio.

### 2.9.3 P-Value

The P-value is another method used to evaluate the relationship between factor input and the response and thereby prove the statistical contribution of each factor. The significance of the F value is called the P-value and tells the probability of the model statistic being as extreme as the one observed given that the null hypothesis is true (NIST, 2012a)

$$H_0 : \beta_1 = 0 \quad (2.21)$$

The alternative hypothesis is that each of the regression coefficients is different from zero:

$$H1 : \beta 1 \neq 0 \quad (2.22)$$

The p-value for which the null hypothesis is rejected is determined by the level of significance. A common value for the level of significance is $\alpha = 5$ %, which means that a p-value lower than 0.05 indicates that the predictor has meaningful effect to the model and that the null hypothesis, eq. 2.21, can be rejected. (NIST, 2012a)

## 2.10 Selection of Experimental Design

The choice of what experimental plan to use depends on the objective of the experiments, and the number of factors to be investigated at different levels.

**Comparative objective**
A comparative objective is if several factors are investigated, but the goal is to make decisions based on only one priority factor, where the question of interest is whether the factor is significant or not. The significance of a factor means whether or not a change in the response is related to different levels of that factor. If this criterion is the main goal of the experiments that means it is a comparative problem, and therefore needs a comparative design solution.

**Screening objective**
A screening objective is preferable when the primary purpose is to identify a few important factors out of many less important ones.

**Response Surface objective**
Response surface objective is relevant when the experiment design aims at estimate interaction and even quadratic effects. These designs are suitable when the goal is to find improved or optimal process settings, finding errors or process problems in weak points and making a process more robust against external and uncontrollable variation.

**Optimal fitting of a regression model objective**
When optimal fitting of a regression is the objective, the aim is to model the response as a mathematical function of a few continuous factors when sufficient model parameters are desired. This is called a regression design.

| Number of factors | Comparative objective | Screening objective | Response surface objective |
|---|---|---|---|
| 1 | One-factor completely randomized design | - | - |
| 2 -4 | Randomized block design | Full or fractional factorial | Box Behnken |
| 5 or more | Randomized block design | Fractional factorial or Placket-Burman | Screen first to reduce number of factors |

*Figure 2.15. A guideline for selection of experimental design*

When it comes to decision making regarding which experimental design to use, there is several factors to take into account. The extent of the design is a critical factor when limited resources in terms of time and cost are crucial. The other aspect of this is, what the cost is to choose the wrong, or more often, a too simple design poor of information. (NIST, 2013g)

# 3. Method

## 3.1 Literature studies

In order to achieve an understanding of the component and gain required knowledge in the area of design of experiment, statistics as well as thermal- and structural analysis, literature studies had to be done. Literature search was done using search engines on the Internet, Chalmers library and internal documentation search at GKN Aerospace Engine Systems. Interviews with experts at GKN Aerospace provided valuable information about the component and the related problems that it was facing.

## 3.2 Data Acquisition

In order to simulate normal flight conditions data has been collected from temperature measurements at different positions around the TS. To reduce simulation time the flight cycle has been reduced to only consider crucial time steps. The flight cycle measurements were provided by the original engine manufacturer. The data provides a foundation to all simulation work that was made in this thesis. How this data is used is mentioned further in chapter 3.5 Thermal analysis.

CAD models were provided by GKN in order to get reliable simulations. The CAD models were of different composition where some features were width, height and length that had been altered between the different model configurations. This will be mentioned further in chapter 3.6 Structural analysis.

## 3.3 Software research

All simulations were done using ANSYS. The simulation setup for both thermal and structural analyses as well as post processing of results are presented further in the following chapters of this thesis. The computer program MATLAB was used for post processing of the results. MATLAB provided useful functions and the ability to plot the results in a convenient way. The built-in functions and statistical features of MATLAB 2014a supported fast evaluation of the data when implementing regression and ANOVA analysis. MINITAB, ModeFrontier and Microsoft Excel was also used to derive important conclusion from simulation data and is useful software when working with design of experiments.

## 3.4 Development process

The team working with the development of the TS at GKN Aerospace is a multidisciplinary team of engineers with different expertise and responsibilities. In order to improve concepts and gain a higher level of technological readiness the team works in a highly iterative manner in order to test and evaluate different concepts. The concepts have a highly demanding specification that needs to be met in order to meet the high certification requirements put on commercial jet engines. EWB evaluates a set of design definitions by performing analyses and post processing the results. Data from all simulations are gathered to be examined and evaluated. The product development process of the multidisciplinary team can be seen in appendix I, only for the internal report, and shows what input and output each process has. The process chart has a strong focus on thermal analysis since the goal of this thesis is to increase the thermal robustness of the component.

The development chart is based on interviews with several of GKN employees working in the EWB start platform. A start platform is created based on findings from earlier designs and a system model of the jet engine. Altogether this becomes a design study. In the next stage the team decides what they want to analyze. A hypothesis is created and several different geometries are tested in an iterative manner. In the next step a mesh is created and provided as input for the thermal analysis, the structural analysis as well as for stiffness- and weld analysis. Subsequent processes run parallel with each other and all the results are collected in a final database which shows trend curves or response surfaces that are used for providing reliable feedback for the specific design case. A so called CUMFAT analysis is performed subsequently of the structural analysis in order to determine the component life. The OEM performance group is the abbreviation for the informative data input given by the original engine manufacturer. A large effort of the process chart was put on the generation of thermal input that is provided for the thermal analysis and this will be further described in the following section.

## 3.5 Thermal analysis

As seen in appendix II the process chart contains the all through process of generating the thermal input. OEM provides data from any given flight cycle supported with measured temperatures at several points in the jet engine. This is compiled to a complete LCF mission showing engine parameters versus time for the given flight mission.

Calculations of airflow, heat transfer coefficients and heat are needed to apply appropriate thermal loads. This is implemented by several computer macros that eventually provide ANSYS input tables with interpolated temperature that are applied on all the surfaces of the CAD model. The flight cycle provided by OEM is divided into a certain amount of time steps that are chosen by experience. Many time steps require long simulation time or high computer power, but in the contrary fewer time steps require shorter simulation time and provide possibilities for multiple simulations at the same time.

## 3.6 Structural analysis

The structural analysis is performed subsequently to the thermal analysis since it requires a thermal result file as input. The thermal result file is applied on the model and the model is locked in its six DOF. ANSYS is also used for this analysis and it calculates elongation and stress levels emerging from thermal expansion at all nodes in the whole model. The result files consist of this information and are used for subsequent post processing of the results.

## 3.7 Post processing of simulation data and use of statistical methods

Evaluation of simulation and DoE data should be performed systematically and in iterative fashion. This section describes how post processing operation is performed for evaluation of DoE data given from structural and thermal analysis. The overall process that was used is presented in figure 3.1 and shows the DoE analysis flowchart. The process follows a methodology presented by NIST Handbook of Statistical Methods (NIST, 2012b)

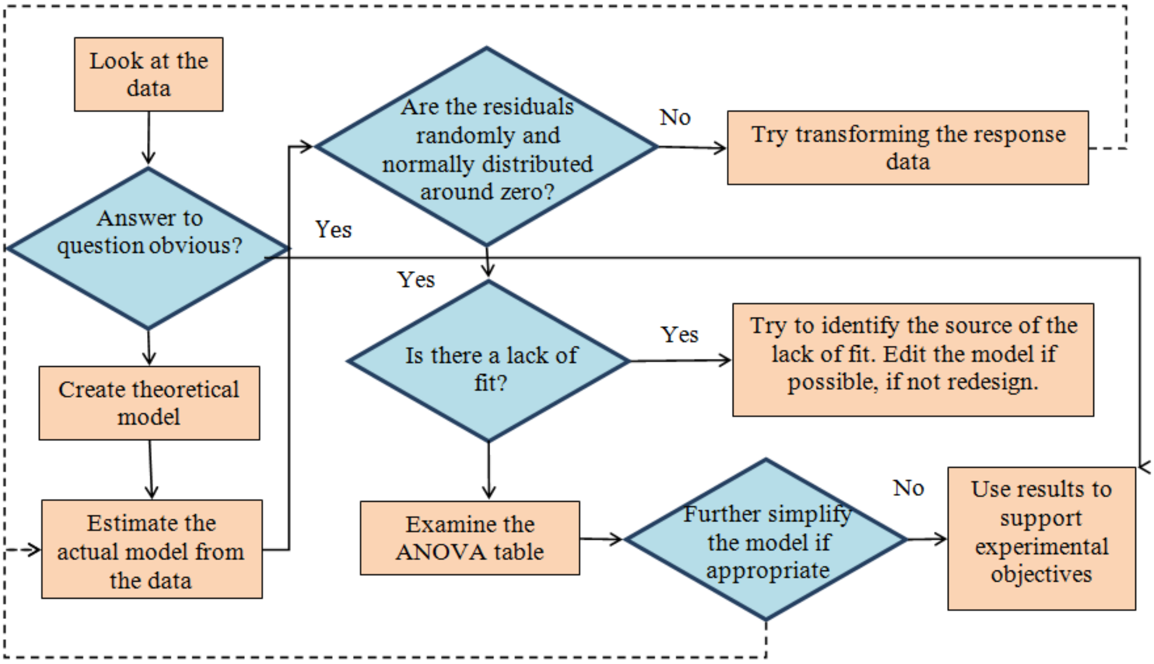



*Figure 3.1. DOE analysis flowchart*

## 3.8 Geometrical variation

To investigate the geometrical variation based on data from older models a neural network function was generated in MATLAB. With aid from a neural network function the data set of varied geometrical models together with the thermal results were mapped as numeric input to a set of output targets. The results derived from the neural network were used for investigation of geometrical and thermal factors.

# 4. Results

This chapter presents the derived results from the thesis work.This chapter also describes how the theory and methodologies earlier presented was implemented in the ongoing thesis work.

## 4.1 Selection of process parameters

An initial research was started to fully understand the presented thermal problem. To gain this knowledge a research study in literature and utilization of internal information was conducted. A mapping of the product development process was made as mentioned in section 3.4. The mapping helped to understand what matters that were actually needed to address. The investigation supported the creation of the research question of this thesis report. Following the procedure of DOE related work mentioned in chapter 2.5, a cause and effect analysis was performed in order to establish the important process parameters to be included in the analysis. A fishbone diagram, shown in figure 4.1 was used to gain an understanding of the underlying causes of stress in the TS. As it can be seen the diagram spreads into four main sectors.

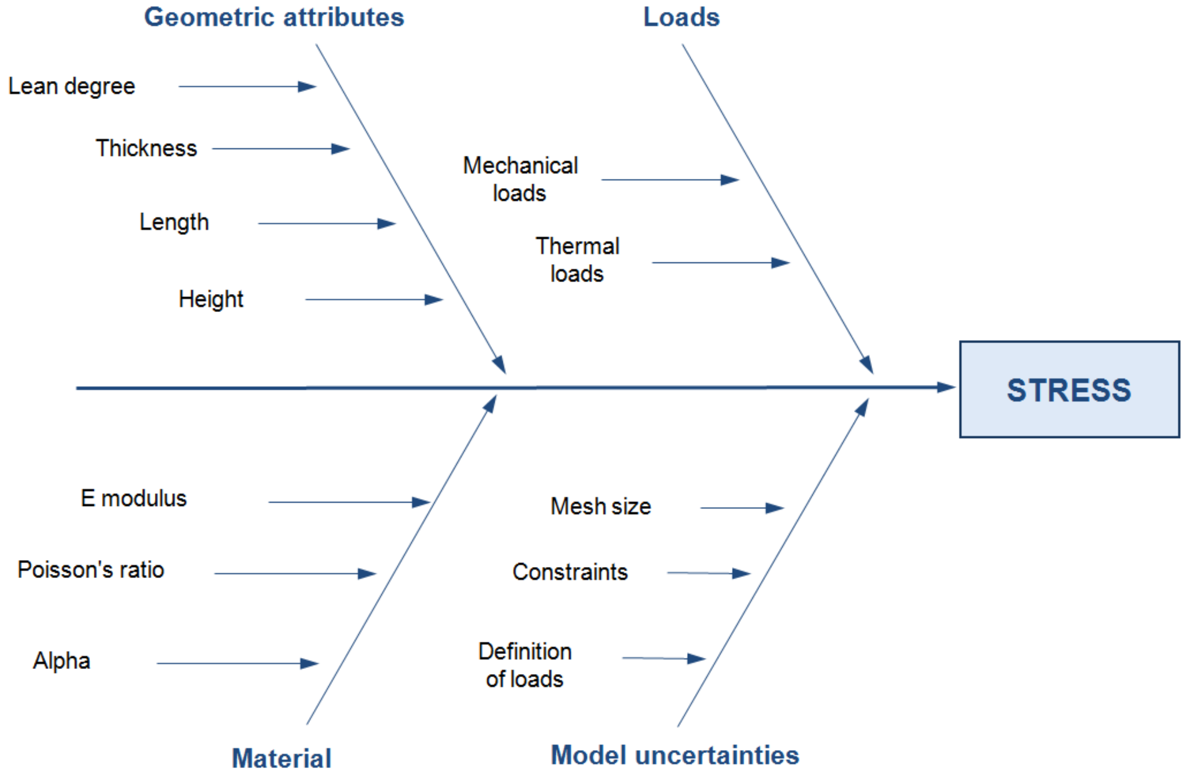



*Figure 4.1 Fishbone diagram showing root causes of stress.*

**Geometric attributes**
This sector covers all the geometrical variables that change the topology and positioning of all the parts in the TS.

**Loads**
This sector covers both the outer forces that act on the TS and the thermal loads emerging from the hot gasses exiting the low pressure turbine. No consideration to mechanical loads was taken during this analysis. The analysis focused on investigating how thermal boundary conditions (Thermal loads) affected stress distributions in the TS.

**Material**
This section regards the material parameters used for the analysis. There are the Young's modulus, Poisons ratio and the alpha value, for all of which was not investigated during this thesis work.

**Model uncertainties**
Model uncertainties originate from what constraints that was used when building the model. It also covers the definition of loads and how these are applied to the simulation model and at last there are model uncertainties emerging from the use of different mesh sizes and shell models.

## 4.2 Thermal Management

As presented in chapter 2.2, the TS is a rotational symmetric component and consists of 14 struts, 11 regular and 3 mount struts, see figure 2.3. When studying the TS it early stood clear that the area of focus had to be reduced in order to grasp the problem in an efficient way. It has been mentioned that GKN experienced shortened life and crack initiation at weld positions. It was therefore decided that the area of research should concern welds around strut sections of the TS. The load bearing mount struts are especially important for this study, where the most stress sensitive area should be investigated. Therefore the examined strut is one of the load bearing mount struts shown in figure 4.2

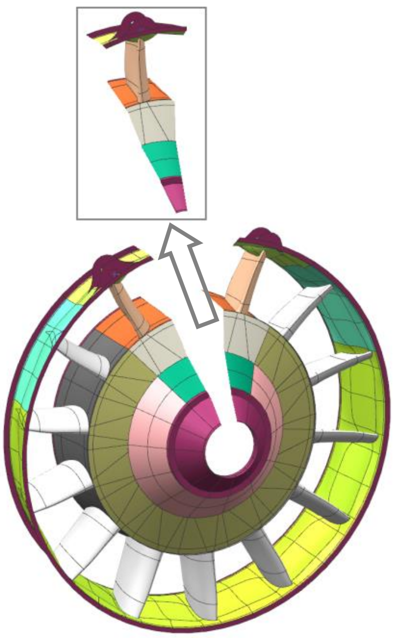
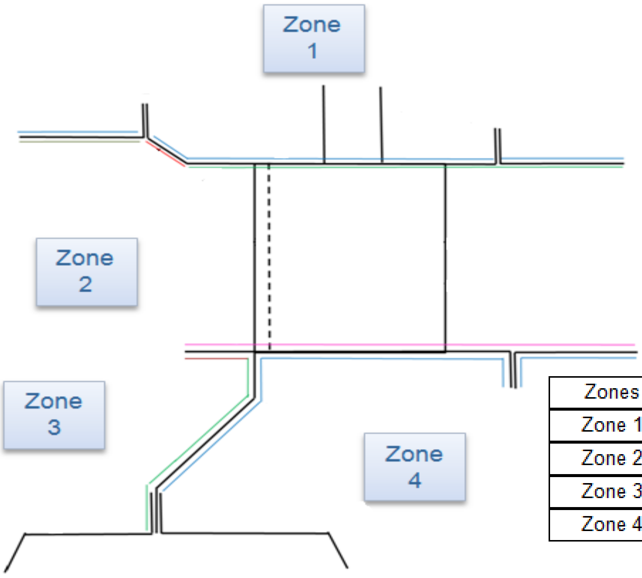


*Figure 4.2 Mount strut position on the TS*

One of the major concerns when it comes to simulation is the limitations in time and processing capacity, which demands an efficient set up of the model. The thermal zones surrounding the TS were divided into four zones. The main zones are the gas canal, the secondary airflow (bleed air), the nacelle and the hub cavity. The arrangement of the thermal zones can be seen in figure 4.3.




| Zones | Position |
| --- | --- |
| Zone 1 | Nacelle |
| Zone 2 | Gas canal |
| Zone 3 | Secondary airflow (Bleed air) |
| Zone 4 | Hub cavity |

*Figure 4.3. The figure shows a side view cross section of the TS and illustrates a side view of one of the 14 struts.*

As seen in the figure different temperatures act on each surface of the TS cross section. The nacelle is a cavity outside the case of the TS and the secondary airflow are a cavity of air emerging within the core of the jet engine. In this report the mentioned temperatures will be named as thermal boundary conditions or thermal BC’s. Temperature measurements for each surface were provided by GKN and as mentioned earlier they are measurements from a real flight cycle. Thermal BC’s were interpolated between measurement points and was provided as ANSYS input tables for thermal analysis as described in chapter 3.5 Thermal analysis.

## 4.3 Experimental plan

The TS is now sectional divided into a segment including one bearing mount strut, see figure 4.3. The selected four thermal zones shown in figure 4.2 could now be used as four different input factors for the upcoming experimental study. To investigate each zones effect and contribution to the resulting stresses in the TS, a design of experiment plan was set up. As an initial study and for the purpose of investigating the thermal robustness of the TS, consideration was only taken to the thermal loads. The division of the TS into four thermal zones now served as four factors for a DoE design. The initial set up was a screening design, where the goal was to identify the main effects (i.e. the effect each factor has on the output response) of the chosen factors. The initial idea was to test the factors at three levels, to be able to identify a quadratic response. This together with four factors generates a Taguchi L9 design, according to table 2.1. The experimental setup is presented in figure 4.5.

| Experiment Run | A | B | C | D |
|---|---|---|---|---|
| 1 | -1 | -1 | -1 | -1 |
| 2 | -1 | 0 | 0 | 0 |
| 3 | -1 | 1 | 1 | 1 |
| 4 | 0 | -1 | 0 | 1 |
| 5 | 0 | 0 | 1 | -1 |
| 6 | 0 | 1 | -1 | 0 |
| 7 | 1 | -1 | 3 | 0 |
| 8 | 1 | 0 | -1 | 1 |
| 9 | 1 | 1 | 0 | -1 |

*Figure 4.5. The design set up for Taguchi L9*

An early assumption for the thermal variation in the experimental plan was 28K. Literature studies suggest that this temperature level should be

$$\Delta T = \mu + 3\sigma \quad (4.1)$$

Where μ is the mean temperature value of all the measurements and $\sigma$ is the standard deviation of all measurements (Vilmart, 2010)

Temperatures are varied between -1, 0 and +1. The assumption was made that the temperature difference established represented a realistic variation in the input parameters from OEM .The complete Taguchi L9 temperature table contains data measurements from a real flight cycle, but will not be shown here for confidential reasons. The data collected from a real flight cycle consists of over 400 time steps distributed from engine start until engine shut down. To minimize simulation time the original data were interpolated and narrowed down to 41 representative time steps.

## 4.4 Positioning of nodes

GKN has found that most of the low cycle fatigue occurs at weld positions on the struts. By investigating how stress and temperature distributions occur in these cross sections it is possible to get an understanding on what effect variation in thermal BC's have on TS life. As initial investigation a general ANSYS-script was created to choose a set of nodes along the lower weld line suction side on one of the struts shown in figure 4.6. The positioning of the weld curve is shown in figure 4.7.

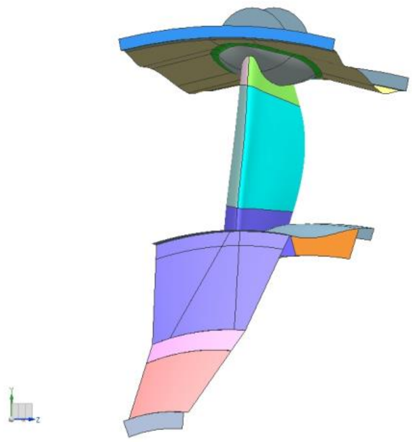


*Figure 4.6 One of the 14 struts*

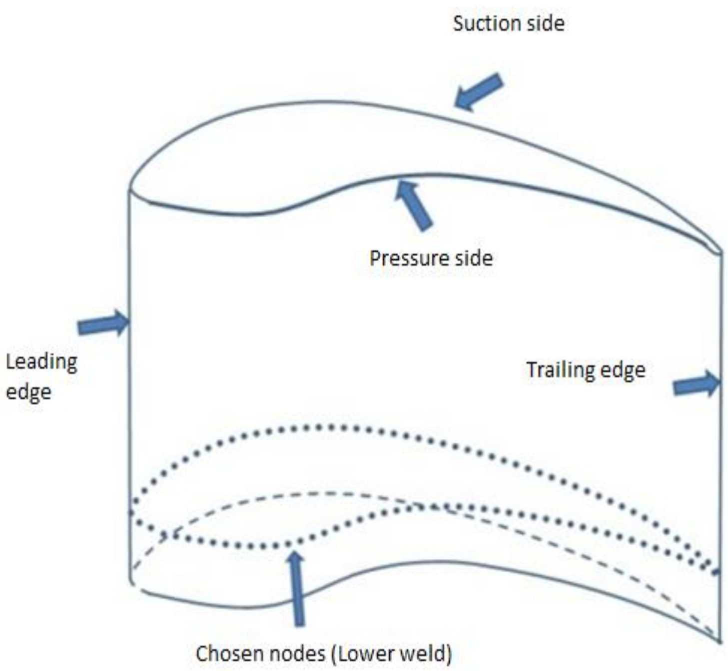



*Figure 4.7. Lower weld edge on one of the 14 struts.*

## 4.5 Simulation setup

The simulations were set up in ANSYS and thermal boundary conditions were applied. ANSYS was set to simulate transient effects of the thermal loads, thus the program creates more time steps within the given flight cycle. The model was a predefined shell model with given mesh size and was given as input to the thermal and structural analyses by GKN. ANSYS input tables provided the thermal loads as described in chapter 3.5 Thermal analysis. The simulations were performed using only thermal loads and no mechanical forces were considered for the analysis. This was due to the request on investigating how thermal loads affected stress distributions in the TS. If consideration to mechanical forces had been taken there would have been interference from these forces within the results, which was undesirable. The result files from the thermal analysis were used as input for structural analysis.

Increased temperatures will result in thermal expansion of the struts in accordance to equation 2.5. In order to derive stress levels at different nodes from the result files. ANSYS-script for post processing was created. The program chooses a set of nodes along the lower suction side weld and range them from the leading edge node (LE) to the trailing edge node (TE).First principal stress for each node are written in a complete result file. Figure 4.8 shows the procedure of the Taguchi L9 simulations.

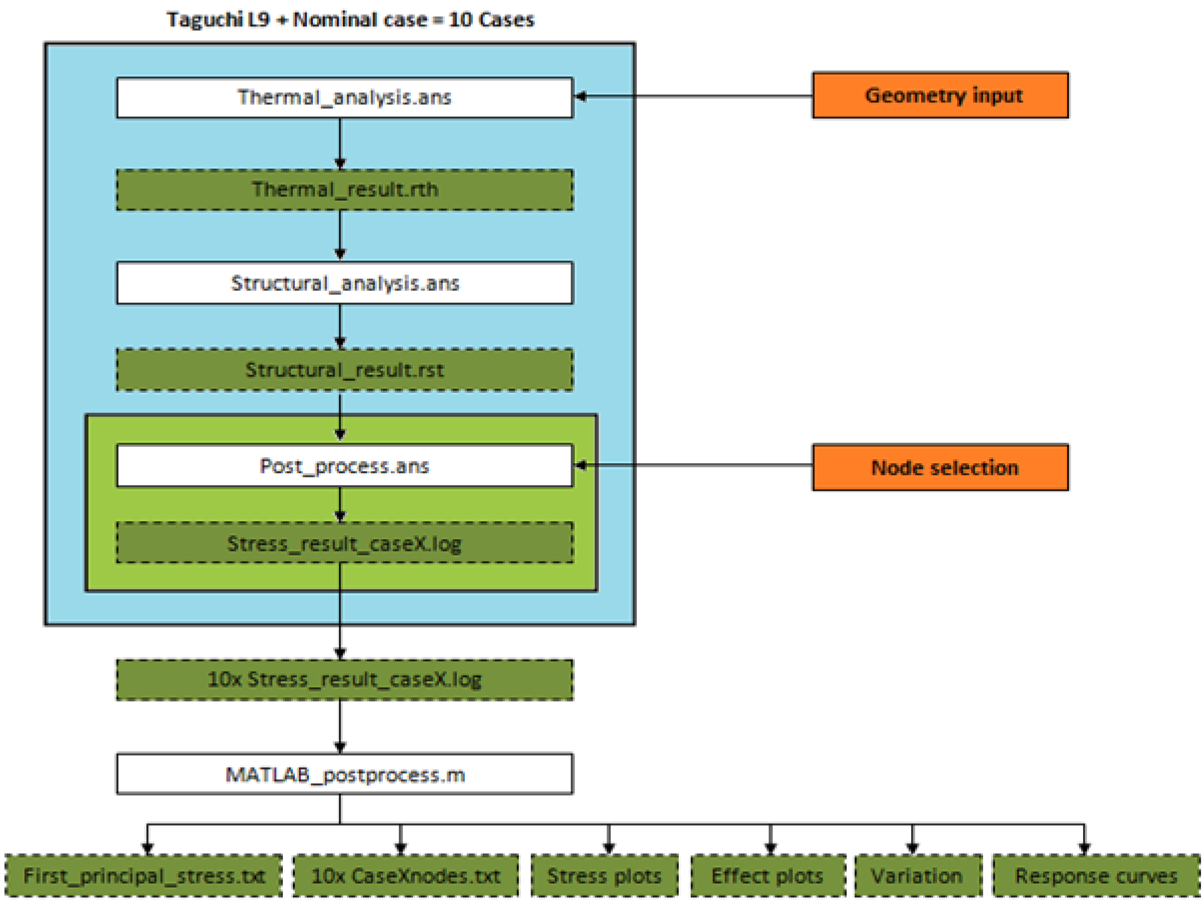



*Figure 4.8. Simulation procedure*

As it can be seen in figure 4.8 the post process script generates result files for each load case simulation performed in order of the Taguchi L9 table presented in chapter 4.3. All results files were then compiled using MATLAB in the MATLAB post process program file. This program was also used to generate response curves, plots of the results, effect plots and to calculate related statistics. To this purpose MINITAB and Excel were also used which will be described later in this report.

## 4.6 Temperature and stress distributions

This section will describe the findings from the thermal and structural analyses performed in ANSYS.

### 4.6.1 Thermal distribution

When performing a thermal analysis on the nominal model some interesting observations could be seen.  When the TS is exposed to hot gasses from the jet stream exiting the LPT, the struts rapidly begin to heat and expand in contrary to the outer shell and the hub that is still cold. Temperature also reaches its highest levels in stages of take-off and there are clear signs of cyclic temperature levels between idle phase and take-off. Figure 4.11 in the following section shows how this affect stress in one of the struts.

### 4.6.2 Stress distributions

In the initial stage a structural analysis was performed in order to get a better understanding of what processes that are taking place during the flight cycle. An example from the structural analysis is shown in figure 4.9 and shows stress distributions in the TS for one time step and for one of the thermal load cases. When the structure is heated the hub tends to rotate due to the thermal expansion taking place in all struts as can be seen in figure 4.9 below. The thermal expansion in the figure is heavily enhanced to show a tendency that eventually leads to buckling of the TS.

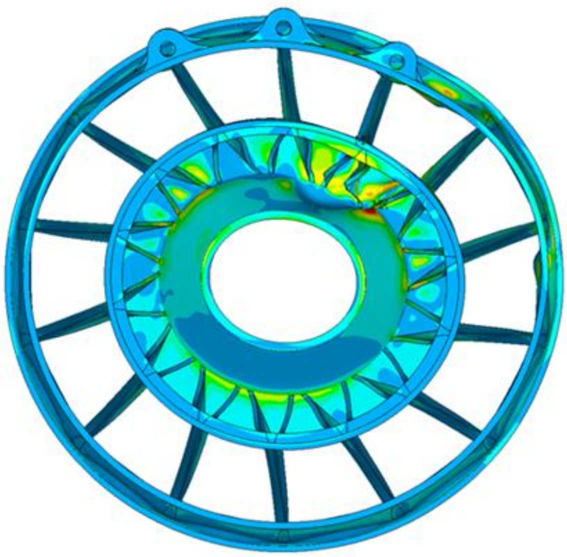


*Figure 4.9 Skewed Turbine Structure.*

ANSYS result files provided stress measurements for all direction as well as von Mises stresses in all nodes. Von Mises stresses provides a value without direction and is therefore not as important for crack initiation. First principal stress was chosen due to the fact that it has a direction and is always pointing in the direction of which the stress has the highest levels. It is therefore a favorable quantity to observe when evaluating risk for crack initiation.

## 4.7 Analysis of the results

Reflecting back on the basic procedure of DOE analysis presented in chapter 2.5 and especially analysis and interpretation of the results, several steps was used in order to examine the data.

The first steps of the result analysis were simply visual examination of the data. Scatter plots was used to interpret any linearity in the response vector. Surface plots were used to identify what section of the weld edge that had the highest stress levels during the whole flight cycle.

Figure 4.10 shows a surface plot of one of the 10 thermal load cases that was applied to the model. The Taguchi L9 experimental plan only consist of 9 runs, but an additional nominal load case with only zeroes was also included in the experimental plan. The X- and Y-axis shows node numbers along the suction side weld on the TS and flight cycle ranging from take-off to landing of the airplane. As it can be seen there are valleys with low stress amplitudes in the middle of weld edge. The highest stress amplitudes occur in the beginning of the flight cycle and can be found close to the leading edge or close to the trailing edge of the vane. This supports that in future post processing operations it is beneficial to only study the nodes furthest to the leading edge and to the trailing edge of the strut.

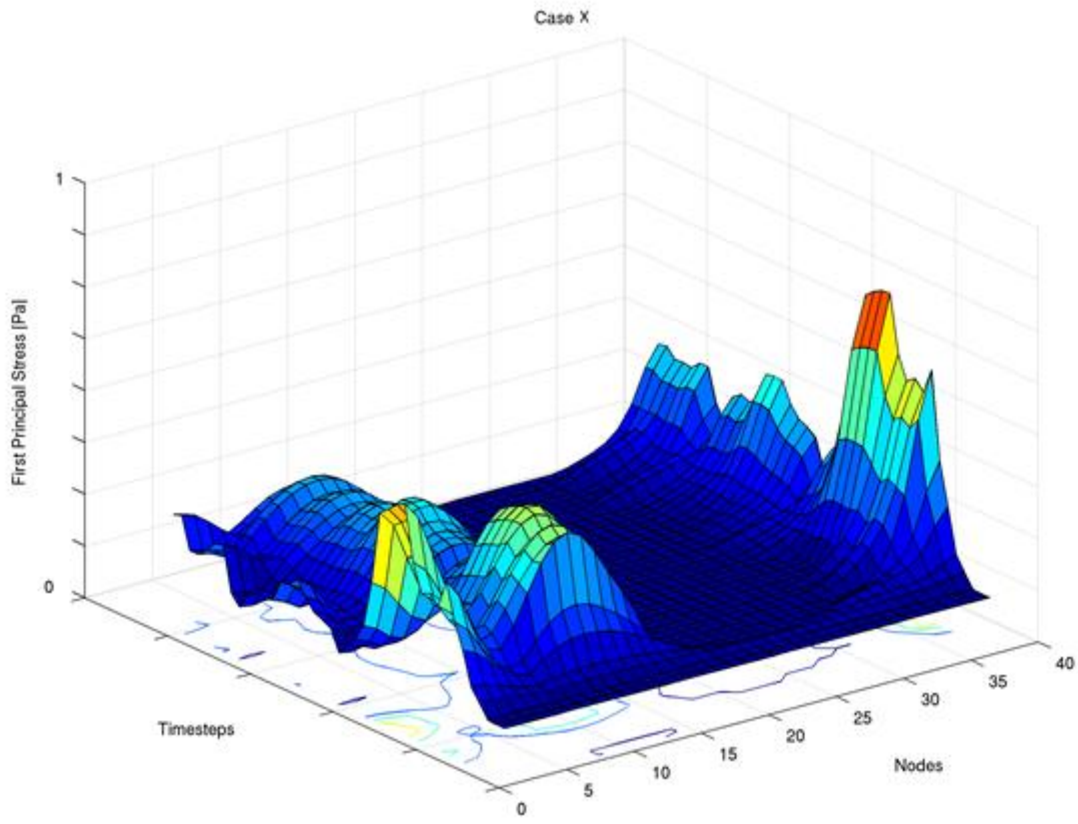



*Figure 4.10. First principal stress on suction side weld vs. flight cycle*

## 4.8 Scatter plots

Scatter plots were created using MATLAB and was used to derive conclusion regarding linear relationships between the response and the zone temperatures. The scatter plot in figure 4.11 shows that temperature differences in zones are related to a response, but the linearity is somewhat unclear. There are some tendencies that show a stress front for temperature increase in zone 2. Temperatures in zone 1 and 4 are clustered in the same way and shows that these zones share some relationship. In zone 3 it can be seen that there is a tendency to a non-linear relationship between zone 3 temperatures and the stress in the LE node. Zone1, 3 and 4 shows that there's an outlier clusters of stress measurements in all of these plots. In overall there was proof that variation in stress amplitudes depended on temperature levels, but there was still unknown what the underlying causes of variation were.

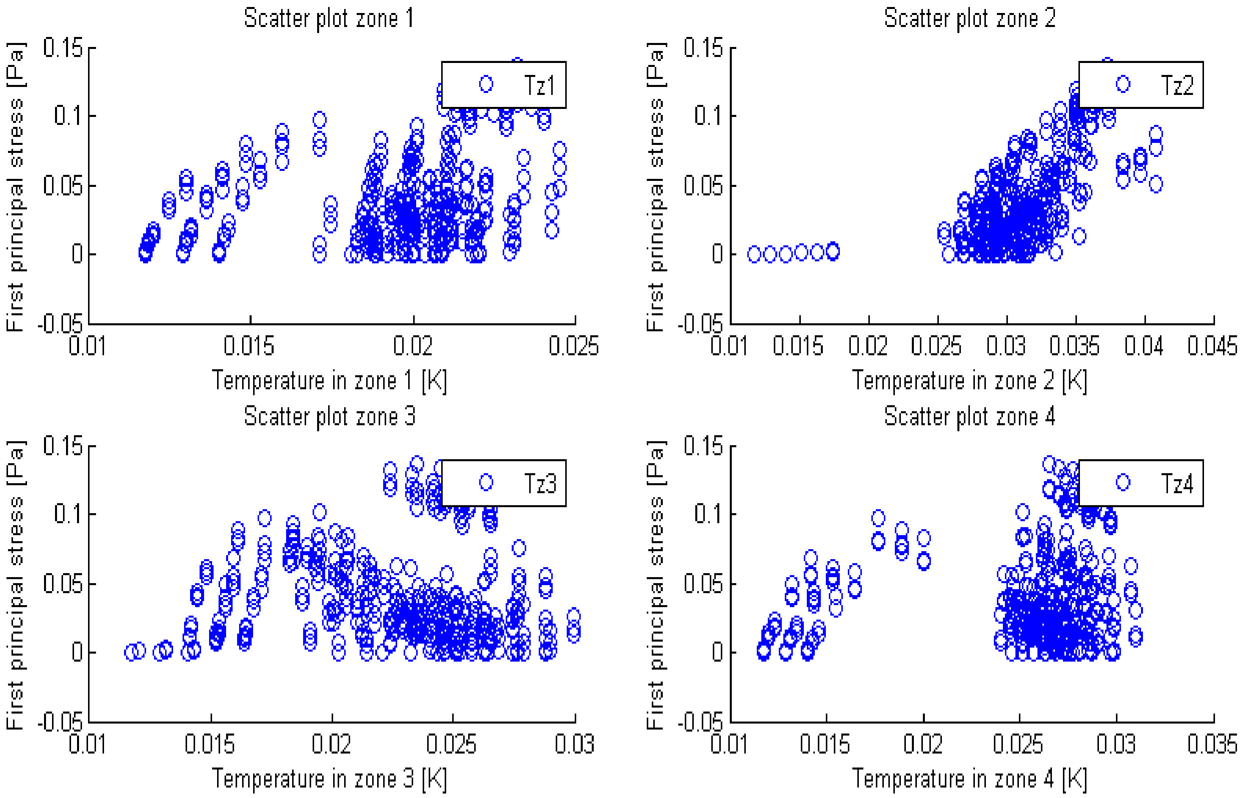



*Figure 4.11. Scatter plots showing stress as a function of temperature in the different thermal zones.*

## 4.9 Normal probability plot

A normal probability plot was used in order to assess if data was normally distributed. The plot of the standard deviations of our response in figure 4.12 showed that there was a spread in data at stresses over the 0.06 normalized stress level and that there was undesirable noise within the process. There are high fluctuations in stress levels over the whole flight cycle and the normal probability plot indicates that there are departures from the normality condition. A conclusion that can be drawn from the plot is that normal distribution is not a good model for these data measurements since there are nonlinear patterns in the results.

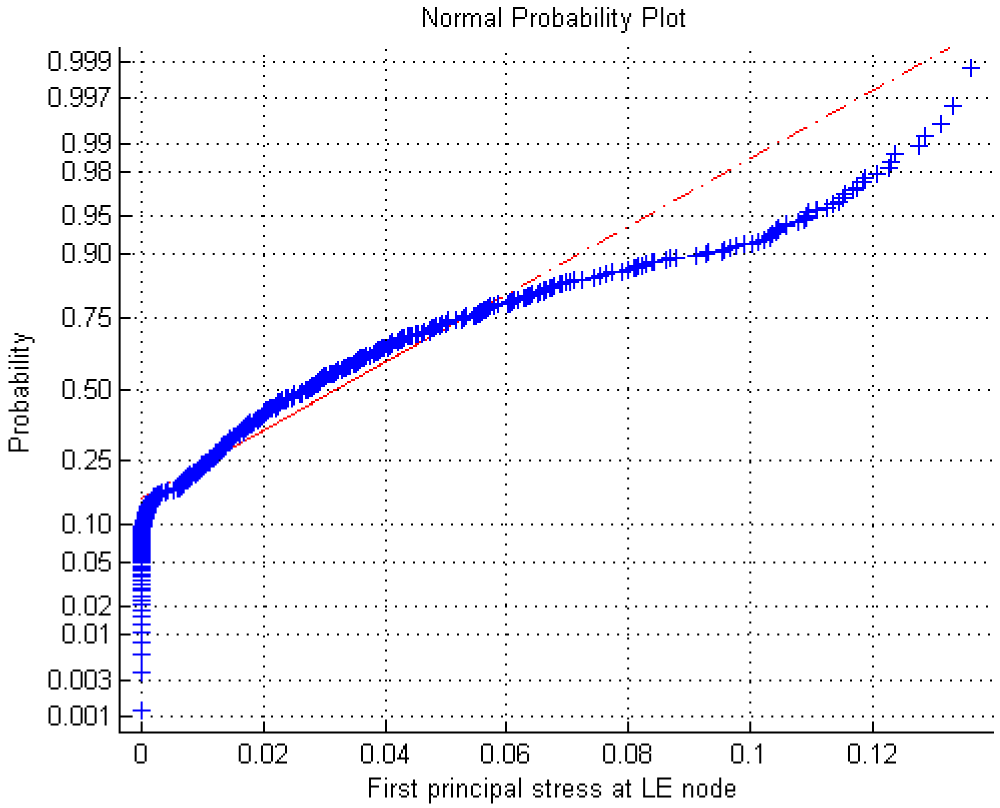



*Figure 4.12 Normal probability plot*

## 4.10 Temperature gradient plots

When having analyzed the data to this point it stood clear that it had to be some underlying process that caused variation in the response. A hypothesis was that there could in fact be linear relationships in temperature gradients. The key issue was then to identify temperature gradients that drive stress increase in the weld. Temperature gradients were calculated using MATLAB and plotted against stress levels in LE node. The result of this procedure can be seen in figure 4.13. Temperature gradient between zone 1 and zone 2 shows some linear relationships or a cluster of points emerging after a specific increase in delta T. Temperature gradients between zone 1 and zone 2, zone 1 and zone 3, zone1 and zone 4 as well as zone 3 and zone 4 show no sign of linear relationship. Temperature gradients between zone 2 and zone 3 shows a linear relationship with increasing stress levels in relation to temperature, but there are still clustered outliers departing from the rest of the results. A nonlinear relationship can be seen for temperature gradients between zone 2 and zone 4.

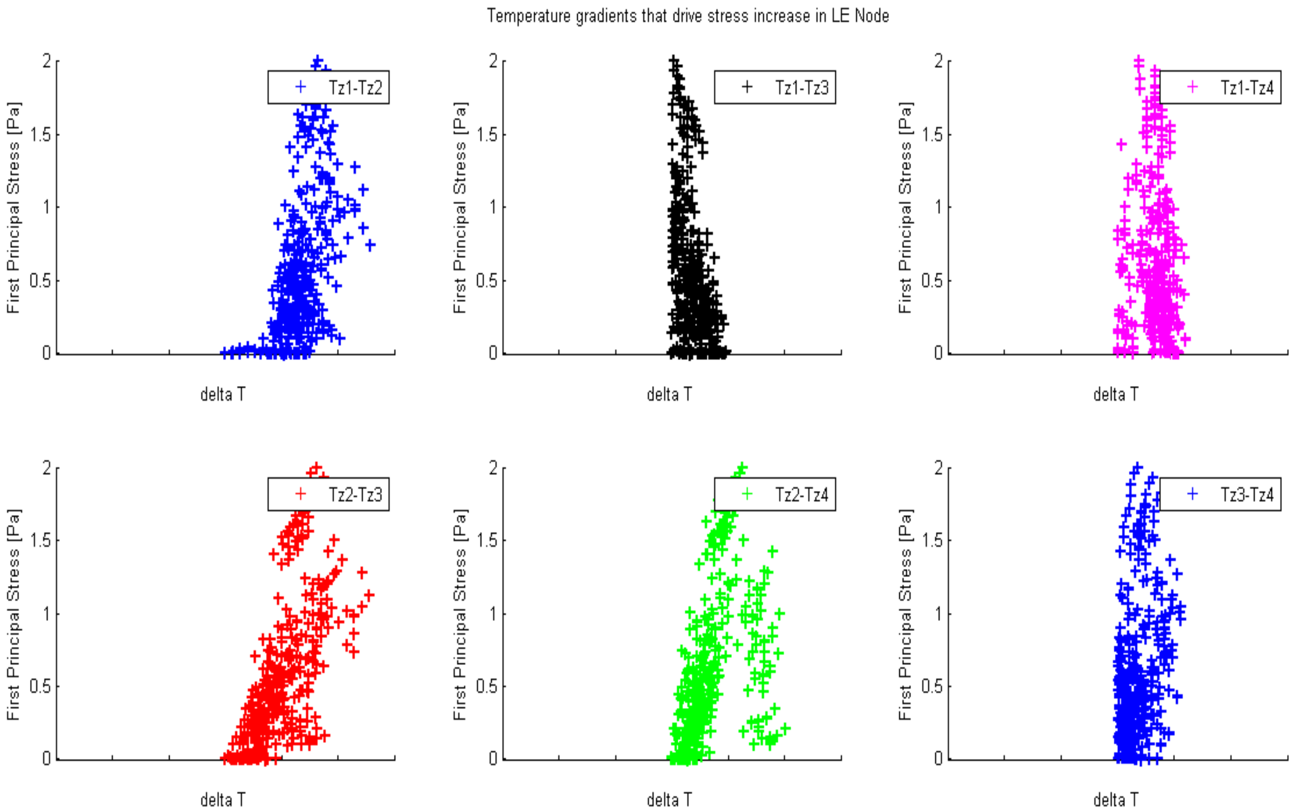



*Figure 4.13. Temperature gradients between thermal zones.*

Conclusions that could be drawn from these results were that there was no clear evidence of specific temperature gradients causing stress increase in LE node. There is also a large variation in the response which indicates that there are other factors causing stress increase in the node. At this point another hypothesis was that there are transient effects within the flight cycle causing "lag" in the stress amplitudes. There is a transient effect between air temperature and actual stress increase emerging from thermal expansion in the TS. It was therefore decided to analyze steady state points within the flight cycle that have a fully developed flow and the transient effects have been overcome.

To account for transient effects occurring along the flight cycle, MATLAB was used to reduce the flight cycle to study steady state points. Figure 4.14 shows temperature gradient plots for reduced flight cycle and doesn't show any signs of linearity. The data points are randomly distributed within the intervals of temperature difference. It is therefore difficult to develop any kind of model that fits to these data points.

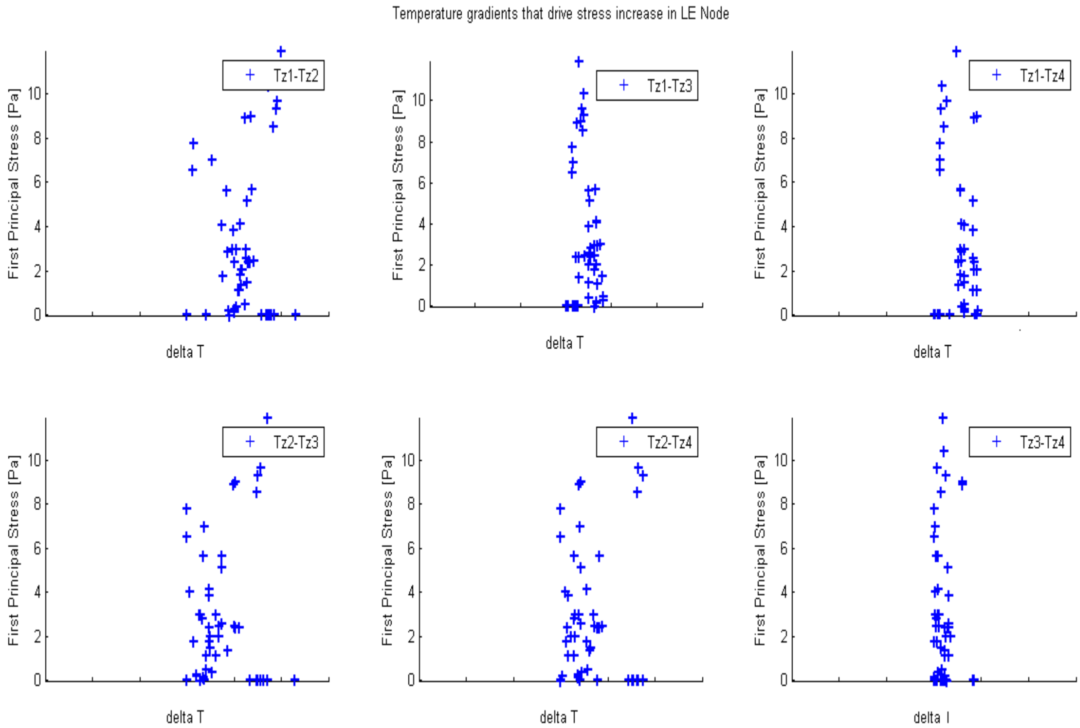



*Figure 4.14. Temperature gradients between thermal zones for reduced flight cycle*

## 4.11 Main effects plots

To gain a further in depth understanding of the data a main effect plot was created in MINITAB in order to identify the most important factor in the experiment. The main effect function calculates the shift in average response as the experiment moves from minus to plus in the experimental plan. The factor with the largest shift is the most important factor and the one with the least are the least important. (NIST, 2012c). The main effects from the different thermal zones are shown in figure 4.15. As it can be seen zone 1 and zone 2 has the most important effect to stress increase when increasing temperature. Zone 4 has the largest negative effect thus an increase in temperature in this zone causes decreasing stress levels. The factor with the least important effect is zone 3 which also has a negative effect on stress amplitudes in the LE node.

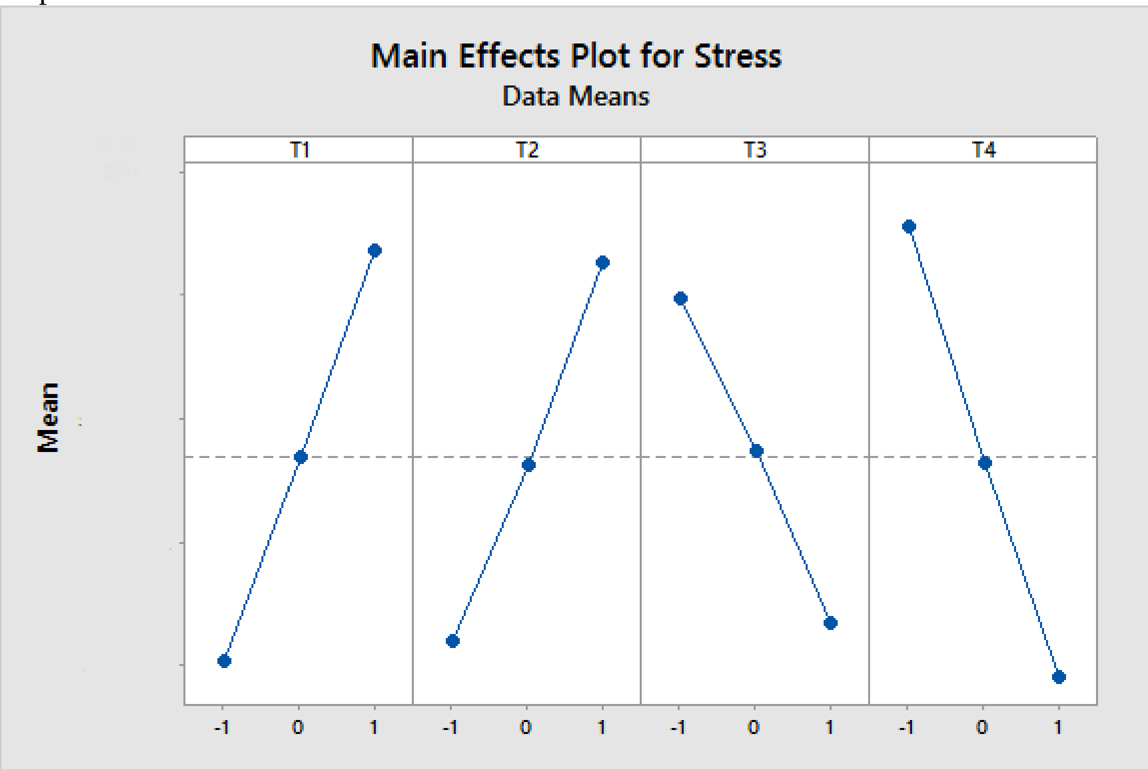



*Figure 4.15. Main effect plots corresponding LE for the different thermal zones*

In close relation to the main effect plot there is the interaction plot which shows the effect of interactions between each thermal zone as the experimental plan moves from minus one to plus one. The interaction plot includes both interaction effects between two factors as well as main effects. Figure 4.16 shows how mean stress levels change depending on the interaction between the thermal zones. Interactions with steep lines show that the factor is more important than for factors with flat lines. The interaction plot provided good understanding on how the thermal zones interact as the temperature levels change. For example it can be seen that for an interaction between zone 1 and zone 2 (T1 & T2), an increase in temperature in the gas canal (T2) will increase the stress in LE node if keeping the nacelle temperature at nominal temperature or at a 28K increase. On the other hand the stress will decrease if the nacelle temperature is lowered. The interaction plots may seem easy to interpret but confounding between factors and interactions needs to be taken into consideration in order to avoid deceptive information regarding the process.

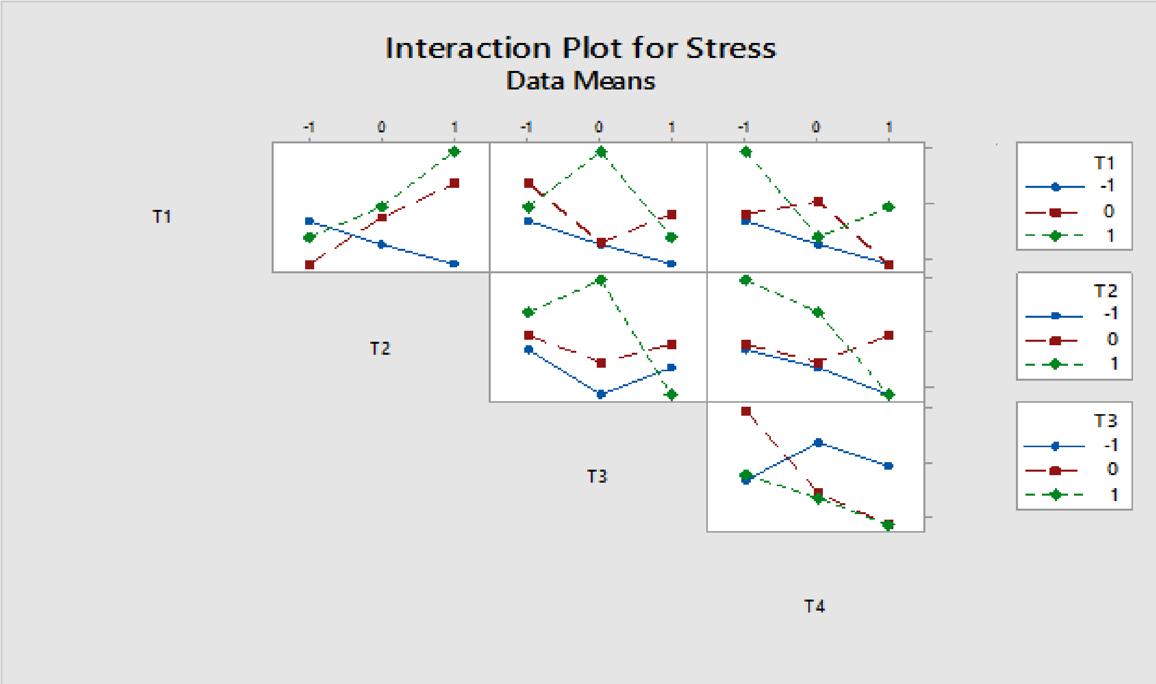


*Figure 4.16. Interaction plot corresponding LE for the different thermal zones*

After having investigated the LE node it was of interest to evaluate if the same relationship could be seen for the trailing edge node. A main effect plot considering the stress at the TE node shows a significant effect for the zones T2 and T4, as seen in figure 4.17. T2 has a positive influence on the response and T4 has similar effect but negative. The effects of T1 and T3 are now less significant. A specific defined limit for what is considered a contributing effect is represented as the P-value described in chapter 4.14.1.

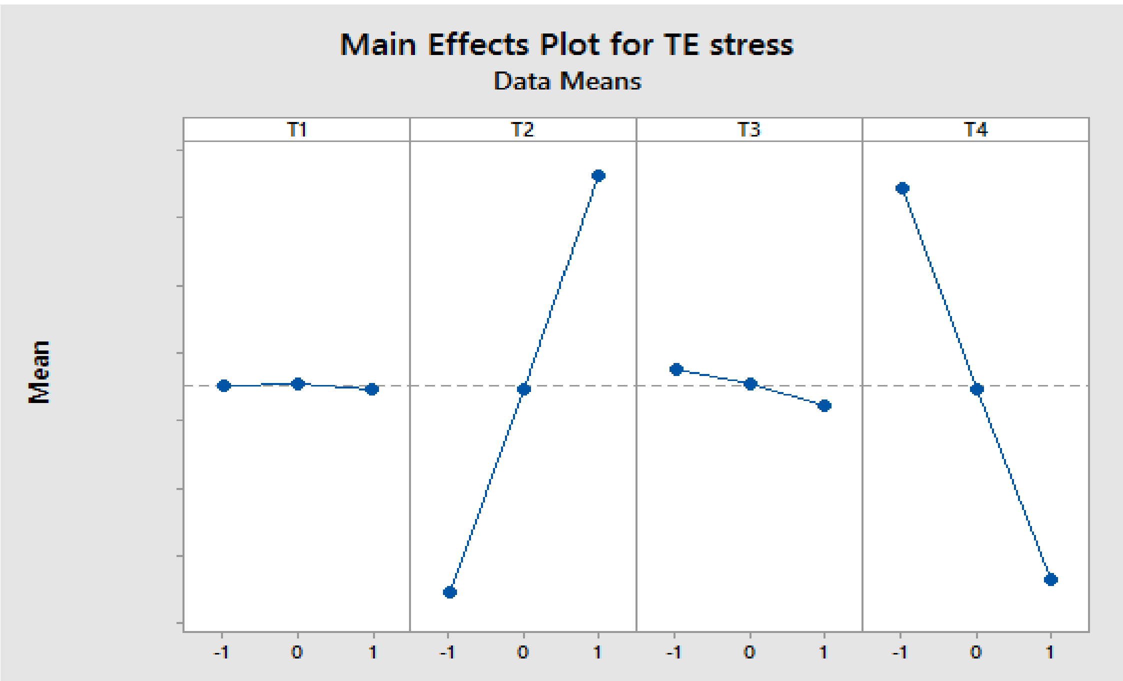



*Figure 4.17. Main effect plots corresponding LE for the different thermal zones*

The interaction plot shows how a temperature difference in any of the thermal zones affect the mean stress values derived from the TE node. For example, It can be seen in figure 4.18 that the stress are decreasing when T2 is held constant for all levels, when T4 the temperature in zone 4 are increased.

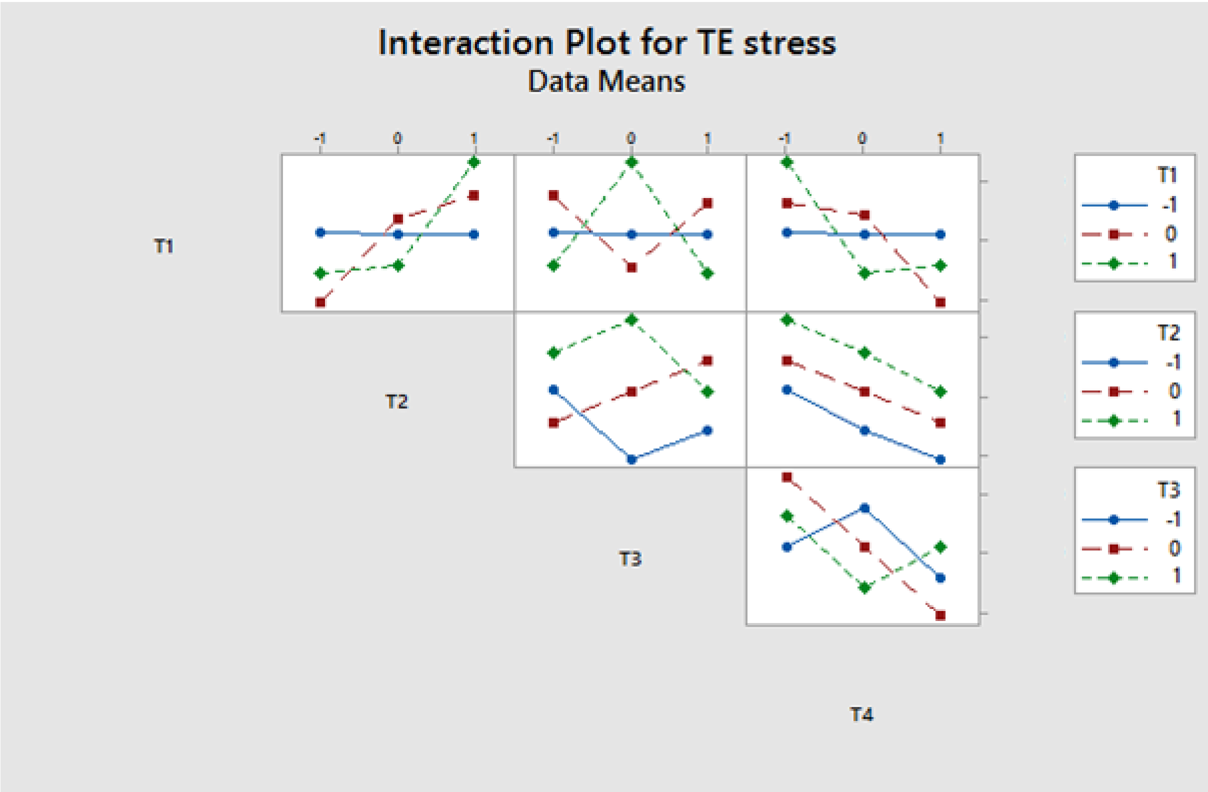



*Figure 4.18. Interaction plot corresponding TE for the different thermal zones*

By help of these graphs it is easier to interpret how a specific temperature gradient will change stress levels in the TS. By only looking at the temperature gradient plots it is difficult to understand that the stress levels in each plot are a combined result emerging from temperature gradients between all zones. Distinguishing the power of certain temperature gradients are therefore easier by help of interaction plots.

## 4.12 Effect of thermal zones

In order to get a greater understanding of how the main effects (eq. 2.6) change during the whole flight cycle, effect plots were created using MATLAB. The effect of all thermal zones was calculated for all previously mentioned nodes, load cases and for the whole flight cycle according to the method described in chapter 3. Figure 4.19 shows how the effect varies at node 1 along the suction side weld of the mount strut during the whole flight cycle. Referring to the different zones mentioned in section 4.1, Zone 1 and Zone 2 has large positive impact on first principal stress in node 1. Zone 3 and 4 on the other hand provides a large negative effect. The effect curves for this geometry have been calculated for all nodes on the suction side weld. In a later stage it will be possible to analyze any predefined weld or edge that is of interest for the analysist.

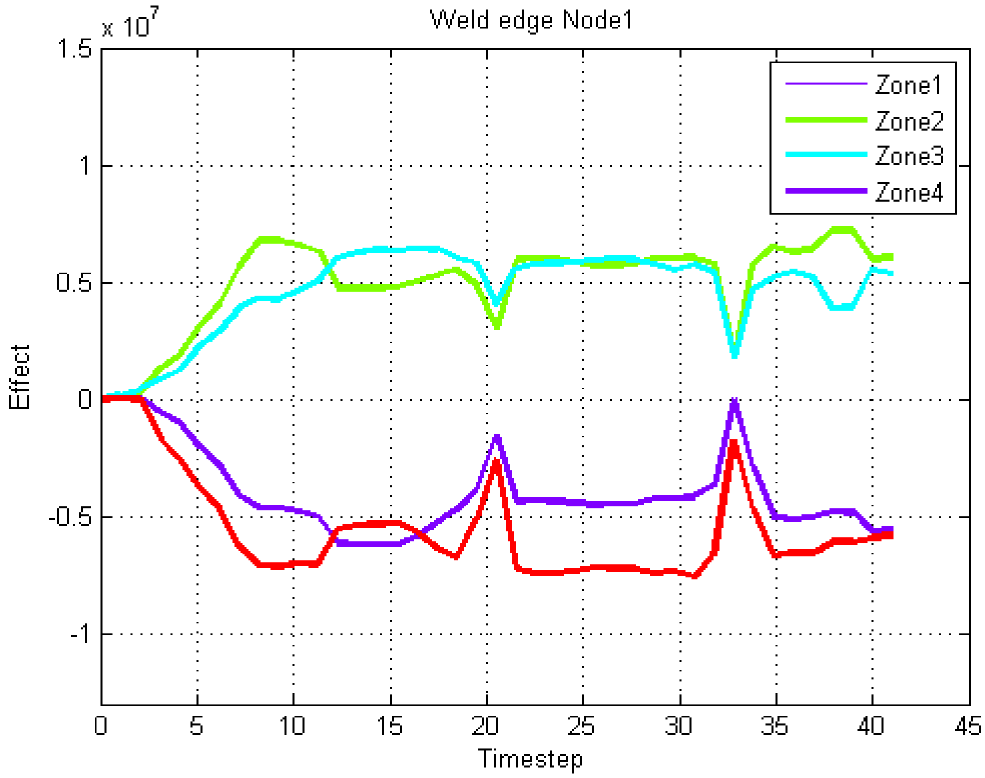



*Figure 4.19. Main Effect levels at the LE node.*

## 4.13 Correlation

The correlation coefficients between the thermal zones and the stress in the leading edge node were calculated in MINITAB using eq.  2.15. Table 4.1 shows the correlation coefficient for the different zones. The correlation coefficient only tells to what range two variables tend to move together and thus it can be seen that there is a strong correlation for increasing stress when increasing the temperature in the gas canal. The least correlation exists between stress and temperature levels in the bleed air cavity of zone 3.

*Table 4.1. Correlation table*

**Correlation: T1; T2; T3; T4; Stress**

| | T1 | T2 | T3 | T4 |
|---|---|---|---|---|
| T2 | 0,709 | | | |
| T3 | 0,866 | 0,614 | | |
| T4 | 0,921 | 0,715 | 0,851 | |
| Stress | 0,385 | 0,618 | 0,078 | 0,249 |

## 4.14 Response surface methodology

When having evaluated the data without seeing any clear relationship the next step was to develop a theoretical model of the data. This section will regard the creation of a theoretical model that is evaluated through statistical tools in order to create a response function that matches simulation results. The theoretical models were used to create response surfaces that could be used for optimization and interpretation of stress amplitudes under certain load conditions.

The built in statistical toolbox package of the mathematical software MATLAB was used do derive the coefficients of the approximation models and especially to derive response surface fits. The functions *fitlm* and *anova* were used to in a fast and effective way evaluate different approximation models, by providing the useful statistics mentioned in chapter 2.12. The regression models that were evaluated were linear, interaction, quadratic and a pure quadratic model. The multiple linear regression models are given by eq. (2.7) through (2.10).

### 4.14.1 Analysis of Variance

An analysis of variance was conducted in order to interpret what thermal zones that gave the largest statistical effect over the whole experiment. The ANOVA analysis uses statistical calculations presented in chapter 2.9. Results from the analysis derived from the LE node are shown in the table 4.2.

*Table 4.2*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 1.6874e+16 | 1 | 1.6874e+16 | 7.2969 | 0.0071976 |
| x2 | 1.4406e+16 | 1 | 1.4406e+16 | 6.2294 | 0.012961 |
| x3 | 1.0586e+16 | 1 | 1.0586e+16 | 4.5777 | 0.032987 |
| x4 | 2.0609e+16 | 1 | 2.0609e+16 | 8.9115 | 0.0030056 |
| Error | 9.3659e+17 | 405 | 2.3126e+15 | | |

From these results it could be derived that zone 3 has the least significant effect on the stress in the LE node. Most significant is zone 4, the hub cavity.

The analysis was also performed on the trailing edge node shown below in table 4.3

*Table 4.3*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 4.1862e+12 | 1 | 4.1862e+12 | 0.00097499 | 0.97511 |
| x2 | 5.788e+16 | 1 | 5.788e+16 | 13.48 | 0.00027335 |
| x3 | 4.0686e+14 | 1 | 4.0686e+14 | 0.09476 | 0.75837 |
| x4 | 5.1452e+16 | 1 | 5.1452e+16 | 11.983 | 0.00059388 |
| Error | 1.7389e+18 | 405 | 4.2936e+15 | | |

The results showed that the significance of zone 2 and zone 4 had increased while significance of zone 1 and zone 3 had reduced. The analysis showed that zone 2 (gas canal) had the greatest effect on stress in all nodes along the weld edge. Significance of zone 1 (nacelle) and zone 3 (bleed air cavity) tends to reduce when analysing nodes closer to the trailing edge.

### 4.14.2 Linear model

A linear regression model was first evaluated for fit against the simulation data. The model was written as according to eq. 2.8 and was calculated for the LE node. The ANOVA analysis showed the following relationship.

*Table 4.4*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 1.8804e+17 | 1 | 1.8804e+17 | 268.4 | 1.2023e-46 |
| x2 | 2.8796e+17 | 1 | 2.8796e+17 | 411.04 | 1.3627e-63 |
| x3 | 1.7298e+17 | 1 | 1.7298e+17 | 246.9 | 8.8005e-44 |
| x4 | 6.8319e+16 | 1 | 6.8319e+16 | 97.518 | 9.466e-21 |
| Error | 2.8373e+17 | 405 | 7.0058e+14 | | |

MATLAB function regstats calculated $R^2$, Adjusted $R^2$ , F-statistics and P-value and these are shown in the table 4.12 . The ANOVA analysis, table 4.4 showed that for a linear regression model all terms become significant and should therefore be included in the model.

The linear model and instead using the TE node as response vector will show the following relationship.

*Table 4.5*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 1.487e+17 | 1 | 1.487e+17 | 108.54 | 1.1227e-22 |
| x2 | 5.3009e+17 | 1 | 5.3009e+17 | 386.91 | 6.0228e-61 |
| x3 | 1.2375e+17 | 1 | 1.2375e+17 | 90.323 | 1.8094e-19 |
| x4 | 3.2522e+16 | 1 | 3.2522e+16 | 23.737 | 1.5871e-06 |
| Error | 5.5487e+17 | 405 | 1.3701e+15 | | |

As same as before it could be seen from table 4.5 that all terms in the model were significant. MATLAB function regstats calculated $R^2$, Adjusted $R^2$ , F-statistics and P-value and these are shown in the table 4.12 in the same manner as before.

### 4.14.3 Interaction model

An interaction model according to eq. 4.2 for LE node will produce the following ANOVA table.

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum\sum_{i<j} \beta_{ij} x_i x_j + \mathcal{E} \quad (4.2)$$

*Table 4.6*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 8.6449e+16 | 1 | 8.6449e+16 | 206.24 | 5.397e-38 |
| x2 | 2.357e+17 | 1 | 2.357e+17 | 562.32 | 3.3801e-78 |
| x3 | 9.1351e+16 | 1 | 9.1351e+16 | 217.94 | 1.1654e-39 |
| x4 | 5.0756e+16 | 1 | 5.0756e+16 | 121.09 | 8.9297e-25 |
| x1:x2 | 1.5078e+16 | 1 | 1.5078e+16 | 35.973 | 4.4884e-09 |
| x1:x3 | 6.0501e+15 | 1 | 6.0501e+15 | 14.434 | 0.00016782 |
| x1:x4 | 1.1204e+15 | 1 | 1.1204e+15 | 2.673 | 0.10285 |
| x2:x3 | 1.5676e+16 | 1 | 1.5676e+16 | 37.398 | 2.2977e-09 |
| x2:x4 | 1.8287e+16 | 1 | 1.8287e+16 | 43.627 | 1.2717e-10 |
| x3:x4 | 2.0423e+15 | 1 | 2.0423e+15 | 4.8724 | 0.027859 |
| Error | 1.6725e+17 | 399 | 4.1916e+14 | | |

MATLAB function regstats calculated $R^2$, Adjusted $R^2$ , F-statistics and P-value was calculated in the same way as before and these are shown in table 4.12. The ANOVA analysis for this model shows that the interaction term between zone 1 and zone 4 is greater than the 5 % significance level and it could therefore be neglected for further evaluation if the model was to be chosen at a later stage.

For the TE node the following relationship is present.

*Table 4.7*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 3.6702e+16 | 1 | 3.6702e+16 | 50.656 | 5.1475e-12 |
| x2 | 4.7152e+17 | 1 | 4.7152e+17 | 650.79 | 7.7372e-86 |
| x3 | 2.8025e+16 | 1 | 2.8025e+16 | 38.68 | 1.2615e-09 |
| x4 | 5.701e+16 | 1 | 5.701e+16 | 78.684 | 2.4713e-17 |
| x1:x2 | 2.011e+16 | 1 | 2.011e+16 | 27.756 | 2.2565e-07 |
| x1:x3 | 1.4148e+15 | 1 | 1.4148e+15 | 1.9527 | 0.16307 |
| x1:x4 | 6.5401e+15 | 1 | 6.5401e+15 | 9.0266 | 0.0028287 |
| x2:x3 | 2.6212e+16 | 1 | 2.6212e+16 | 36.178 | 4.0756e-09 |
| x2:x4 | 1.4411e+16 | 1 | 1.4411e+16 | 19.889 | 1.0683e-05 |
| x3:x4 | 5.7552e+15 | 1 | 5.7552e+15 | 7.9432 | 0.0050671 |
| Error | 2.8909e+17 | 399 | 7.2454e+14 | | |

MATLAB function regstats calculated $R^2$, Adjusted $R^2$ , F-statistics and P-value was calculated in the same way as before and these are shown in table 4.12. Table 4.7 show that relationships are switched when evaluating the trailing edge node. The interaction term between zone 1 and zone 3 becomes insignificant and could therefore be neglected at a later stage.

### 4.14.4 Purequadratic model

A purequadratic regression model was also chosen for evaluation. The model is the same equation 2.9 but without interaction terms, and thus only include the quadratic terms. The ANOVA analysis, table 4.8, shows the following relationship for the different thermal zones and stress in the LE node.

*Table 4.8*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 1.18093e+17 | 1 | 1.18093e+17 | 232.229 | 1.08027e-41 |
| x2 | 2.19456e+17 | 1 | 2.19456e+17 | 431.559 | 1.34588e-65 |
| x3 | 9.88822e+16 | 1 | 9.88822e+16 | 194.451 | 2.59821e-36 |
| x4 | 3.94765e+16 | 1 | 3.94765e+16 | 77.630 | 3.80246e-17 |
| x1^2 | 1.94543e+16 | 1 | 1.94543e+16 | 38.256 | 1.53051e-09 |
| x2^2 | 5.33518e+16 | 1 | 5.33518e+16 | 104.916 | 5.01636e-22 |
| x3^2 | 8.86640e+15 | 1 | 8.86640e+15 | 17.435 | 3.64828e-05 |
| x4^2 | 1.03436e+16 | 1 | 1.03436e+16 | 20.340 | 8.52217e-06 |
| Error | 2.03916e+17 | 401 | 508519187481845 | | |

MATLAB function regstats calculated $R^2$, Adjusted $R^2$ , F-statistics and P-value was calculated in the same way as before and these are shown in the table 4.12. All the terms in this model proved to be significant since all terms are below the 5% threshold.

The analysis of the TE node showed the following relationship.

*Table 4.9*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 6.5722e+16 | 1 | 6.5722e+16 | 72.727 | 3.0785e-16 |
| x2 | 4.4541e+17 | 1 | 4.4541e+17 | 492.88 | 8.4648e-72 |
| x3 | 3.4044e+16 | 1 | 3.4044e+16 | 37.672 | 2.0126e-09 |
| x4 | 4.0305e+16 | 1 | 4.0305e+16 | 44.6 | 8.0836e-11 |
| x1^2 | 8.4603e+15 | 1 | 8.4603e+15 | 9.3619 | 0.0023642 |
| x2^2 | 1.5098e+17 | 1 | 1.5098e+17 | 167.07 | 3.436e-32 |
| x3^2 | 2.8105e+15 | 1 | 2.8105e+15 | 3.11 | 0.078573 |
| x4^2 | 5.0251e+16 | 1 | 5.0251e+16 | 55.607 | 5.5222e-13 |
| Error | 3.6238e+17 | 401 | 9.0369e+14 | | |

MATLAB function regstats calculated $R^2$, Adjusted $R^2$ , F-statistics and P-value was calculated in the same way as before and these are shown in table 4.12 for later evaluation. The ANOVA analysis, table 4.12, shows that the quadratic term of zone 3 is insignificant and could therefore be neglected at a later stage.

### 4.14.5 Quadratic model

A quadratic regression model was also implemented in accordance to eq. 2.9. The analysis shows that all the interaction has significant effect since the p-value is less than the significance level of 5 %.

*Table 4.10*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 9.2496e+16 | 1 | 9.2496e+16 | 241.59 | 7.5398e-43 |
| x2 | 2.2838e+17 | 1 | 2.2838e+17 | 596.5 | 5.9254e-81 |
| x3 | 9.2333e+16 | 1 | 9.2333e+16 | 241.17 | 8.606e-43 |
| x4 | 4.8265e+16 | 1 | 4.8265e+16 | 126.07 | 1.4118e-25 |
| x1:x2 | 4.2532e+15 | 1 | 4.2532e+15 | 11.109 | 0.00094021 |
| x1:x3 | 2.4873e+15 | 1 | 2.4873e+15 | 6.4967 | 0.011185 |
| x1:x4 | 2.7738e+14 | 1 | 2.7738e+14 | 0.72449 | 0.39519 |
| x2:x3 | 1.0561e+16 | 1 | 1.0561e+16 | 27.584 | 2.4642e-07 |
| x2:x4 | 1.2227e+15 | 1 | 1.2227e+15 | 3.1936 | 0.074694 |
| x3:x4 | 4.9735e+15 | 1 | 4.9735e+15 | 12.99 | 0.00035311 |
| x1^2 | 1.9939e+14 | 1 | 1.9939e+14 | 0.5208 | 0.47093 |
| x2^2 | 2.6226e+14 | 1 | 2.6226e+14 | 0.68501 | 0.40837 |
| x3^2 | 8.6875e+15 | 1 | 8.6875e+15 | 22.691 | 2.6766e-06 |
| x4^2 | 6.3212e+15 | 1 | 6.3212e+15 | 16.511 | 5.8383e-05 |
| Error | 1.5123e+17 | 395 | 3.8286e+14 | | |

MATLAB function regstats calculated $R^2$, Adjusted $R^2$ , F-statistics and P-value was calculated in the same way as before and these are shown in table 4.12. The ANOVA table shows terms with underlined insignificant values. These could be neglected at a later stage when full model evaluation has been made.

For the TE node the ANOVA table for the quadratic model looks as following.

*Table 4.11*

| | SumSq | DF | MeanSq | F | pValue |
|---|---|---|---|---|---|
| x1 | 4.0925e+16 | 1 | 4.0925e+16 | 59.11 | 1.196e-13 |
| x2 | 4.4178e+17 | 1 | 4.4178e+17 | 638.09 | 1.7487e-84 |
| x3 | 3.1812e+16 | 1 | 3.1812e+16 | 45.947 | 4.4364e-11 |
| x4 | 5.4737e+16 | 1 | 5.4737e+16 | 79.06 | 2.1768e-17 |
| x1:x2 | 5.4058e+15 | 1 | 5.4058e+15 | 7.8079 | 0.0054553 |
| x1:x3 | 3.1301e+15 | 1 | 3.1301e+15 | 4.521 | 0.0341 |
| x1:x4 | 1.1036e+15 | 1 | 1.1036e+15 | 1.594 | 0.2075 |
| x2:x3 | 1.8752e+16 | 1 | 1.8752e+16 | 27.085 | 3.1369e-07 |
| x2:x4 | 1.6204e+14 | 1 | 1.6204e+14 | 0.23404 | 0.62881 |
| x3:x4 | 2.8604e+15 | 1 | 2.8604e+15 | 4.1314 | 0.042762 |
| x1^2 | 1.6139e+15 | 1 | 1.6139e+15 | 2.331 | 0.12762 |
| x2^2 | 7.1464e+14 | 1 | 7.1464e+14 | 1.0322 | 0.31027 |
| x3^2 | 3.8911e+15 | 1 | 3.8911e+15 | 5.6201 | 0.018235 |
| x4^2 | 6.3408e+15 | 1 | 6.3408e+15 | 9.1584 | 0.0026379 |
| Error | 2.7348e+17 | 395 | 6.9235e+14 | | |

MATLAB function regstats calculated $R^2$, Adjusted $R^2$ , F-statistics and P-value was calculated in the same way as before and these are shown in table 4.12. The ANOVA analysis, table 4.11, shows terms with underlined insignificant values. These could be neglected at a later stage when full model evaluation has been made. The interaction term between zone 1 and zone 4, zone 2 and zone 4, squared term of zone 1 and zone 2 all proved to be insignificant.

*Table 4.12*

| ***Summary*** | **R-square** | **Adjusted R-square** | **F-statistics** | **P-value** |
|---|---|---|---|---|
| **Linear model (LE)** | 0.716000212648254 | 0.713195276476879 | 2.5526e+02 | 2.8921e-109 |
| **Linear model (TE)** | 0.699850211128788 | 0.696885768769566 | 2.3608e+02 | 2.0674e-104 |
| **Interaction model (LE)** | 0.832596603228597 | 0.828401029374677 | 1.9844e+02 | 4.6304e-148 |
| **Interaction model (TE)** | 0.843620359329262 | 0.839701070089394 | 2.1524e+02 | 6.1138e-154 |
| **Purequadratic model (LE)** | 0.795892838160611 | 0.791820874832144 | 1.9545e+02 | 2.9643e-133 |
| **Purequadratic model (TE)** | 0.803977001184381 | 0.800066317916240 | 2.0558e+02 | 9.2465e-137 |
| **Quadratic model (LE)** | 0.848629044854405 | 0.843263998342916 | 1.5817e+02 | 3.9337e-152 |
| **Quadratic model (TE)** | 0.852065989304816 | 0.846822758546000 | 1.6251e+02 | 4.3182e-154 |

The regression models show that we have the least fit in a linear model and the best fit in a quadratic model with $R^2 = 0.8521$ as seen in table 4.12. The ANOVA table also shows that there are some insignificant factors that can be discarded from the model since they show a probability greater than the 5% significance level. An interaction factor between zone 1 and zone 4, between zone 2 and zone 4, as well as quadratic factors of zone 1 and zone 2, can be discarded since the p-value is greater than 0,05. Before reducing the model it was important to know if the residuals are normally distributed around zero.

It was seen in table 4.11 that some terms in the model could be neglected due to their insignificance. The reduced LE node model then became

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \beta_5 x_1 x_2 + \beta_6 x_1 x_3 + \beta_7 x_2 x_3 + \beta_8 x_3 x_4 + \beta_9 x_3^2 + \beta_{10} x_4^2$$

The reduced model for the TE node was also the same since same terms was reduced from the model. A new ANOVA analysis was conducted which showed that the terms for zone 1 and zone 4, as well as interaction term for zone 1 and zone 3 were insignificant for the model and thus could be neglected.

*Table 4.13*

| ***Summary*** | **R-square** | **Adjusted R-square** | **F-statistics** | **P-value** |
|---|---|---|---|---|
| **1st reduced model(LE)** | 0.8395 | 0.8354 | 208.6213 | 1.1425e-151 |
| **1st reduced model(TE)** | 0.8491 | 0.8454 | 224.5952 | 4.7907e-157 |
| **2nd reduced model(LE)** | 0.8368 | 0.8346 | 344.9113 | 2.5905e-155 |
| **2nd reduced model(TE)** | 0.8408 | 0.8389 | 421.8732 | 7.2820e-158 |

The results from these analyses, table 4.13, revealed that reduced models did produce better F-values with greater significance. The critical F-value for a model of 6 terms and 410 observations was found in tabulated data for a 5 % significance level giving
$F_{(0.05,5,405)} = 2.236$. The F-value for the second reduced model of both LE and TE node are much larger than the critical value thus the null hypothesis can be rejected. A lack of fit test was used to evaluate the model. The residuals from a fitted model tell a lot about how well the model performs. If the residuals behave randomly without any curvature it is a sign that the

model fits the data well (NIST, 2012d). If not it might be beneficial to transform the response data as mentioned in the DoE analysis flowchart. A residual plot, shown in figure 4.20, for run order revealed the following relationship for the 2nd reduced model of the LE node.

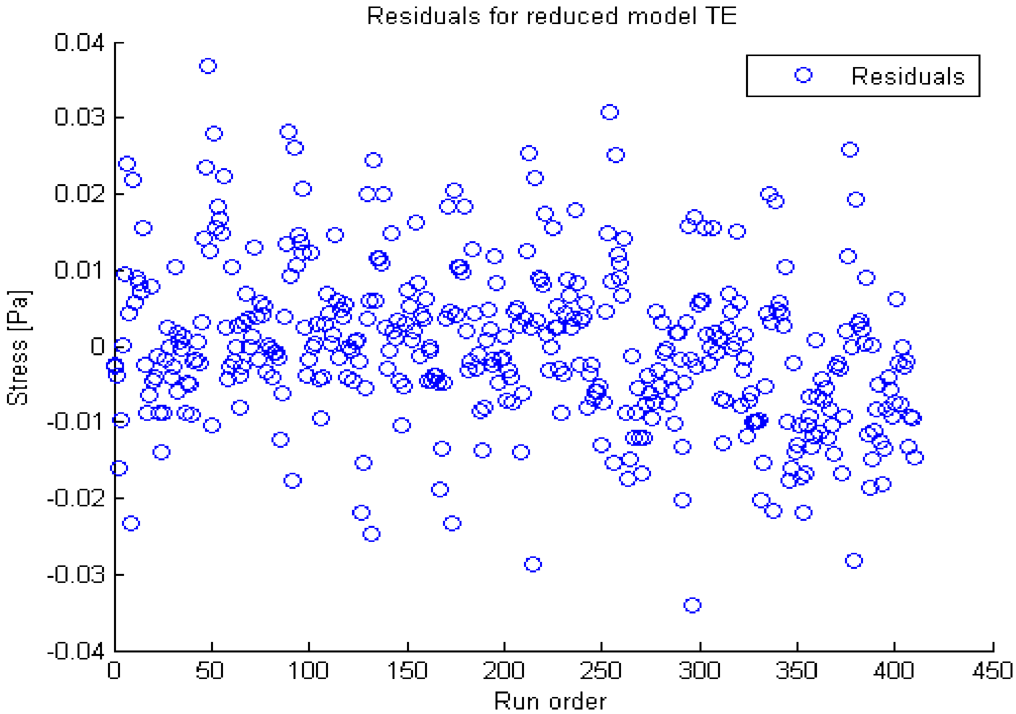



*Figure 4.20. Residual plot for second reduced model of LE node*

As seen the residuals behave randomly and thus the model behaves well. The relationship was also shared with the model of the TE node. A normal distribution plot of the errors can also help to determine if the model behaves in a stable way. Deviations from the normality will in that case show that the model is unstable as also mentioned by (NIST, 2012e). A normal probability check of the residuals was conducted and the result is seen in figure 4.21 which show that the residuals are placed along the normality, and thus it is a stable model.

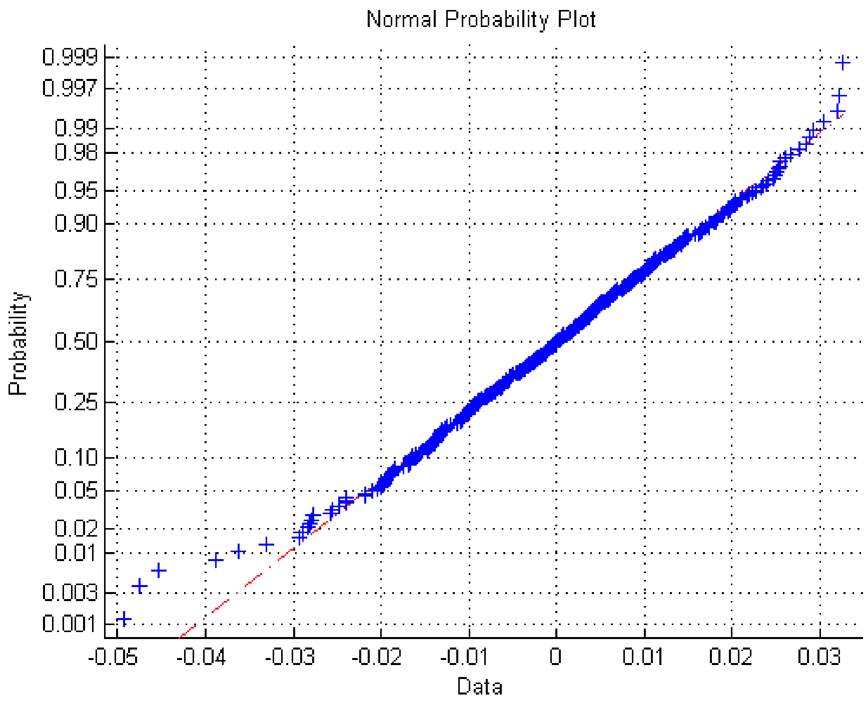



*Figure 4.21. Normal probability plot for second reduced model of LE node*

For the TE node there was deviating residuals at larger stresses, as seen in figure 4.22, thus indicating that the model behaves somewhat more unstable at the higher stress levels.

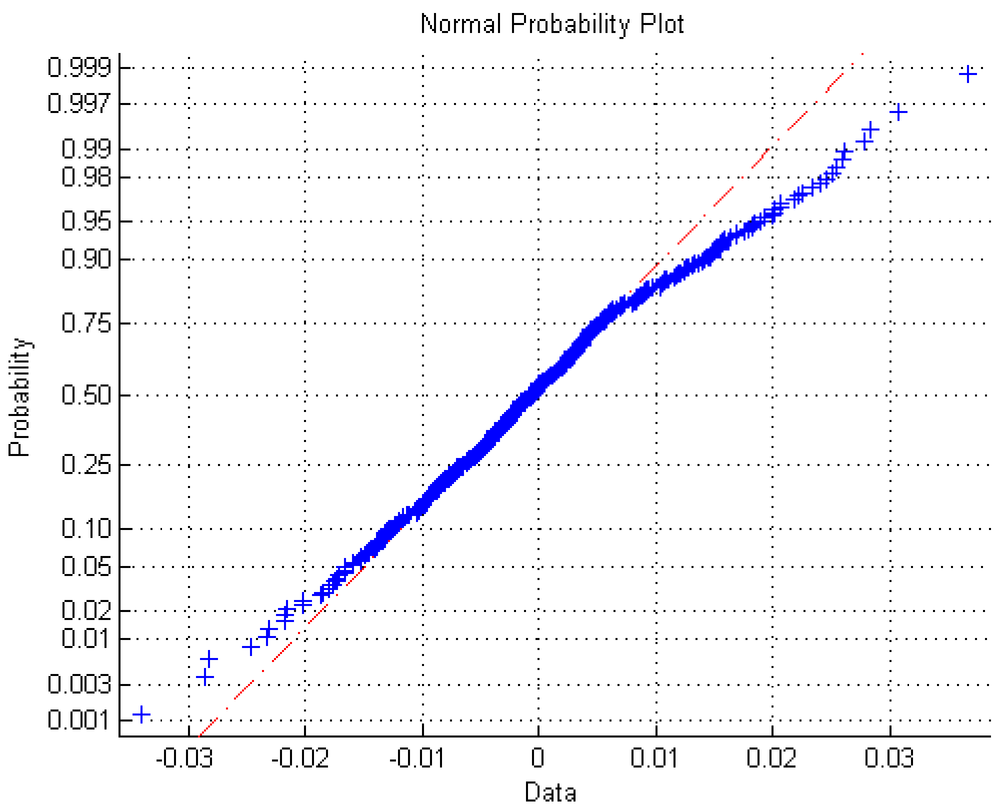



*Figure 4.22. Normal probability plot for second reduced model of TE node*

The stresses in the LE node could be described with the following approximation model, but there is still lack of fit in the model since the $R^2$ value is 0.8368

$$y_{LE} = 10^7(\,7.5553 - 0.0416\mathrm{x}_2 + 0.0001\mathrm{x}_1\mathrm{x}_2 + 0.0001\mathrm{x}_2\mathrm{x}_3 + 0.0002\mathrm{x}_3\mathrm{x}_4 - 0.0002\mathrm{x}_3^2 - 0.0001\mathrm{x}_4^2)$$

The stresses in the TE node could in the same way be described with the following approximation model, but there is still lack of fit in the model since the $R^2$ value is 0.8408

$$y_{TE} = 10^5(\,-9.051x_2 + 8.8817x_4 + 0.0057x_1x_2 + 0.0263x_2x_3 - 0.0196x_3^2 - 0.0111x_4^2)$$

## 4.15 Box-Behnken setup

The initial screening design with a modified Taguchi L9 gave the contributing main effects from the four tested factors. Next step was to investigate higher order interactions and establish a response surface model to evaluate the relationship between the input parameters (temperature) and output response (stress). For this the Taguchi design was inadequate because of the saturated design, where not enough experiments are generated to evaluate those interactions. The approximation models were also inadequate as response functions and thus more experimental runs were needed in order to determine more appropriate models. With few experimental runs it was also difficult to determine what temperature differences that were important stress drivers. When looking at the main effect plots in figure 4.15 and 4.17 it can also be seen that the center points of each factor deviates from the average of the response, which indicate that there is some curvature in the system. Thus a response surface experimental plan is advisable as also seen in figure 2.15.

| Zone 1 | Zone 2 | Zone 3 | Zone 4 |
|---|---|---|---|
| 0 | 1 | -1 | 0 |
| -1 | -1 | 0 | 0 |
| 0 | 0 | 1 | -1 |
| 1 | -1 | 0 | 0 |
| -1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| -1 | 0 | 1 | 0 |
| 0 | 0 | -1 | -1 |
| 1 | 0 | -1 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | -1 |
| -1 | 0 | -1 | 0 |
| 0 | 0 | -1 | 1 |
| -1 | 0 | 0 | 1 |
| 0 | -1 | 0 | -1 |
| 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 |
| 0 | -1 | 0 | 1 |
| 0 | 1 | 0 | 1 |
| -1 | 0 | 0 | -1 |
| 0 | -1 | -1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | -1 |
| 0 | 0 | 0 | 0 |
| 0 | -1 | 1 | 0 |

*Figure 4.23. The proposed Box Behnken design matrix, with 3 center points, unblocked.*

In the same manner as before the analysis was now performed with 27 different thermal load cases. A main effect plot was studied for LE node using MINITAB. The results are shown in figure 4.24.

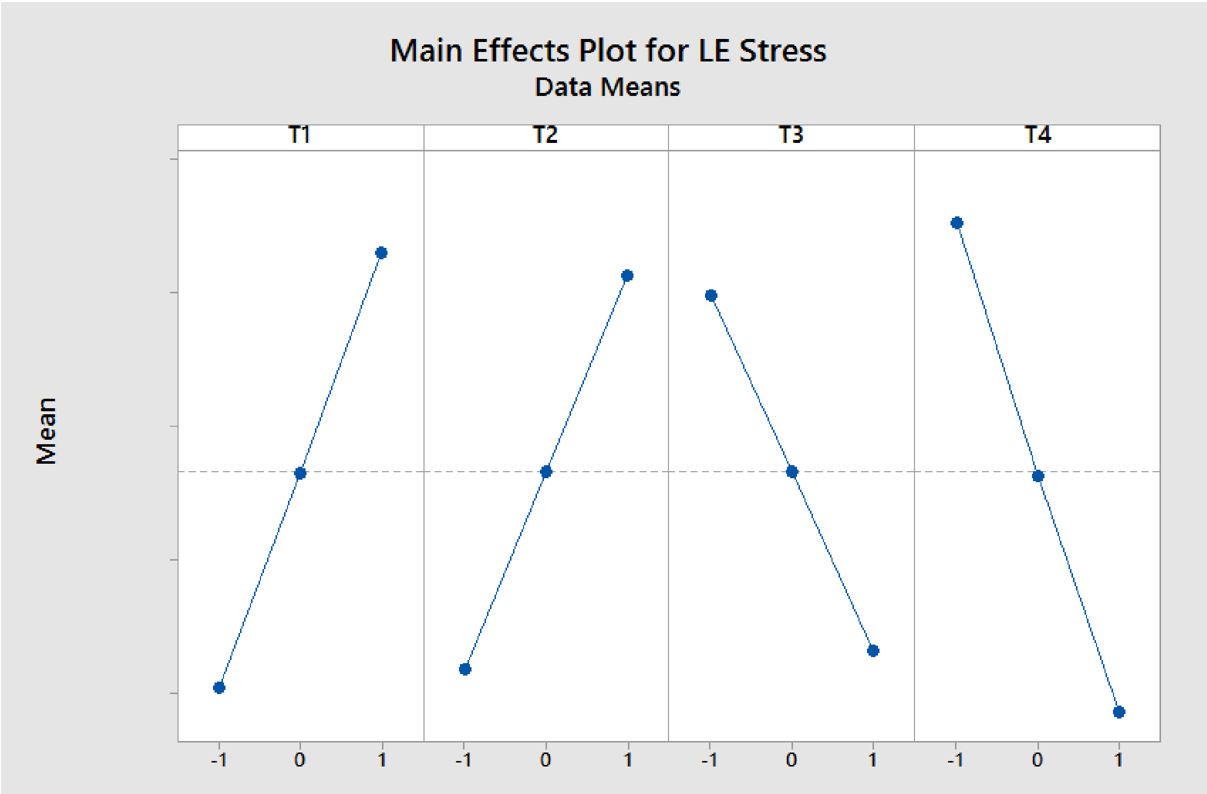



*Figure 4.24. Main effect plot from Box-Behnken analysis.*

The main effects were investigated in the same manner as described in earlier sections of this report. It could be seen that the largest contributing positive effects came from zone 1 and zone 2. The contributing negative effect came from zone 3 and zone 4. The next logical step was to look at the two-way interaction effects from an interaction plot, shown in figure 4.25. Analyzing how the thermal zones interact with each other showed that the lines within the interaction plot are parallel to each other. This was an indication that there are no interaction effects between the thermal zones.

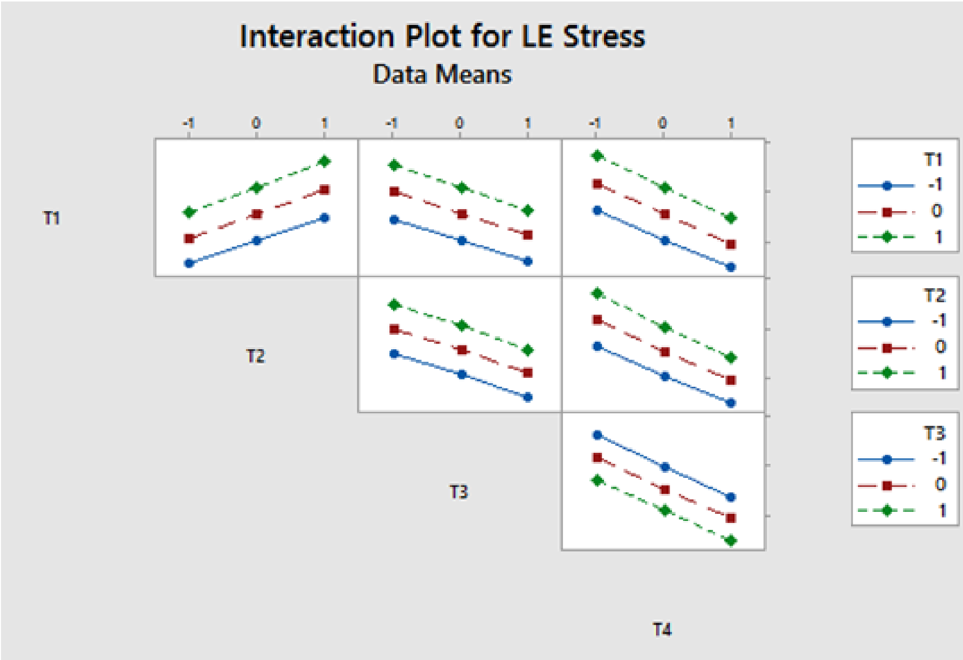



*Figure 4.25. Interaction plot from Box-Behnken analysis*

A quadratic model as described in eq. 2.9 was used as input for ANOVA analysis and stepwise regression as described in section 4.14. Insignificant terms within the model were removed due to their insignificance in the same manner as earlier stated. The results of the ANOVA analysis are summarized in table 4.14.

*Table 4.14*

| **Summary** | **R-square** | **Adjusted R-square** | **F-statistics** | **P-value** |
|---|---|---|---|---|
| **BB quadratic model(LE)** | 0.8588 | 0.8570 | 474.4818 | ~ 0 |
| **BB quadratic model(TE)** | 0.8611 | 0.8593 | 483.4674 | ~ 0 |
| **BB reduced model (LE)** | 0.8587 | 0.8571 | 553.8805 | ~ 0 |
| **BB reduced model (TE)** | 0.8609 | 0.8592 | 520.3571 | ~ 0 |

From this analysis it could be seen that the R-square and adjusted R-square values slightly increased and less terms were removed from the reduced model. The F-values increased compared to the second reduced model that was derived from simulation data from the Taguchi L9 analysis. Since the values did not get close enough to 1, this indicated that there were other underlying effects that also produced stress other than just temperature. The distribution of the residuals was also studied and verified that the model behaved well as also mentioned in section 4.15.5.

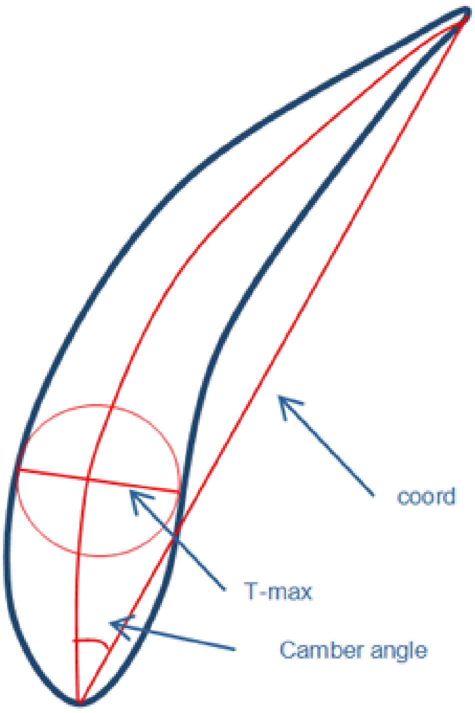
The stresses in the LE node derived from the data provided by Box Behnken could be described with an approximation model with 13 terms, but there is still lack of fit in the model since the $R^2$ value is 0.8587

The stresses in the TE node could in the same way be described with an approximation model with 14 terms, but there is still lack of fit in the model since the $R^2$ value is 0.8609.

## 4.16 Geometry variation

In order to identify what geometrical attributes that were insensitive regarding thermal variation, a geometry study was performed with the Taguchi L9 approach. The geometrical study was performed in the same sequence as before, but for all changes in geometry there was a Taguchi L9 thermal analysis as input.

The study only focused on six features that were changed in between models for a total set of 41 geometries. Additional to the 41 geometries there were 10 thermal simulations performed for a total of 410 simulations in ANSYS. The output file was written as a text file for easy post processing operations. The geometry input table can be seen in appendix V, but is not available in the public version of this report due to confidential reasons. Figure 4.26 shows what geometrical attributes that was varied in the geometrical DoE study.



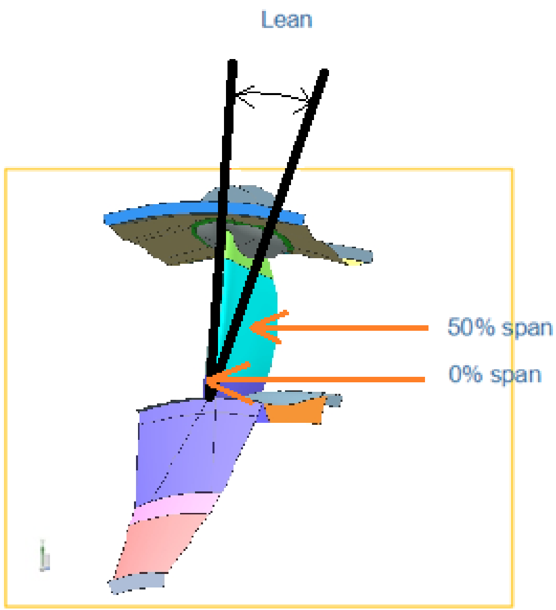




*Figure 4.26. Definition of geometrical factors*

The axial chord at 0% defines the length of the axial chord at the lower weld edge and 50% represents the length of the chord when positioned in the middle of the strut. The same relationship is present for the maximum thickness at 0 % and 50%. Lean angle of the strut and the camber angle are also geometrical factors that were considered in the study.

### 4.16.1 Interpolated nodes

The mesh of the CAD models varies between geometries, thus placing nodes at different positions along the weld for each simulation. Instead of writing all consecutive stresses for all nodes along the weld it was therefore determined that a fixed set of observation points should be written into the result file. This proves useful when different geometries are evaluated and thus the result file will always contain the set of interpolated observation points. The amount of observation points was for this analysis determined to be 11.

### 4.16.2 Determining sensitive parameters

The result file from ANSYS simulations was evaluated in MATLAB and Excel for determination of sensitive geometrical parameters. MATLAB constructed a complete table showing how geometrical and thermal factors varied as input for the analysis. The program then identified the max stress of the LE node and the TE node during the whole flight cycle. A neural network function, mentioned in chapter 2.6.1 was implemented in Mode Frontier to combine the two DoE experimental plans. The results are visualized later in this section. Mode Frontier also produces a Pareto chart and visualizes it in the following way. This type of chart shows quantitative values of the effects in relative terms, as well as cumulative percentage for all the effects. The Pareto chart shows how large effect a factor or combinations of factors have on the result. The results are presented as bars for each factor, thus the largest bar having the largest effect on the results.

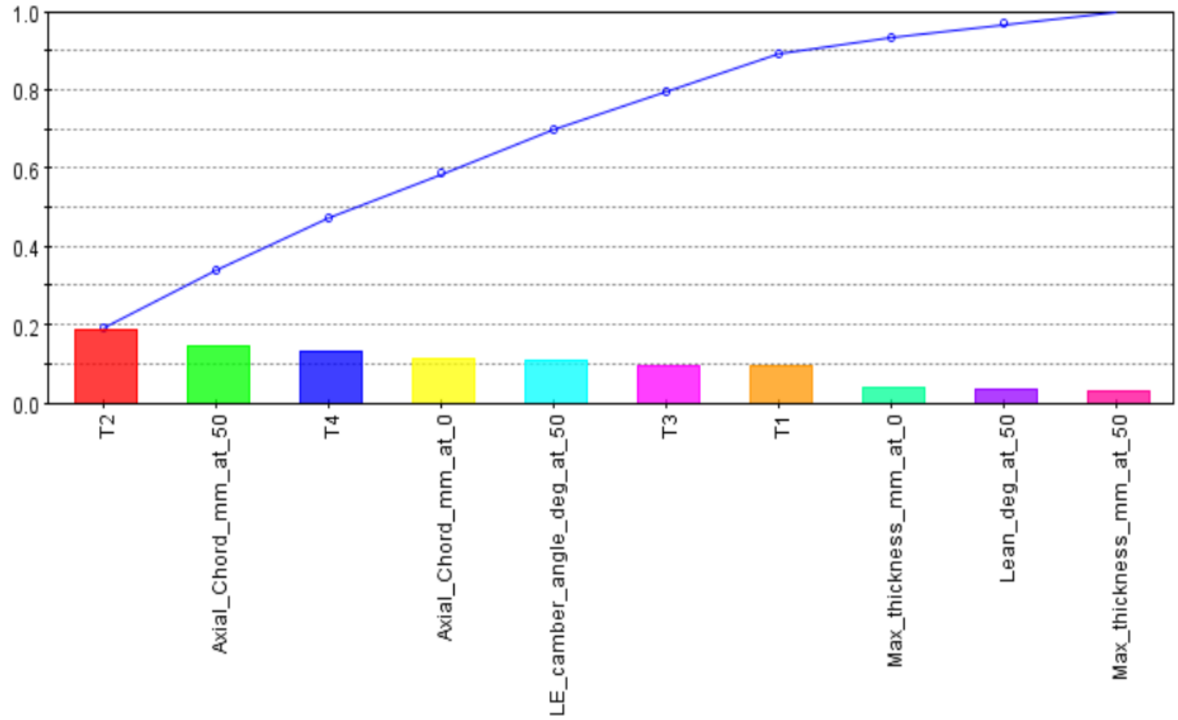



*Figure 4.27. Pareto Chart of main effects for all parameters tested.*

Figure 4.27 chart shows that the largest effect is produced from zone 2 and the thickness of the strut at 50 % have the least effect on the results. The line represents the cumulative percentage of the effects and thus increases as effects are added together. Before interpreting these results as the reality, consideration has to be taken to the creation of the neural network. Validation of the fit of the response function is important for further use of this methodology.

### 4.16.4 Robustness plots

By help of the response function developed by the neural network it was possible to plot the response as a function of the geometrical factors for each temperature level in the Taguchi L9 experimental plan. The plots were compiled in Excel and are shown in appendix III. There are 12 plots as total showing the 6 geometrical factors for the LE node and the TE node. Figure 4.28 shows how the stress in the LE node changes when increasing the leading edge camber angle. It can be seen that the LE camber angle is insensitive to thermal variation when lower than 60 degrees. Increasing the camber angle from this point will result in large stress differences between high and low temperature input. Thus it can be concluded that the LE camber angle is more robust or insensitive to thermal variation.

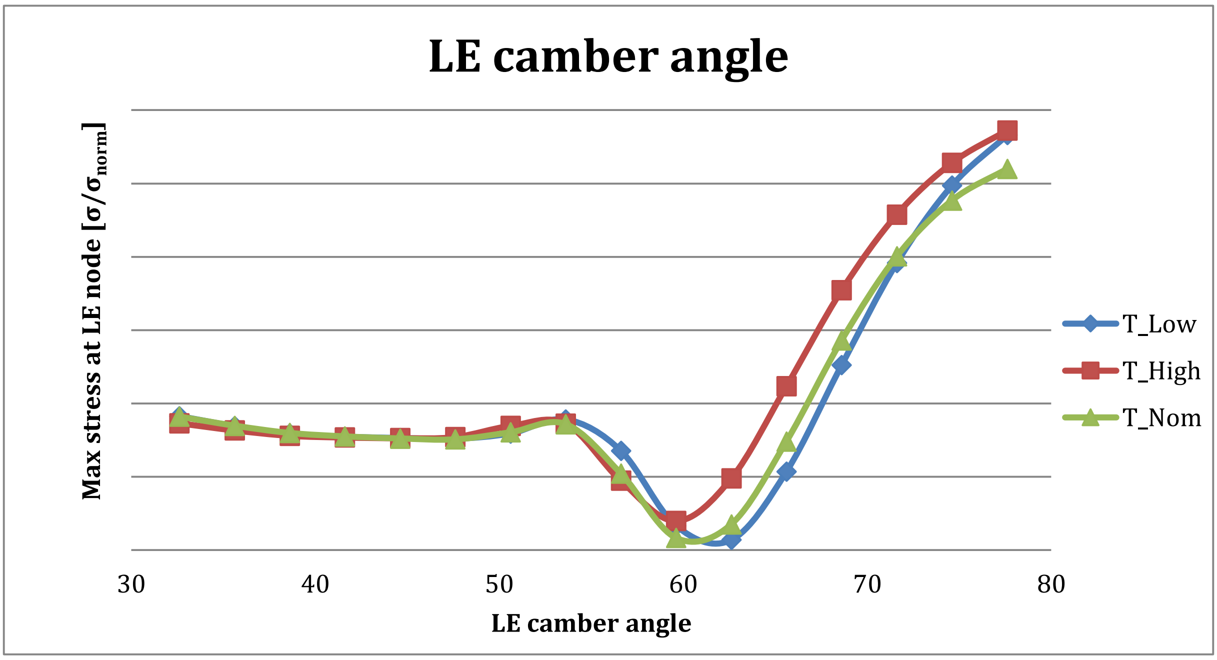



*Figure 4.28. Stress as a function of LE camber angle derived for the LE node*

For the trailing edge node, figure 4.29, it can be seen that the threshold for insensitive camber angle is slightly above 45 degrees which tells that other nodes needs to be investigated before conclusions regarding the complete weld can be stated.

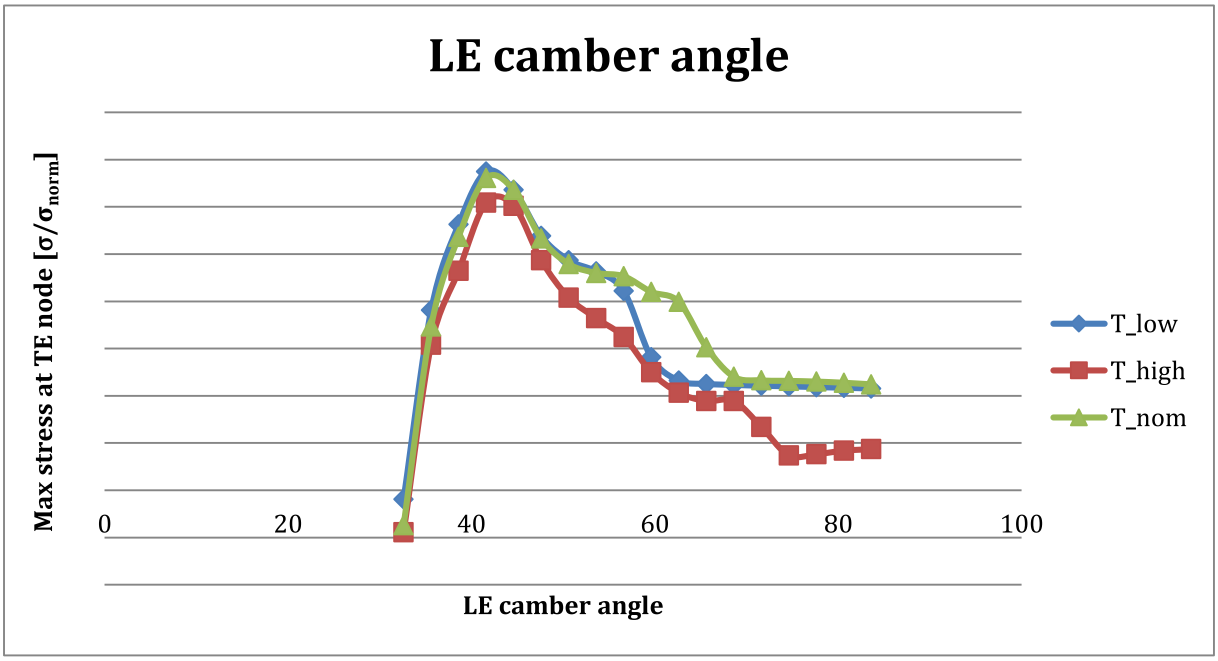



*Figure 4.29. Stress as a function of LE camber angle derived for the TE node*

The same methodology can be used to investigate the remaining geometrical factors shown in appendix III. A large span between low and high temperature reveals that the particular geometrical setting is sensitive to variation in temperature and therefore the topology needs to be defined with caution.

# 5. Discussion

This chapter will include a discussion based on how well the results in this work really answers the research questions stated in chapter 1. The chapter deals with the discussion about the meaning of the concluded results, and debates about how relevant the findings are in an engineering manner.

Consequences of limited analysis of the underlying intentions of the experiment often result in limited findings or even useless data. Therefore the intention of design of experiment needs to be evaluated early in the planning of the study, which was found not to have been made in an appropriate way in earlier studies. Usually the preparatory work before setting up your DoE is of greater importance than the actual matrix design. The preparation may include a description of the process in form of a flowchart and identifying potential sources of variation. When it comes to decision making regarding which experimental design to use, there are several factors to take into account. The extent of the design is a critical factor when limited resources in terms of time and cost are crucial. The other aspect of this is, what the cost is to choose the wrong, or more often, a too simple design poor of information. (NIST, 2013g)

The number of experiment runs required for a full factorial design is way too extensive in terms of resource utilization. Processor calculation time rises as more simulation runs needs to be conducted. The initial Taguchi design set up doesn't allow estimations of more than main effects from the tested factors. That means that the evaluated main effects may be confounded with two-way interactions. The early adoption of the Taguchi L9 design didn't raise the question about whether or not there were substantial interactions between the selected thermal zones. The four selected thermal zones all had a substantial effect, so therefore in hindsight a "screening oriented" design as the Taguchi wasn't the most feasible design. If the objective was to investigate *which* factors that actually are appropriate for further studies, i.e. the base for example, geometrical studies, a screening design would have a great meaning in identifying those factors with greatest main effects. If a screening is necessary to sort out a vital few important factors, with in mind that there may exist confounding effects, a Placket Burman design is the most economical one, where the setup of resolution III reveals the most important factor effects.

The Taguchi design doesn't generate enough runs required to properly investigate curvature and the set up a satisfying response function. The early assumption that some interactions are negligible led to an initial screening design, where a minimum of experimental runs were conducted. This fractional factorial design, as earlier stated, doesn't generate enough information to create a feasible response surface, which is needed to predict a design optimum.

The strength of Taguchi L-arrays are instead when the objective is to investigate how noise factors affect the response in relation to the control factors, and a screening is necessary in relation to this. The Taguchi L-array design allows for investigating several factors main effects in a reasonable amount of time, even if this isn't the main purpose of using a Taguchi design. Usually a surface response study is made after a potential screening to gather more information and further understanding of the effects of input variables on the outputs. (Taylor, 2004).

When deciding what nodes that was to be chosen for analysis it quickly became clear that area of analysis had to be reduced in order to grasp the problem. The response function developed in this report does only provide a model for stress amplitudes in the leading edge node. Taking other nodes into consideration will change the composition of the model, but this report suggest a methodological approach on how such a model can be created.

When looking on a main effect plot it is important to remember that the plot only shows results in terms of data means and does not represent the actual effect at a certain time step in the flight cycle. Another time step will most certain show another relationship for the main effects. A better understanding was gained by interpreting the effect of different thermal zones in each time step for all nodes along the suction side weld.

The effect of the thermal zones cannot be estimated independently from each other, in conflict with the assumptions for the initial DoE set up. The interaction plots for the Taguchi L9 experimental plan showed that there were interactions between the thermal zones, but the additional study with the Box-Behnken experimental plan showed that there weren't any interaction effects, which provided a puzzling result. Nevertheless the Taguchi L9 interaction plots cannot be trusted since there are confounding effects within the experimental plan. For future work there should be an initial DoE without confounding factors to detect main effects and at least second order interactions. This was presented with the presented Box-Behnken set up. With few experimental runs it was difficult to determine what temperature differences that were the most significant stress drivers. The data behaved stochastic and thus it was difficult to see any patterns in the plotted data. Further experimentation and a larger experimental plan with no confounding effect are needed to search for these stress drivers in a beneficial way. Validation test runs needs to be performed in order to be sure that the model behaves as predicted. One also needs to evaluate to what level the R-squared statistics actually answers whether a model is good or not. An overall consensus in the engineering society is that a high R-square and a low P-value guarantees that you have a good model, which in fact is not always the case. It is important to analyze how the residuals are distributed and perform test runs with other data in order to verify that the model behaves well. A model for the reduced flight cycle was not considered in this thesis, but is also an interesting area of investigation.

## 5.1 Improvement of analysis procedure

Together with the experienced engineers in the EWB group an analysis procedure was developed which significantly reduced simulation time and facilitated the post processing of data. A Python-script was written that performs all analyses in an automated way and gathers the results in a collected result file. Procedures on how to interpret DoE data have been presented which eases post processing of the DoE data in a structured way. Predefined edges and lines have been implemented in the CAD models to ease the choice of nodes in the ANSYS-scripts. The new predefined edges are shown in figure appendix IV.

The thermal analyses can also be performed in stationary mode which also saves simulation time to a tenth of the original. For an initial study based on DoE related work this is a beneficial way to shorten the simulation time.

The use of MATLAB showed weaknesses in automatic post processing since MATLAB code could not be executed on a remote server. The graphical interface had to be used to run the scripts which hampered post processing operations. Other software might function in a more beneficial way, which needs to be investigated.

The use of R-square statistics needs to be performed with caution. The overall consensus regarding the R-square and the P-value is that a high R-square and low P-value automatically shows that the model is good, which is not the actual case. A low R-square value can be calculated for a model that actually fits the data well and a high R-square value can be calculated for a model that does not fit the data at all. Similar relationships exist for the P-value. A low p-value does not necessary mean that the model has a good fit. A further analysis of the residuals and the actual F-value gives a greater understanding of the model and are powerful techniques for model evaluation. Validations test runs are also needed to make sure that the model behaves well.

# 6. Sources of error

The simulations performed in this thesis work used predefined finite element models with predefined mesh sizes. The results of this analysis could have been more precise if another mesh size had been used. If smaller elements had been used the results would have been more precise with the drawback of longer simulation time. The finite element method models used in the analysis were shell models which try to replicate solid models by applying stiffness to each element. A solid FEM model would on the other hand increase the simulation time. No consideration has been taken to how stiffness is affected by increasing temperature levels. Thus the effect of changing material properties has not been investigated.

The initial settings for thermal variation set points was ±28K, based on engineering experience and considered feasible in line with knowledge from earlier projects. (Arroyo, 2014)
A different approach for this would be to set the levels for the temperature variance as a limit at a certain distance from the standard deviation of the thermal distribution. A suggestion is the setting for $\Delta T$ as;

$$\Delta T = \mu + 3\sigma$$

Here μ is the mean temperature value of data and $\sigma$ is the standard deviation.

The applied temperatures for the analyses are based on air temperatures in the different thermal zones and originally from OEM, which is the "source of variation". The transient temperatures that occur between the air temperature and the actual temperature in the metal surfaces will affect the results. If calculations are made directly from metal temperatures more reliable results would have been obtained for the stresses. For this particular study the air temperatures represent the input data from OEM, which makes it relevant as the investigated variance parameters. Next question may be if the received OEM data really is best suited as base for a DoE, where a distinct effect from a certain parameter is wanted. Maybe a heavier weighting (larger values on the selected levels) of the original data or a more "extreme" data would have uncovered effects and behaviors that are hard to discover with the current data from a real flight cycle.

The choice of DoE setup in this thesis work is not fully investigated and there may be better design settings. A different design setup is suggested in future works, but everything depends on what one wants to accomplish with the study and what the objective is. As discussed, the initial choice of a Taguchi design left out some important information that could have been investigated at an earlier stage.

# 7. Conclusion

Consequences of limited analysis of the underlying intentions of the experiment often result in limited findings or even useless data. Therefore the intention of the experiment and a clear objective of the study need to be evaluated early in the planning of the experimental design set up.

The conflict between the economic benefits of running factorial screening designs and the lack of information about possible two-factor interactions is ultimately a question of priorities and what the impact may be if the interpreted results are not correct. As stated in discussion chapter, the Taguchi design methods have the benefits that you can sort the experimental plan in controllable and uncontrollable variation. This was used as an initial approach to the project, but later the idea of Taguchi design was considered unnecessary because of the insight that in computer simulations the experiments are deterministic, which means that all parameters are controllable, even the noise factors (Forslund, 2012). This leads to the conclusion that the Taguchi L9 could be replaced with a regular suitable fractional factorial design, if the main effects are of interest and a screening design is needed. For a highly saturated factorial screening the Placket Burman design is more feasible when it comes to the investigation of many factors, and the so called Pareto principle applies. Pareto principle states that, in relation to the investigation of factors, that around 80% of the effects comes from 20% of the factors, even called “law of the vital few” or “the principle of factor sparsity”. The Pareto principle is applicable when it comes to the investigation of many different factors, where it is most likely only some of the factors that have substantial effect. This could be the case when you, for example, have a lot of different geometrical parameters to try out. For future work there should be an initial DoE without confounding factors to detect main effects and at least second order interactions if this is the objective of the study.

The investigated effect plots from the different thermal zones indicated a non-linear behavior, which indicate that Designs of Experiments with at least three levels should be used to investigate this even further. A model that best describe what type of curvature that is present within the response is needed.

A study of a Box-Behnken design was conducted which showed that the models were slightly improved, but that there were still an insignificant fit against the response data. Conclusion that could be drawn from this was that there are other factors contributing to increased stress levels other than only temperature gradients. The Box-Behnken experimental plan showed equal results in main effects as the Taguchi L9 experimental plan.

During the initiating phases of this thesis work it was early found that there was no standardized way of investigating stress levels at different welds on the TS. The nodes on the lower suction side weld edge had to be picked manually in ANSYS, making it time consuming to investigate other weld edges or positions of interest on the TS.

The factors with substantial main effects are chosen for a more in depth analysis in response surface methodology where it is possible to study the factors with help of statistical tools such as regression and ANOVA analysis. The results can also be used for optimization purposes. The performed screening design also confirms this with the given results in 4.17.3. Which implies that in our system, there are a few driving factors as can be seen in figure 4.17. As presented in the figure with help of the Neural Network Function the axial chord length has a large effect on the generated stress level, and should therefore be further investigated in additional studies. There are confounding effects within the response function which also needs to be addressed before using the Neural Network Function for decision making purposes. The geometrical study does not consider maximum or minimum stresses at other positions in the TS and the neural network function has not been evaluated for its lack of fit.

From the geometrical study it could also be concluded that the LE chamber angle is more robust or insensitive to thermal variation for degrees lower than 60. The curvature within the robustness plots can be a result of the interpolation occurring within the neural network function. Therefore the threshold for insensitive geometrical factors settings needs to be investigated further before conclusions regarding overall robustness can be stated.

A large span between low and high temperature reveals that the particular geometrical setting is sensitive to variation in temperature and therefore the topology needs to be defined with caution.

ANSYS-script, Python-script and post processing code has been written that supports further investigation of other welds at predefined nodes in the TS. It was also not possible to use MATLAB for the automated post processing operations, which will be mentioned in following chapters.

# 8. Developed methodology for investigation of thermal robustness

Addressing the research question in this thesis report the major outcome of this thesis is the methodology implemented and investigated during this work. The following procedures serve as a suitable approach on how to increase thermal robustness of the TS:

1. Gather information about the problem and investigate what thermal loads those are to be studied.

2. Define the objectives of the study, determine what experiment approach is appropriate and select the process variables. The result of design of experiment is much depending on what choices that are made on this level. A support for additional DoE studies has been created and recommendations for different experimental plans have been shown that serve different experimental purposes.

3. Predefined node positions which support possibility to investigate different geometries and different weld or edges on the struts have been created.

4. An automated analysis procedure developed for the purpose of performing many experimental runs in a short time are always needed. Simulation time can be shortened by the use of ANSYS-script, Python-scripts and automated post processing operations mentioned in this thesis report. Automated generation of BC-tables also support in the reduction of simulation time.

5. The key design parameters can be identified by analysing main effect plots and interaction plots, but the key issue is that the chosen experimental plan has to be chosen with care, depending on what objective you have with the study. If a large set of potential parameters needs to be investigated, a screening design may be appropriate to distinguish key parameters for further, more in-depth investigation. When those important factors are identified, a surface response objective may be appropriate to get a more extensive understanding of the behaviors of those parameters and their effects.

6. Regression or ANOVA analysis can be performed to develop response functions that can be used in many ways to answer the objectives of the experiment.

7. The method used for the geometrical attributes can be used to gather knowledge on how to increase robustness with respect to thermal and geometric attributes. Neural network functions serve as a good way of identifying which geometrical parameters that are sensitive together with the effects of the thermal zones and is also beneficial for combining results from several studies.

8. This thesis work suggests DoE experimental plans that can be used for screening and development of response functions all of which are applicable early in the product development processes.

# 9. Further work

Suggestions for further work are here presented, based on conclusions drawn from this thesis work.

A methodology was developed to be able to analyse several predefined welds. Unfortunately these welds could not be studied within this thesis due to the lack of time for multiple studies. In the interest of GKN it would be beneficial to study how thermal BC's affect stress distribution along all of these welds. It would support a better understanding of the design and also prove beneficial in terms of optimizing the component. For this work the material data wasn't specified to simulate a real weld, which in some way should be the goal for future studies. Weld simulations could be implemented and data from such simulations could be used in further studies.

Validation test runs need to be implemented in the future to validate that the response function works as anticipated.

The DoE analysis process, that has been presented, is an extensive and iterative process. It is reprehensible to draw conclusion based on experimental results if the experimental plan itself from which the runs are generated are incorrect processed. A methodology for analysis of DoE data has been presented as well as techniques that can be used for development of a response function. Stepwise regression can be used to develop response functions for different response variables of choice by using the DoE analysis flowchart. A single response function including data measurements from several nodes from the model have not been developed, but would be of high interest for a follow up on this thesis. A response function that shows stress as a function of several observation points at different positions on the TS is well suited for multi-objective optimization purposes. That is as long as the underlying DoE analysis have been done in a correct way and that reasonable conclusions have been made, a work that has been much regarded in this thesis, but needs further investigation.

As stated in conclusion, today the BC-tables that correspond the input from OEM, have to be updated manually if changes are needed. In the future an easy-operated automatic update of the BC-tables is appropriate for a more efficient iterative process. A proposal is implementations of BC-tables in the python-script, where a single command could be used to, for example, change the weightings of all temperatures. In this thesis work the data from a real flight cycle was used, but in further projects the practitioner may want to use more extreme data, or custom made data to investigate certain interests.

All of the work related to MATLAB was done in the graphical interface of the software. Today, post processing is made manually in this interface which doesn't support a fast evaluation of the results. In order to speed up post processing, investigation is needed to answer if it is possible to use MATLAB on a remote server for post processing purposes. Other software's might be more useful for this purpose and those need to be investigated and weighted against their pros and cons. Some built in functions of the MATLAB code doesn't support other DoE setups than Taguchi L9. There are many possibilities for improvements of the post processing MATLAB code.

In this study the main effects for the geometry parameters were generated from ModeFrontier, but in future studies a correct design of experiment set up should be implemented. Design set ups suitable for future research could be Latin Hypercube, space filling or Definitive screening design in Jmps, to name a few. The current Python-script works well for a Taguchi L9 design, but still needs further work to implement other experimental designs. During this project a wide range of software was utilized, but the potential of this software should be more investigated. To name an example the comparison between MINITAB and Jmp, for DoE generation and statistical tools, should be investigated. Even the exploration of other, not utilized software, should be made in future projects.

To minimize simulation time, in future studies, only the temperature difference with the highest impact on stress levels should be chosen as a factor. For example, an increase in the temperature difference between Zone 2 and Zone 4 has strong correlation with higher stresses in the component, and should therefore be tested as the only contributing factor in the DoE setup. This factor would then be given a high or a low value, and through how distinct the interaction effect is, to each different geometrical factor, one can investigate which geometrical configuration would result in a high sensitivity to generated stresses.

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# Appendix I

**CONFIDENTIAL MATERIAL**

## Appendix II

**CONFIDENTIAL MATERIAL**

## Appendix III

**CONFIDENTIAL MATERIAL**

## Appendix IV

**CONFIDENTIAL MATERIAL**

## Appendix V

**CONFIDENTIAL MATERIAL**