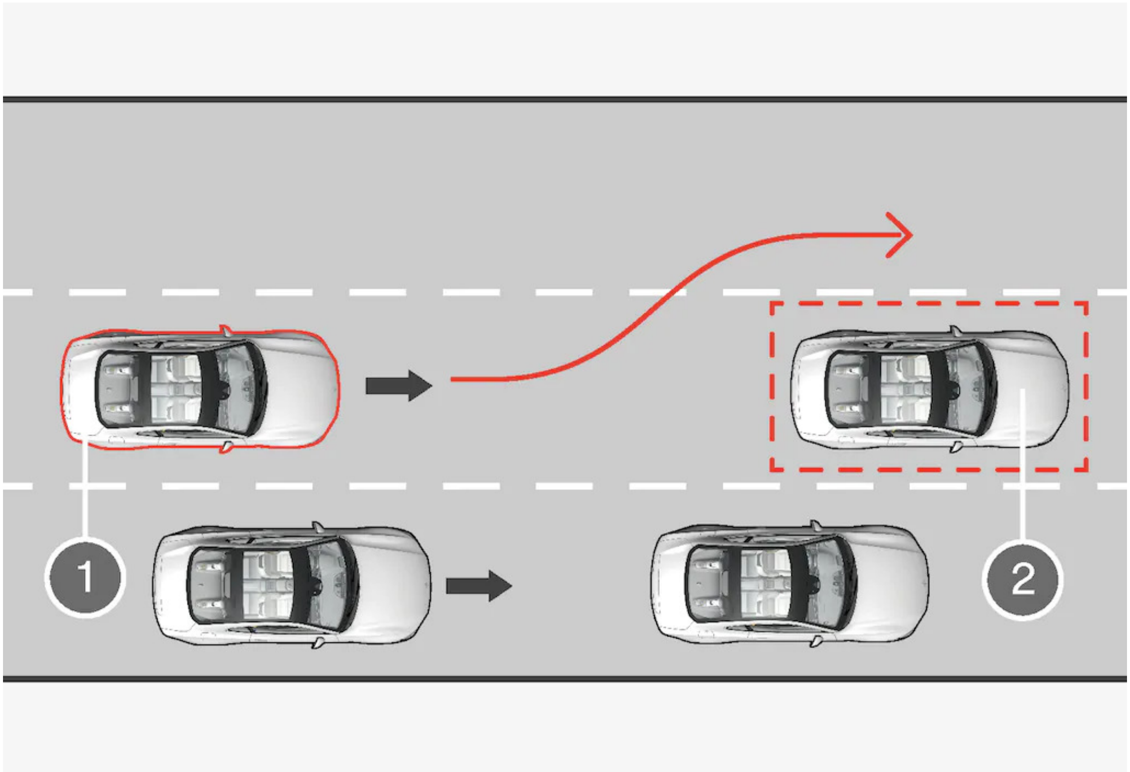


# Comparing vehicle dynamics models of different complexity for comfortable and emergency steering in virtual assessment of active safety systems

Master's thesis in Automotive Engineering

AASHISH UDAYKUMAR KODGI
RAGHAVENDRA RAMIREDDY GANGADHARA REDDY

Department of Mechanics and Maritime Sciences
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2020

Master's thesis 2020:78

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Different vehicle models to determine the last point of steering for evaluating active safety systems

AASHISH UDAYKUMAR KODGI
RAGHAVENDRA RAMIREDDY GANGADHARA REDDY

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Supervisor: Pinar Boyraz Baykas, Mechanics and Maritime Sciences
Examiner: Jonas Bärgman, Mechanics and Maritime Sciences

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Department of Mechanics and Maritime Sciences
Division of Vehicle Safety
Chalmers University of Technology
SE-412 96 Gothenburg
Telephone +46 31 772 1000

Cover: Steering assistance system helping the driver to perform evasive maneuvering in order to avoid the collision. [24]

Typeset in LaTeX
Printed by Chalmers Reproservice
Gothenburg, Sweden 2020

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# Abstract

The World Health Organization (WHO) states that every year, 1.35 million people die as a result of road traffic crashes [1]. According to the National Highway Traffic Safety Administration (NHTSA), rear-end collisions are the type of crashes that occur most frequently. Actually, 29 percent of all collisions are rear-end crashes [2]. It is the responsibility of the automotive sector to make use of the technologies available to the full extent, to reduce the number of fatalities caused every year due to road traffic crashes. The development of Advanced Driver Assistance Systems (ADAS) has led to the prevention or mitigation of many collisions. Therefore, testing of active safety systems plays a vital role in improving these systems. Rigorous analysis under different conditions can provide insight to further development of these systems. Development of active safety and automated driving systems requires safety evaluation during the development stage by using different tools for testing and evaluation. One such tool for evaluation of active safety systems is performed using computer simulations virtually, as evaluation of these systems in real traffic scenarios is not only dangerous but also expensive. Since vehicle models take part in simulations, it is important to study the impact of vehicle models of different complexities for evaluating active safety systems. The main goal of the thesis is to select different vehicle models of varying complexity levels and study what impact the selection has on the last point of steering to avoid a stationary obstacle while performing evasive manoeuvring. In the increasing order of their complexities, a single-track model, a two-track model without load transfer, and a two-track model with load transfer, were selected. These models were further subjected to changes by considering different longitudinal speed, different cornering stiffness at the front and rear axles of the vehicle, and different tire models. Analysis was performed in terms of comparing the timing of the point of no return (last time to avoid the obstacle) where no substantial difference was noticed among the different vehicle models. Finally, based on considering the deviations in the last point of steering in seconds and computational time required for 10,000 iterations, the conclusion is made that the most complex model is not always necessarily the best model that can be used in simulations.

Keywords: Active safety, safety evaluation, virtual evaluations, vehicle models, evasive steering, lateral dynamics, the last point of no return.

# Acknowledgements

We thank our examiner Jonas Bärgman and supervisor Pinar Boyraz Baykas, for providing guidance and feedback throughout the duration of this thesis. We offer our heartfelt gratitude to the Department of Mechanics and Maritime sciences, Division of Vehicle, and traffic safety at Chalmers University of Technology for providing us with the wonderful opportunity of working on this thesis. And lastly, we would like to extend our gratitude to our parents and friends for being a constant source of support throughout this thesis work.

Aashish Udaykumar Kodgi, Gothenburg, September 2020
Raghavendra Ramireddy Gangadhara Reddy, Gothenburg, September 2020

**In memory of Pinar Boyraz Baykas, who sadly passed away before the end of this thesis project, we would like to offer special thanks to her. She continues to inspire us by her example and dedication to us and other students she guided over the course of her career.**

# Contents

# Notations

| | Notation | Unit |
|---|---|---|
| Vehicle position | x,y,z | m |
| Forces on the vehicle | $F_x, F_y, F_z$ | N |
| Moments on the vehicle | $M_x, M_y, M_z$ | Nm |
| Roll, Pitch, Yaw angle | $\phi_x, \phi_y, \phi_z$ | rad |
| Roll, Pitch, Yaw angular velocities | $w_x, w_y, w_z$ | $rad/s$ |
| Steering angle | $\delta$ | rad |
| Time | t | s |
| Gravity | g | $m/s^2$ |
| Steering angle of steering wheel | $\delta_{sw}$ | rad |
| Tyre slip angle | $\alpha$ | rad |
| Tyre lateral slip | $s_y$ | - |
| Mass of the vehicle | m | Kg |
| Moment of inertia about Z-axis | $I_{zz}$ | $kgm^2$ |
| Wheelbase | L | m |
| Distance from CoG to front axle | $L_1$ | m |
| Distance from CoG to rear axle | $L_2$ | m |
| Front axle cornering stiffness at origin | $ca_1$ | $N/rad$ |
| Rear axle cornering stiffness at origin | $ca_2$ | $N/rad$ |
| Sprung mass | s | kg |
| Vehicle acceleration, in inertial system | $a_x, a_y$ | $m/s^2$ |
| Vehicle inertia about X axis | $I_{xx}$ | $Kgm^2$ |
| Steering ratio | k | - |
| Height of CoG | h | m |
| Front roll center height | $h_1$ | m |
| Height of CoG above roll axis | $h_2$ | m |
| Magic formula parameter | C | - |
| Magic formula parameter | E | - |
| Road tyre friction coefficient | $\mu_0$ | - |
| Tyre load based non-linearity parameter for friction | $\mu_1$ | - |
| Tyre stiffness parameter | $c_0$ | - |
| Tyre load based non-linearity parameter for stiffness | $c_1$ | - |
| Rated load for the tyre | $F_{z0}$ | N |

# Abbreviations

ABS- Anti-lock braking system

ESC- Electronic stability control

ADAS-Advanced driver assistance systems

NDD-Naturalistic driving data

FOT- Field operational tests

DOF-Degrees of freedom

SAE- Society of Automotive Engineers

CARE-Community database on Accidents on the Roads in Europe

TEM-Trans-European North-South Motorway

ISO-International Organization for Standardization

NHTSA- National Highway Traffic Safety Administration

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# 1
# Introduction

## 1.1 Problem Definition

Development of active safety and automated driving systems requires safety evaluation during the development stage. In a typical rear-end scenario where an additional Advanced Driver Assistance Steering System along with the main steering system, is beneficial for drivers to perform evasive manoeuvres with some assistance offered by such systems. A typical rear-end scenario is shown in Figure 1.1, which represents the different levels of risks involved as the ego-vehicle travels towards lead-vehicle. Therefore, active safety systems play a crucial role in assisting the driver in manoeuvring to avoid a collision; such systems must be evaluated effectively by adopting different tools for evaluation.

Previous work conducted on the combined effect of braking assistance and steering into an integral advanced driver assistance systems for collision avoidance [25], suggests that generally, the preferred measure to avoid the collision at low velocities is through braking. However, at higher velocities, the stopping distance increases with the square of the relative velocity, and therefore at relatively higher velocities the collision can be avoided with the help of steering. When considering only steering to avoid a collision, the last point of steering in time plays an important role, based on which active safety systems such as emergency steering assist can be developed and evaluated. The last point of steering in time can be defined as the latest possible time, starting from an initial position, at which the driver should start steering in order to successfully avoid collision with the stationary vehicle. Previous work carried out on emergency steering assist for collision avoidance [29] provides much information regarding the last point of steering and last point of braking. The paper suggests that when the ego vehicle is approaching a stationary obstacle, the last point of steering occurs latter than the last point of braking in order to avoid the collision. Since, our thesis focuses on steering and lateral dynamics of the vehicle to successfully avoid the collision, the last point of steering plays a vital role, based on which different vehicle models with varying complexities can be compared for evaluation of active safety systems before incorporating them into the vehicles.

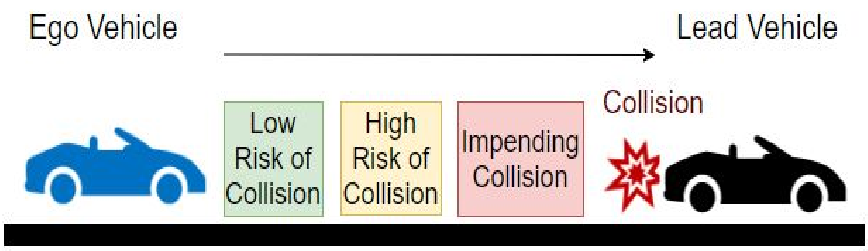


**Figure 1.1:** Collision avoidance by steering or braking when the ego vehicle is approaching the lead vehicle

## 1.2 Background

As mentioned in the problem definition, there are many tools which can be adopted to develop and evaluate active safety systems such as bench tests, virtual tests, driving simulators, test tracks and Field Operational Tests (FOT) [17]. Human factors drive the latter three tests for developing and evaluating active safety systems. The safety systems can be evaluated using Naturalistic Driving Data (NDD), which may be collected in field operational tests. Field operational Test is large collections of real-traffic data, often performed in a naturalistic fashion, but to evaluate an active safety system. As a consequence, a treatment and a baseline phase are often present in an FOT. FOT provides genuine driver behaviour and unique insights into human factors, but they are expensive to carry out, and it involves complex situations. The data used for FOT is a collection of big data in real-traffic, by road users performing their usual daily activities. This naturalistic data can be recorded with instrumented vehicles specified to collect the data. Previous research [3] on this topic gives insight on different strategies for the collection of naturalistic driving data (NDD), also provides valuable information on identifying crash situations using NDD. One more tool which can be adopted for testing and evaluation of active safety systems is to make use of test tracks where the one or more vehicles are equipped or installed with the new prototype system, and they are tested in a test track. The environment is well known, but it generally involves professional drivers to carry out the testing.

The next tool which can be adopted are driving simulators; they play a vital role in developing different ADAS, with the help of a simulator software the driver can perceive how the vehicle is behaving under different conditions. It helps the driver to feel the motion and vibrations during the simulation, where multiple vehicles can be tested. Model fidelity plays an important role while choosing driving simulators, as the testing is done in a virtual environment, therefore, the fidelity of the model is directly proportional to the cost required to test the systems; also it is essential to meet the simulator feature to that of the evaluation needs. Similar research on this topic [4], involves the development of a vehicle dynamics model for the driving simulator, and this research provides excellent insight on model validation, to ensure that the developed vehicle dynamics model behaves just like a real car. Where the model validation is checked based on pre-defined manoeuvres such as steady-state cornering, transient response test and straight driving tests were performed in a test track in a vehicle equipped with data-collecting systems as mentioned before. The results from the simulator and the results from the test rack are compared to check the fidelity of the model.

The safety evaluation of active safety systems is performed using computer simulations virtually as evaluation of these systems in real traffic scenarios can be dangerous, ineffective and may bring high-costs. The advent of computers with very high capabilities, it is now possible to consider, model-based approach in the testing of automotive applications by performing simulations and evaluation before rendering them into the hardware. Therefore, it allows testing and validation engineers to work with modelling and integration of different types of vehicle subsystems and components under one simulation framework. Examples of a virtual environment for system evaluation are Vehicle simulations and Traffic simulations. Vehicle and application simulation offers high fidelity and technical evaluation. One such virtual testing method counterfactual simulation where, it predicts the effectiveness of active safety systems based on mathematical models of the driver, the vehicle and the environment.

The simulations involved in the safety evaluation will require many components such as vehicle models, driver models and intelligent system functionalities. The main idea of the thesis is to perform a comparative study of vehicle dynamics models for determining the necessary level of complexity for a realistic assessment of active vehicle safety systems; this leads to the selection of simulations as a tool for evaluation by comparing trajectories for performing collision avoidance manoeuvres [5]. Therefore, the main focus is on the lateral dynamics of the vehicle. Hence, the last point of steering plays a crucial role in this comparison among different vehicle models.

## 1.3 Aim

The main aim of the thesis is to compare vehicle dynamics models of varying complexities used in virtual safety assessment simulations for the development of active safety systems, with respect to lateral dynamics. Specifically, we aim to understand if increasing vehicle model complexity changes the timing of point-of-no return with respect to steering. That is, we compare the last point in time (distance) to start steering to avoid a collision, across the different models.

## 1.4 Deliverables

The thesis involves literature survey on vehicle models in the initial phase and then selecting different types of vehicle models such as one-track model, two-track model without load transfer and also to increase the complexity, two-track model with load transfer is also considered. All the selected models are integrated into a simulation framework which will be used to evaluate the performance of an active safety system designed to avoid a rear-end collision. Counterfactual simulations with relevant mathematical models can help us to analyze or evaluate the safety benefits of an active safety system. The analysis of the vehicle model will be extended to analyze the sensitivity of safety outcome to variation in the vehicle model parameters. The implemented vehicle models will be considered to evaluate the vehicle trajectory and as mentioned in the previous section, the latest possible point in time (distance) to start steering to avoid collision with a stationary vehicle. In order to mimic collision avoidance manoeuvre, the vehicle models considered are simulated to perform a single lane-change manoeuvre on approaching the stationary vehicle. The single lane-change manoeuvre is based on the S-shape manoeuvre recommended by the SAE for the evaluation of collision avoidance behaviour of different vehicles [5]. The avoidance manoeuvres will be implemented as an open loop, meaning the driver will apply a pre-defined or pre-calculated steering angle profile rather than interactively controlling the trajectory of the vehicle in a closed-loop. The thesis will begin with literature survey on vehicle models with Different Degrees of Freedom (DOF) and complexity as well as selecting the most suitable software by analyzing their pros and cons, to achieve the best possible results, which is followed by vehicle model implementation and integration into the simulation framework. The final step will be to evaluate the effectiveness of vehicle dynamics parameters on safety outcomes of the active safety system.

## 1.5 Tools

After performing a literature survey on different vehicle models, it is necessary to represent them with different complexities. There are several software packages available for this purpose, which has its own pros and cons concerning things such as availability, modularity, flexibility and workload.

For modelling of the vehicle models, there are several software packages like SIMULINK, IPG car-maker, Modelica, Dymola, Car-sim tools etc. There are a lot of pros and cons associated with every software that we consider.SIMULINK is a graphical programming environment for modelling, simulating, and analyzing dynamic systems. SIMULINK is used to model different vehicle models as it offers to be more flexible and user friendly. It is flexible because it allows the user to describe the systems in mathematical form; in other words, the systems can be described using equations. This allows an additional degree of freedom to change design parameters in a computer model quickly. For better visualization, a simulation environment is created using the automated driving toolbox, which is also a part of MATLAB/SIMULINK environment. For complex models such as multi-body dynamics model, after thorough research, open integration and test platform known as IPG car-maker was selected as it is also very flexible and robust when compared to other multi-body dynamics software. Car-Maker includes a complete model environment comprising an intelligent driver model, a detailed vehicle model and highly flexible models for roads and traffic. Car-Maker makes it easier to integrate subsystems into the vehicle and facilitates early testing for smooth interactions between subsystems. It also provides an additional benefit where the import/export of models from car-maker to SIMULINK and vice-versa is possible.

## 1.6 Planning

- With the start of the thesis, the literature survey was carried out by both the team members and acquired information regarding different models, software packages and methodology to carry out the simulations.

- Both the team members involved in building the basic models required for the simulations.

- Once the basic models were built, we planned to simulate different vehicle models individually and compare the results.

- The task of report writing was an ongoing process throughout the thesis and was divided equally.

- The later part of the thesis it was emphasized on interpreting results for different conditions. Refer **??** in the appendix section.

# 2

# Vehicle Dynamics Theory

In the section, a general description regarding vehicle modelling and concepts of different sub-components used to model the vehicle are discussed and how these sub-components are altered to increase or decrease the complexities of vehicle models is understood with detailed explanation.

## 2.1 Co-ordinate System

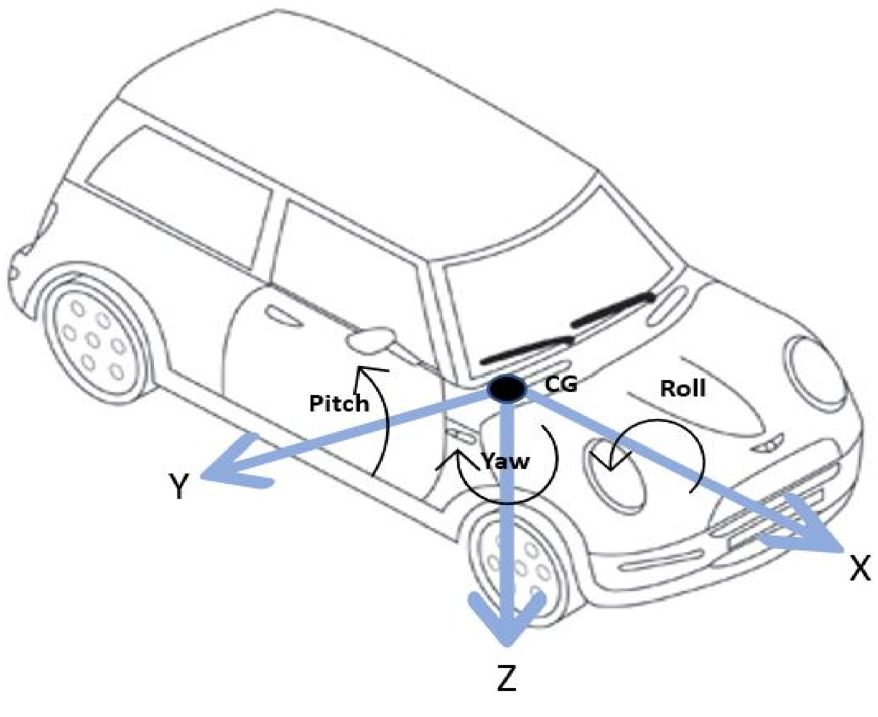


**Figure 2.1:** SAE-Vehicle Axis System

The vehicle system coordinates used in the entire project is shown in figure 2.1, which is following SAE standards for vehicle coordinate system. CG is the centre of gravity of the vehicle, where the mass of the vehicle is concentrated at this point. The motions of the vehicle are defined in accordance with the right-hand orthogonal coordinate system. By SAE convention the coordinate X is the forward motion of the vehicle and on the longitudinal plane of symmetry. Whereas the coordinate Y is the lateral motion towards the right of the vehicle, coordinate Z is the downward motion concerning the vehicle. All three axes are mutually perpendicular to each other. Also, the roll rotation is defined around the X-axis, the pitch rotation is defined around the Y-axis and the yaw rotation around the Z-axis of the vehicle coordinate system. However, since the thesis is concerned only with steering at a constant longitudinal velocity to avoid a collision, the vertical dynamics of the vehicle are not taken into consideration due to the absence of braking or acceleration.

## 2.2 Tires

The dynamic performance of the vehicle largely depends on the tire. The longitudinal, lateral, and vertical behaviour of the vehicle depend largely on tires. Tires, therefore, have a key role to play in influencing the dynamic behaviour of the vehicle as they are solely responsible for producing the lateral and longitudinal forces required for changing the speed and direction of the vehicle. Given the significant role, tires play in vehicle dynamics; they also are the most difficult to model. There are many different tyre models of varying complexities used for vehicle dynamics analysis and simulations.For this thesis, two types of tire models have been chosen, namely linear tire model and Packeja or magic formula tyre model. Most automated highway systems make use of linear tire model for simplicity [33], However [31] in their research suggest that a more complex tire model is important when the severity of the manoeuvre increases as the saturation of tire force at high slip angle is not predicted accurately by the linear tire model. Hence the vehicle dynamics models chosen are simulated with both linear tire model and nonlinear tire model as inputs, and the results are analyzed in terms of the last point in time to start steering.

### 2.2.1 Tire Slip

The angle between the direction of heading of the tire and the direction of travel is known as tire slip angle. Figure 2.2 shows the slip angle for a single tire.

The tire slip for each wheel can be calculated as follows:

$$\alpha_{fl} = \delta_{fl} - \left( \frac{v_y + l_f r}{v_x - l_f r \frac{W}{2}} \right) \tag{2.1}$$

$$\alpha_{fr} = \delta_{fr} - \left( \frac{v_y + l_f r}{v_x + l_f r \frac{W}{2}} \right) \tag{2.2}$$

$$\alpha_{rl} = -\left( \frac{v_y + l_r r}{v_x - l_r r \frac{W}{2}} \right) \tag{2.3}$$

$$\alpha_{rr} = -\left( \frac{v_y + l_r r}{v_x + l_r r \frac{W}{2}} \right) \tag{2.4}$$

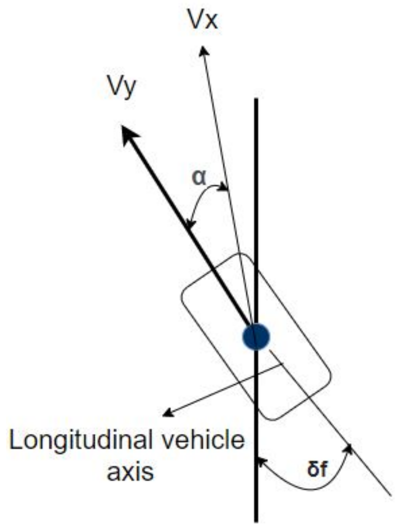


**Figure 2.2:** Tire slip calculation

## 2.3 Linear tire model

To study the behaviour of different vehicle models with different sub-components, the sub-components of a vehicle model can be altered by modelling desired tire model, as there are many tire models available such as linear tire model, non-linear tire model and brush-type model.

In our thesis, since the sub-components of the model is modelled solely based on a mathematical approach, it allowed some room for testing different tire models. Therefore, linear tire model is selected to study the effect of a linear relationship, between lateral forces generated by the tires and lateral acceleration of the vehicle. Therefore, the lateral acceleration plays a vital role in driver's comfort levels during evasive manoeuvring.

In linear tire model, the lateral forces from each tyre are generated by taking tire slip angle $\alpha$ as input. The relationship between tire lateral force and slip angle is linear and half axle cornering stiffness is assumed for each tire:

The lateral force generated by each tire is calculated as follows:

$$F_{fly} = \frac{C_{af}}{2}\alpha_{fl} \tag{2.5}$$

$$F_{fry} = \frac{C_{af}}{2}\alpha_{fr} \tag{2.6}$$

$$F_{rly} = \frac{C_{ar}}{2}\alpha_{rl} \tag{2.7}$$

$$F_{rry} = \frac{C_{ar}}{2}\alpha_{rr} \tag{2.8}$$

## 2.4 Magic Formula tire Parameters

Similarly, for vehicle models, the non-linear nature of the tire can be modelled mathematically, to study its effect on trajectory and driver comfort level during evasive manoeuvring. It generally uses trigonometric functions to fit the curve, where the lateral forces generated by the tires is proportional to the lateral acceleration of the vehicle up to a point, and then it saturates. The main purpose of selecting a non-linear tire model is to study this non-linear behaviour of the tire model since our thesis mainly focuses on the lateral dynamics of the vehicle.

Non-linear tire model representing Pacejka's is also called as Magic Tire Formula. It is probably the most well-known curve fit called 'Magic Formula'. It generally uses trigonometric functions to fit the curve.

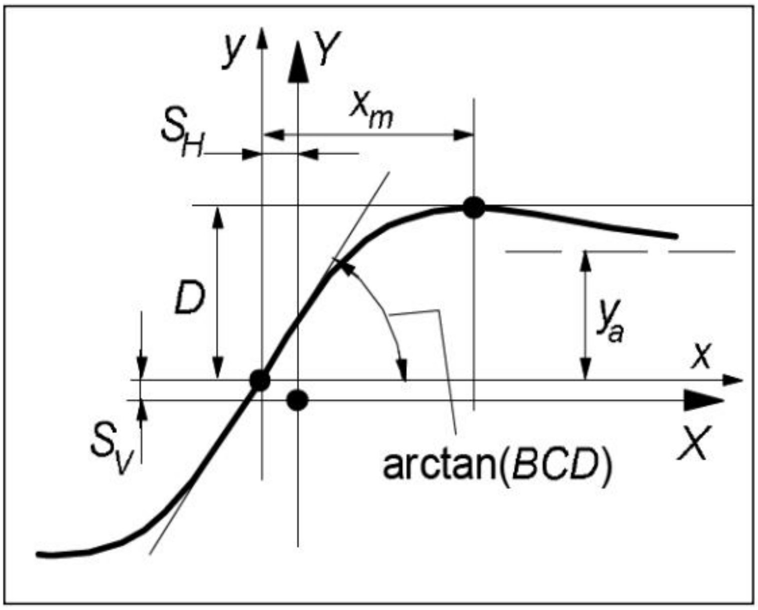


**Figure 2.3:** Magic formula tire parameters [7 with permission from the author]

$$F_y = \mu F_z \sin\Big(C \arctan\Big(B\alpha - E\big(B\alpha - \arctan\big(B\alpha\big)\big)\Big)\Big) \tag{2.9}$$

Where,

$$B = \frac{c_y}{\mu C} \tag{2.10}$$

B is a stiffness parameter, C is a shape parameter, D is a peak value parameter, E is a curvature parameter describing the curve.
The figure 2.3 shows the relationship between different parameters involved in magic

tire formula.

The cornering stiffness and the friction coefficient depend on the normal load, which is different for front and rear axle and are calculated as follows:

$$\mu = \mu_0 \Big(1 - \mu_1 \Big(F_z - F_{z0}\Big)\Big) \tag{2.11}$$

$$c_y = c_{y0} \frac{\Big(1 - c_{y1}\Big(F_z - F_{z0}\Big)\Big)}{2} \tag{2.12}$$

The normal force for the front, $F_{fz}$, and normal force for the rear, $F_{rz}$, are calculated as follows:

$$F_{fz} = mg\big(1 - \frac{l_f}{l_f + l_r}\big) \tag{2.13}$$

$$F_{rz} = mg\big(1 - \frac{l_r}{l_f + l_r}\big) \tag{2.14}$$

By using equation 3.3, the lateral force generated by each tyre can be computed as mentioned below:

$$F_{fly} = \mu_f F_{fz} \sin\Bigg(C \arctan x\Big(B_f \alpha_{fl} - E\Big(B_f \alpha_{fl} - \arctan\Big(B_f \alpha_{fl}\Big)\Big)\Big)\Bigg) \tag{2.15}$$

$$F_{fry} = \mu_f F_{fz} \sin\Bigg(C \arctan\Big(B_f \alpha_{fr} - E\Big(B_f \alpha_{fr} - \arctan\Big(B_f \alpha_{fr}\Big)\Big)\Big)\Bigg) \tag{2.16}$$

$$F_{rly} = \mu_r F_{rz} \sin\Bigg(C \arctan\Big(B_r \alpha_{rl} - E\Big(B_r \alpha_{rl} - \arctan\Big(B_r \alpha_{rl}\Big)\Big)\Big)\Bigg) \tag{2.17}$$

$$F_{rry} = \mu_r F_{rz} \sin\Bigg(C \arctan\Big(B_r \alpha_{rr} - E\Big(B_r \alpha_{rr} - \arctan\Big(B_r \alpha_{rr}\Big)\Big)\Big)\Bigg) \tag{2.18}$$

## 2.5 One-Track Model

Since the primary focus of the thesis is to select different vehicle models for virtual assessment of active safety systems. The selected vehicle models are expected to reasonably and realistically simulate the dynamics of the vehicle during evasive manoeuvring. The different complexities of the models play a crucial role in the assessment of active safety systems. Therefore, it is important to test different vehicle model with varying complexities.

In the single-track model or bicycle model, the front and rear tires are represented as one tire on each axle, unlike the two-track model where two tires are considered at the front and rear axles. The motivation behind selecting the single-track model is to primarily study the lumped-mass effect on vehicle trajectory and driver's comfort level. Further, it allows a fair comparison when the vehicle is considered to have two tires at the front and rear axles, respectively.

This section motivates the complete modelling used for the motion of the FV and describes its lateral dynamics.

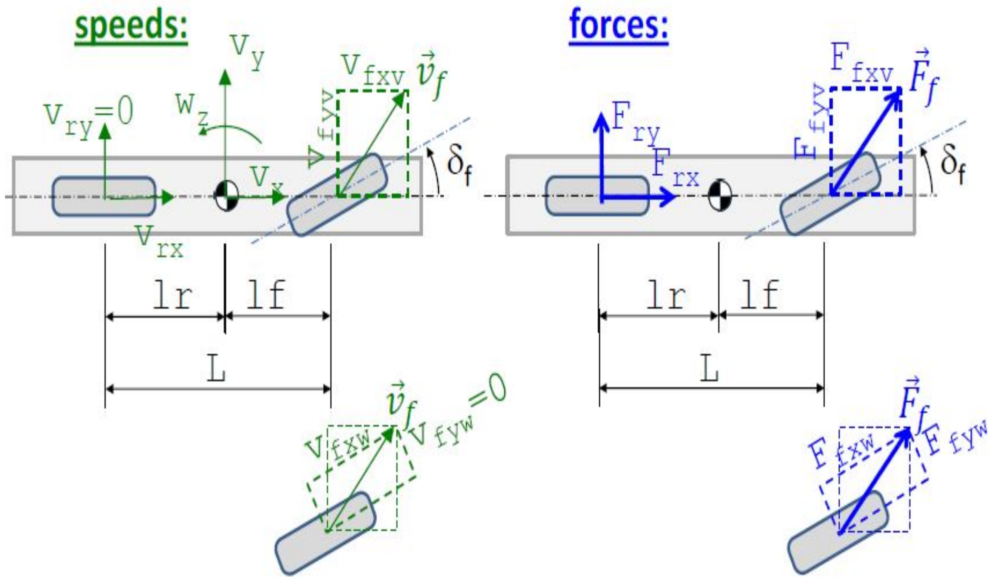


**Figure 2.4:** One-track model with ideally tracking axles [8 with permission from the author]

From figure 2.4 the mathematical model can be derived as follows;

Equilibrium (longitudinal, lateral and yaw-rotational)

$$0 = F_{fxv} + F_{rx} \tag{2.19}$$

$$0 = F_{fyv} + F_{ry} \tag{2.20}$$

$$0 = F_{fyv} \cdot l_f - F_{ry} \cdot l_r \tag{2.21}$$

Transformation between vehicle and wheel coordinate systems:

$$F_{fxv} = F_{fxw} \cdot cos(\delta_f) - F_{fyw} \cdot sin(\delta_f) \tag{2.22}$$

$$F_{fyv} = F_{fxw} \cdot sin(\delta_f) + F_{fyw} \cdot cos(\delta_f) \tag{2.23}$$

$$V_{fxv} = V_{fxw} \cdot cos(\delta_f) - V_{fyw} \cdot sin(\delta_f) \tag{2.24}$$

$$V_{fyv} = V_{fxw} \cdot sin(\delta_f) + V_{fyw} \cdot cos(\delta_f) \tag{2.25}$$

Compatibility between CG and axles:

$$V_{fxv} = V_x, V_{fyv} = V_y + l_f \cdot \omega_z \tag{2.26}$$

$$V_{rx} = V_x, V_{ry} = V_y - l_f \cdot \omega_z \cdot \omega_z \tag{2.27}$$

Ideal tracking(constitutive relation, but without connection to forces)

$$V_{fyw} = 0, V_{ry} = 0 \tag{2.28}$$

Path with orientation(compatibility):

$$\dot{x} = V_x \cdot cos(\psi_z) - V_y \cdot sin(\psi_z) \tag{2.29}$$

$$\dot{y} = V_y \cdot cos(\psi_z) + V_x \cdot sin(\psi_z) \tag{2.30}$$

$$\dot{\psi}_z = \omega_z \tag{2.31}$$

## 2.6 Two Track Model

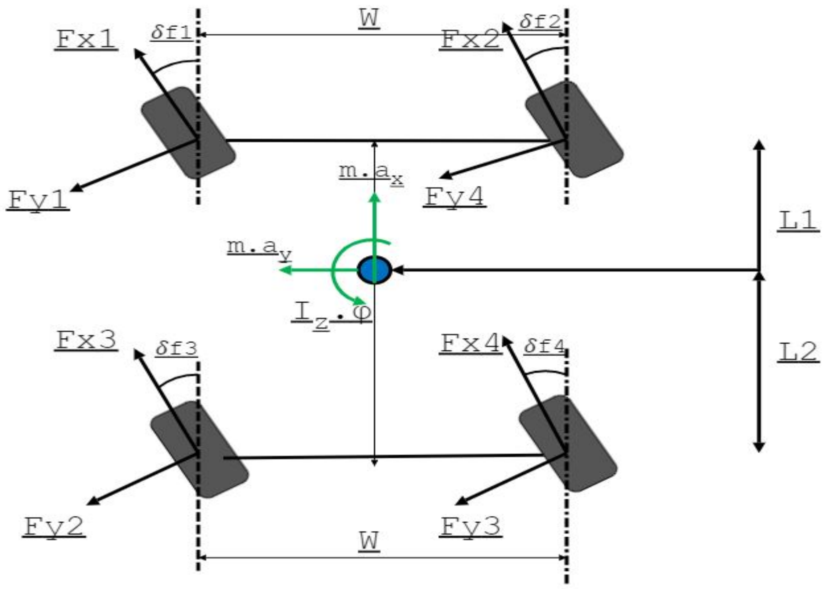


**Figure 2.5:** Two-track model acting in the X-Y plane

From the figure 2.5, the equations describing the chassis motion can be derived as follows:

The equilibrium along X-direction is given by:

$$\begin{aligned} m \cdot a_x = Fx_1 \cdot cos(\delta_{f1}) - Fy_1 \cdot sin(\delta_{f1}) + Fx_2 \cdot cos(\delta_{f2}) \\ -Fy_2 \cdot sin(\delta_{f2}) + Fx_3 \cdot cos(\delta_{f3}) - Fy_3 \cdot sin(\delta_{f3}) \\ +Fx_4 \cdot cos(\delta_{f4}) - Fy_4 \cdot sin(\delta_{f4}) \end{aligned} \quad (2.32)$$

In addition to these generated forces by the vehicle, there are few extra forces such as aerodynamic force $F_{drag}$ and the longitudinal force which acts in the opposite direction of the moving vehicle when a road slope is present known as $F_{slope}$, for simplicity purpose these forces are neglected.

The lateral acceleration of the model is calculated as follow:

$$a_y = \dot{v_y} + v_x \cdot \dot{\phi} \quad (2.33)$$

The speed $v_y$ is no longer a parameter but a variable.

### 2.6.1 Load transfer effect in two track model

Lateral load transfer is a phenomenon which accounts to the change on the vertical loads of the tire because of the lateral acceleration acting at CG of a vehicle. In order to study and analyze the effect of load transfer on the vehicle, Two-track model is modified into Two-track model with load transfer by adding the following equations mentioned below:

$$\ddot{\phi} = \frac{m_a \cdot a_y \cdot h_o - k_\phi \cdot \phi - (c_\phi - m_a \cdot g \cdot h_0)}{I_{xx} + m_a \cdot h_o^2} \cdot \phi \tag{2.34}$$

$$F_{zyi} = \pm \frac{1}{W} \cdot \frac{m \cdot (l - l_i) \cdot ay \cdot h_i}{L} + c_{\phi i} \cdot \phi + k_{\phi i} \cdot \ddot{\phi} \tag{2.35}$$

$$F_{zxi} = \pm \frac{m \cdot a_x \cdot h}{2 \cdot L} \tag{2.36}$$

### 2.6.2 Selected Vehicle Models and Sub-systems

The thesis aims at studying point of no return during evasive manoeuvring; therefore, lateral dynamics plays a vital role in assessing the behaviour of different vehicle models. To study this effect, different vehicle models explained earlier, such as simple one-track model, a two-track model with and without load transfer are considered mainly. Also, the tire sub-system of the different models are altered by using Pacejka tire model which is also known as non-linear tire model, and in addition to this linear tire model is also used to study the effect, as the lateral force generated by the tires is directly proportional to the lateral acceleration generated by the vehicle, unlike non-linear tire model where the lateral acceleration increases up-to a point and then saturates.

# 3
# Methods

The method section aims at giving a detailed overview on how the simulation process was carried out and what are the constraints selected to set-up simulation framework and also explains the procedure in detail which is based on the iteration approach.

## 3.1 Simulation set-up

As discussed earlier, different models such as the one track model, two-track model without load transfer, and two-track model with load transfer are considered. Simulation time of 30 seconds with a sample rate of 0.01 is considered for all the models. In order to calculate the last point of steering during evasive manoeuvre similar to single-lane change, a stationary obstacle is placed at a fixed point in all the three models. The entire modelling and simulation of the vehicle models is performed using MATLAB and Simulink.

## 3.2 Lane width selection

A properly designed infrastructure is very much necessary for smooth mobility and also for passenger's safety. Different countries have different administrations which are responsible for the development and maintenance of these infrastructures. The Swedish National Road Administration (SNRA) is the highway agency responsible for planning, designing, constructing and maintaining the transportation infrastructure in Sweden [9]. Similarly, The Road Directorate is the state agency of the Ministry of Transport responsible for roadways in Denmark. Therefore, every country's standard for lane width differs from each other, and it is very important to adopt the lane width completely on the basis of thorough literature review.

High speeds rural highways experience a very high number of run-off crashes. In Sweden, for example, run-off crashes on two-lane road accounts to a third of all the crashes on these roads, if we go by the numbers around 115 out of 339 crashes are of the type of run-off crashes [10], Where head-on crashes accounted to the second most type of crashes. A similar trend was noticed in other countries. Community database on Accidents on the Roads in Europe (CARE) is a community database which collects detailed data of individual accidents as collected by the member states of the European Union(EU). Annual accident report in 2018 by CARE records that

about 25.600 lives are lost and more than 1.4 million people are injured in 2016[11]. Considering all these things, it is essential to adopt lane width, which could provide high fidelity with the real world.
By thorough research on different lane width standards, the typical cross-section of rural roads is 13 meters in Sweden and Denmark, where it is divided into two lanes of 5.5 meters each with a shoulder width of 1 meter[9], while in England a narrower cross-section of 9.2 meters, where it is divided into two lanes of 3.75 meters each with a shoulder width of 1 meter on each sides[9]. However, in Germany, the typical cross-section of 10.5 meters, which is an average of Sweden's and England's road width, where 3.75 meters is designated for travel lanes and 1.5 meters is designated for shoulder width [9]. Also, according to the Trans-European North-South Motorway (TEM), it suggests that in the case of two carriageways, it is preferable to consider traffic lanes with a width of 3.75 meters[12]. Therefore, 5.25 meters is the maximum available lateral distance by considering the total width of one and a half standard European highway lane widths which would be sufficient in order to complete the manoeuvre successfully.

## 3.3 Selecting Speed Limit for virtual simulation

The thesis mainly focuses on lateral dynamics where the vehicle is intended to travel towards the stationary obstacle before undergoing the evasive manoeuvre to avoid the obstacle.

Since it is an open-loop simulation, the speed plays an important role, which influences lateral dynamics of the vehicle such as yaw rate and lateral acceleration of the vehicle. Therefore, selecting a constant speed for the simulation is very important. Also, the main motive is to check the last point of no steering on rural highways where the speed limit is usually higher and speed urban roads where the speed limit is comparatively less. As mentioned in the Introduction section, the research paper [25] illustrates how the collision avoidance is much more effective at higher velocities when only steering action is considered to avoid the obstacle. At higher velocities, if the driver has missed the last point of braking, there is still the opportunity to evade the obstacle by steering action. Therefore, the function of designing an active safety system such as emergency steer assist systems exists on motorways, where the speed limit is usually high.

As in the case of selecting a standard lane width, even the standardized speed limits for different scenarios must be taken into consideration. The speed limits for highways vary for different countries. Generally, the national government decides on different speed limits for different road types. According to European standards, a common approach known as V85 is adopted in order to decide the speed limits, V85-speed is the speed that is not exceeded by 85 percent of the vehicles[13]. The current speed limits for highways in the European Union(EU) member states is 120 km/hr to 130 km/hr[13]. The generalized speed limit for rural motorways is 80-90 km/hr, whereas, for urban motorways, the speed limit is 50km/hr[13]. Therefore, according to the European Standards, we adopted speed limits as 100 km/hr and 50 km/hr for rural and urban motorways, respectively.

## 3.4 Constraints on lateral acceleration and Maximum possible lateral displacement

According to the study conducted on the combined longitudinal braking assistance and steering for collision avoidance [26], it reflects on the importance of lateral acceleration on the last point of steering, where the last point to steer $d_e$ depends on lateral displacement $s_y$, lateral acceleration $a_y$ and relative velocity of the vehicle $v_{rel}$. [26] The paper provides a relationship between these parameters and is given by the expression:

$$d_e = \sqrt{\frac{2 \cdot s_y}{a_y}} \cdot V_{rel} \tag{3.1}$$

Since the simulations must mimic realistic conditions, ride comfort during evasive manoeuvre needs to be taken into consideration. Lateral accelerations significantly influence ride comfort during the evasive manoeuvre, and hence a restriction on lateral acceleration levels during the evasive manoeuvre was considered. In the previous research [14] on the magnitude of lateral acceleration levels pertaining to moderate levels of driver discomfort was restricted at $5m/s^2$. The longitudinal acceleration is constant as the vehicle travels at constant longitudinal velocity when the manoeuvre is taking place. Previous research by [15] shows that normal steering inputs from drivers corresponds to 1.4 $m/s^2$ and can reach up to 4 $m/s^2$ for 90th percentile cases. Hence our decision to set the lateral acceleration to 5 $m/s^2$ can be seen as the inclusion of hard steering behaviour. Previous research on the evaluation method of highway alignment comfortableness, states that threshold values of comfort levels being 1.8 $m/s^2$ and medium comfort and discomfort values being around 3.6 and $5m/s^2$, respectively[16]. So arguably the threshold chosen for our thesis can be considered on slightly discomfort range, but it can be assumed that during a manoeuvre the driver only feels such high levels of lateral accelerations for a brief moment and hence can be realistic enough without too much compromise on the driver comfort level.

Considering that active safety systems have to act when the driver is unaware of the situation ahead, a fair compromise in the driver comfort level for increased safety can be reasonable. Hence, we considered our chosen threshold value to be justified for performing simulations.

## 3.5 Selected cornering stiffness for vehicle models

As mentioned in the scope of the thesis, it is very vital to consider different parameters which influences lateral dynamics of the vehicle and which in turn the last point of steering depends on the lateral dynamics of the vehicle. From the previous work on cornering stiffness [27], the paper suggests based on Bergman's data, cornering stiffness co-efficient is a linear function of normal load, and cornering stiffness is a quadratic function of the normal load. The paper further reflects on how the cornering stiffness for the stability of the vehicle depends on the normal load. It suggests the behaviour of lateral stability of the vehicle for one specific loading condition. The value of the cornering stiffness is required for only one specific or designated loading condition. However, if lateral stability information for the same vehicle for a different loading condition is required, then it is important to obtain new values for the cornering stiffness.

Further, in our thesis, we consider linear tire model and non-linear tire model, to investigate the effect of lateral dynamics which influences the last point of steering. Since we are considering Saab 9-3 specifications [22] for all the vehicle models, the rated tire load for the vehicle is 4000 Newtons. When we consider the Linear tire model, the theory on linear tire model [28], where it provides a relationship for linearized force and slip angles which is represented as:

$$F_y = c_\alpha \cdot \alpha \tag{3.2}$$

The lateral force depends mainly on cornering stiffness and slip angles, unlike the non-linear tire model where the lateral force does not directly depend on cornering stiffness. The lateral force generated for non-linear tire model is given by the expression:

$$F_y = \mu F_z \sin\left(C \arctan\left(B\alpha - E\Big(B\alpha - \arctan\left(B\alpha\right)\Big)\right)\right) \tag{3.3}$$

where D is given by the expression:

$$D = \mu \cdot F_z \tag{3.4}$$

Further, the research paper on tire characteristics and vehicle handling [28], illustrates how the cornering stiffness is calculated based on the rated tire load. The shape factors C and E, as well as tire parameters $c_1$ and $c_2$ and the friction co-efficient which mostly depends on the vertical load and speed, maybe estimated through regression techniques, therefore in this thesis, since the nominal rated load is fixed to 4000 Newtons as mentioned previously, the cornering stiffness is not altered in the case of non-linear tire models. The relationship mentioned by Hans Pacejka for estimating cornering stiffness is given by the expression:

$$C_\alpha = B \cdot C \cdot D = c_1 \dot{s}in \cdot (2 \arctan \cdot \cdot \frac{F_z}{c_2}) \tag{3.5}$$

## 3.6 Steering input for evasive maneuver

The primary input to the simulation is the steering angle. Since the purpose of the simulation is to evaluate the impact of vehicle models in determining the last point of steering in time and distance to avoid a collision upon detection of a stationary vehicle successfully, the steering profile is intended to guide the motion of the vehicle to perform a lane change manoeuvre. The lane change manoeuvre is based on ISO 3833[5].

The primary focus of the thesis being the evaluation of the different vehicle models representing the planar dynamics of the vehicle, the steering input was decided to be an open-loop steering model (a reference to thesis proposal). However, it is still important to discuss the differences between open loop and closed loop steering system and evaluate if there are any costs or benefits associated with the use of open-loop steering model over closed-loop steering model.

Open-loop steering is implemented with the front wheel steer angle as a function of time. The examples of open-loop steering inputs include step/ramp, one period sinusoidal, random input and continuous sinusoidal. While closed-loop steering input according to ISO constitutes of the driver and a prevailing environment, where the driver interactively controls the steering input to guide the vehicle along a predetermined trajectory.

Open-loop steering input can be designed by developing a simple function between steer angle and time while implementing a closed-loop steering system without having the driver in the loop will involve the use of complex control theory algorithms. For example, research undertaken by [18], a closed-loop steering model was developed using a controller generating a reference signal of the steering wheel angle and a Kalman regulator to correct the differences between actual vehicle parameters influencing the desired trajectory and the reference vehicle parameters. Using such a complex controller will undoubtedly increase the complexity and run time of the simulations used to evaluate dynamic vehicle behaviour in driving scenarios. During track tests for lane change manoeuvres usually, human drivers are used with a defined path, implying a closed-loop steering input. However, in the research paper by [19], suggest that such an approach will result in poor test repeatability due to the presence of variation in driver steering behaviour and provide a design for open-loop steering profile to follow a double lane change trajectory. ISO also proposes open-loop methods for standardized tests to determine transient behaviour of vehicles in lane-change manoeuvres, for example, in ISO tests like ISO 3833 [5]. In doing so, they suggest that evaluating dynamic behaviour of vehicles is highly complex due to the inherent complexities present in driver-vehicle-environment interactions and hence conclude that data intended to correlate vehicle dynamics properties/behaviour and accident avoidance should be collected from many individualistic tests [5]. From this theory provided by ISO, it can be safely interpreted that individualistic tests or simulations used in the development of active safety systems should have good repeatability. Based on previously mentioned research by [19], it can be suggested that open-loop steering inputs help improve the property of repeatability in such tests and simulations.

## 3.7 Implementation of open loop steering for single lane and double lane change maneuver

The implementation of open-loop steering input started with some research for trajectory representing double or single lane-change manoeuvre. Previous work undertaken by [23] aimed at developing a fuzzy active steering controller and evaluate its performance in hardware in loop driving simulator. The driving simulator performed a double lane change manoeuvre as an open-loop driving test. In order for conducting the open-loop test, [23] in their work provides a graphical representation of required steering input for an open-loop double lane change manoeuvre. The following figure 3.1 is replicating the graphical representation for an open-loop double lane change manoeuvre provided by previous research [23].

Based on the open-loop steering input idea provided by the above figure a time function based steering input was developed to be used in the simulations. The steering model developed comprises of two, single-period sinusoidal functions of front-wheel steer angle (deltaf) and simulation time, separated by a dwell duration.
The following image represents the steering model (front-wheel steer angle as a function of simulation time) used in the simulations.

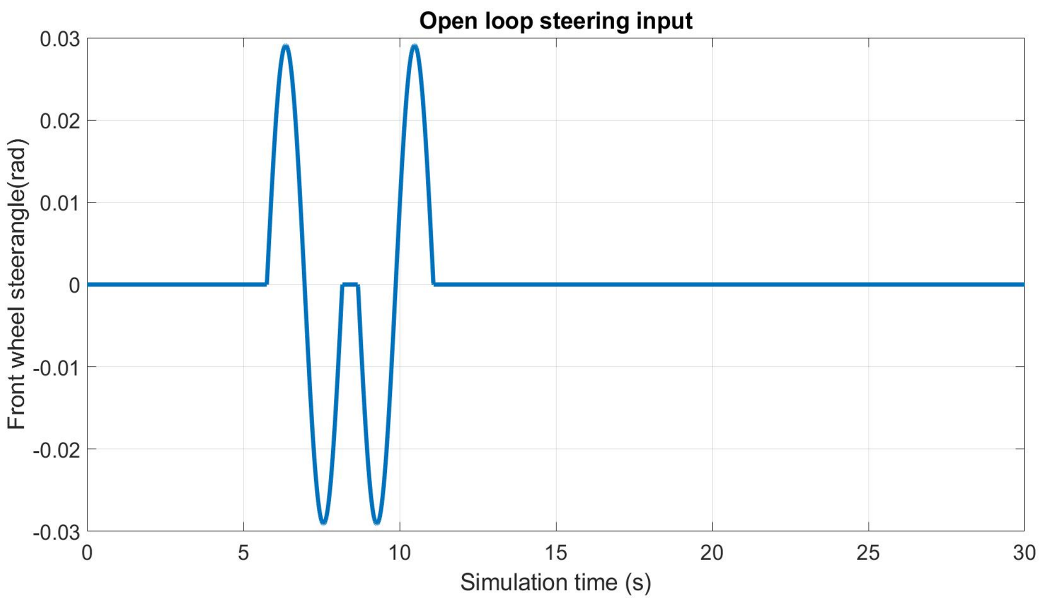


**Figure 3.1:** Open-loop steering inputs used in simulations

Figure 3.1 represents the open-loop steering model developed and used in the simulations. Thus, it is expected that with this model as the input, the vehicle should

travel along a trajectory that approximately mimics the double lane change manoeuvre. The start of the first sine wave represents the start of the lane change manoeuvre. The time period of each of the single period sine waves represents the duration of each manoeuvre.
The following figure shows the obtained trajectory of the vehicle after implementing the above-shown steering model into the simulation model.

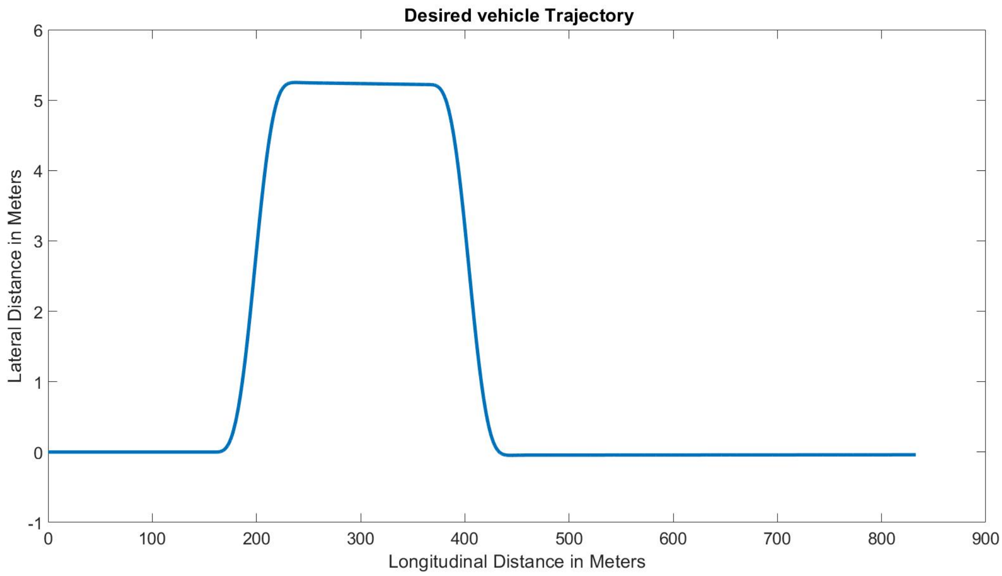


**Figure 3.2:** Vehicle trajectory obtained after implementation of open loop steering system

From the previous figure, it is reasonable to assume that the steering model used produces a trajectory that approximately mimics a double lane change manoeuvre. Hence based on the reasonable accuracy of the designed steering model it was implemented suitably to determine the last point of steering in time(distance) to avoid collisions, for each of the previously mentioned vehicle models. Although the steering model is designed to mimic double-lane change, the main focus is on the first complete sine wave which represents the first lane change of the ego-vehicle as shown in the figure 3.3. In other words, it can be termed as a single-lane change of the ego vehicle in order to avoid the stationary lead vehicle.

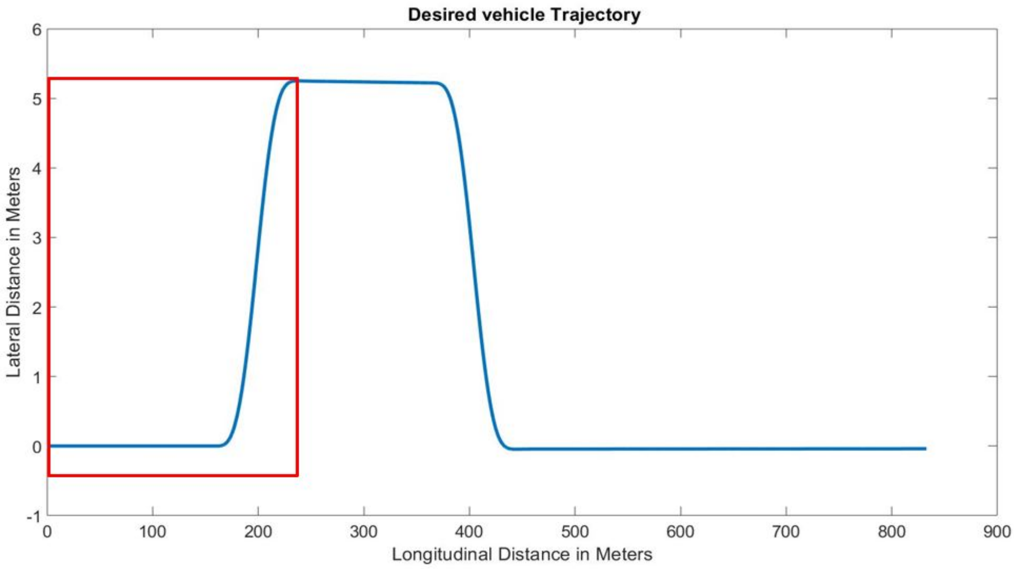


**Figure 3.3:** First sine wave representing the first lane change

## 3.8 Inputs-outputs for the simulation frame

By making use of this simulation set-up and considering the vehicle specifications of Saab-9-3 as mentioned earlier, different models are simulated. The inputs to the model are the steering profile and longitudinal velocity and tire model. By providing these inputs to the vehicle model, it helps us to calculate the lateral acceleration, heading angle and vehicle trajectory, as shown in figure 3.1. This process is repeated for every model and based on the output the analysis is made to suggest which vehicle model allows for the latest point of steering in time, in order to avoid the collision while satisfying the pre-set criteria successfully.

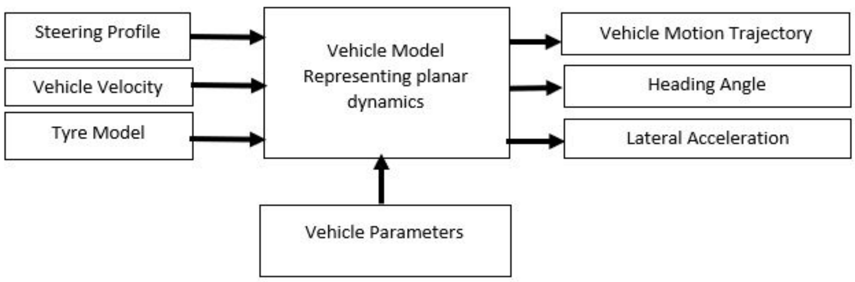


**Figure 3.4:** Simulation process

## 3.9 Procedure

Previous research on the development of an emergency steering system for collision avoidance [29], the paper reflects on the structure and function adopted for developing emergency steering systems. The paper explains how the emergency steering system functions to initiate an emergency steering manoeuvre and how different it is from AEB systems that are triggered by time-to-collision. Further, it reflects on how the system calculates the danger level using the maximum lateral acceleration of the planned evasion trajectory; later the emphasis is made on the lateral displacement where it is determined through a process of iteration, each iteration checks whether the calculated avoidance path leads around the obstacle at a safe distance. It also stresses on the importance of an additional lateral safety tolerance of lateral distance equal to half the width of the vehicle must be added considering the vehicle's yaw angle. As soon as the avoidance path satisfies the set criteria, the iteration ends beneath the critical point. A similar approach has been adopted in our thesis to find the last point of steering in time.

### 3.9.1 Stationary vehicle as an obstacle

Once the values of the parameters front-wheel steer angle (delta) and manoeuvre time are set to meet the necessary constraints of lateral acceleration and lateral displacement, the value of manoeuvre start time is varied iteratively to arrive at the last point of steering in time(distance) to avoid a collision. For each of the iterative simulations carried out, the manoeuvre start time was incremented until the point when the lateral distance between the stationary vehicle and the manoeuvring vehicle was less than 2.7m. Having 2.7m between centres of the two vehicles will imply that the approximate lateral clearance between the two vehicles is about 1m. This value of manoeuvre start time was recorded and was noted to be the last point in time to start steering and compared among the 3 different vehicle models chosen for different conditions.

Figure 3.5 shows the dimensions of the stationary vehicle in terms of wheelbase and track-width. The dimensions adopted for the stationary lead vehicle matches with the dimensions of ego-vehicle which performs evasive manoeuvring with a lateral clearance of 1m as mentioned earlier.

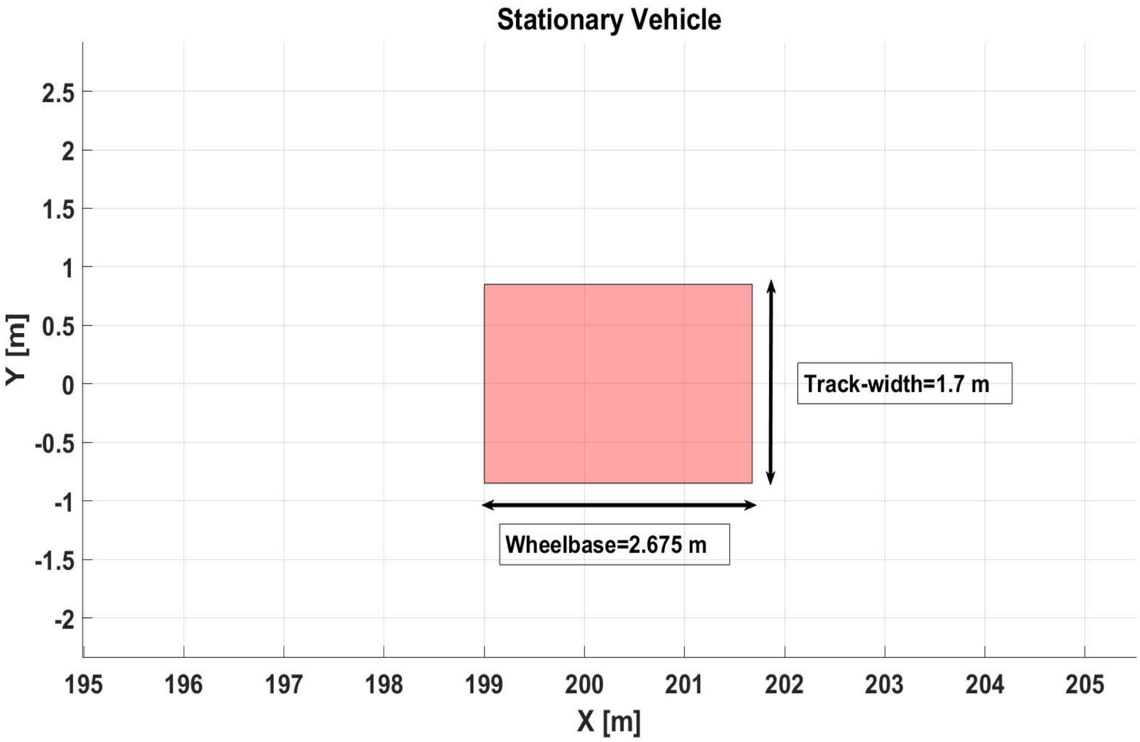


**Figure 3.5:** Stationary vehicle as an obstacle with dimensions

### 3.9.2 Determining the last point of steering in time

The following is a description of the procedure used in the carrying out the simulations on an iterative basis to find the last point of steering for each of the 3 different vehicle models taken into consideration.

From figure 3.4, it can be observed that the primary input to the vehicle model is the steering profile. The steering profile used in the simulations had been embedded within various parameters that dictated the desired trajectory mimicking the collision avoidance manoeuvre. The parameters present within the steering profile are as follows:

- **Front-wheel steer angles (delta)**

- **Maneuver time:** Total time taken to complete the evasive manoeuvre otherwise the duration within which the vehicle avoids the target vehicle by moving left and returns to straight-line motion and performs the second manoeuvre to return to the same lane as it was previously travelling in.

- **Maneuver start time:** Dictates the instantaneous time at which the vehicle starts performing the evasive manoeuvre. The purpose of iterative simulations is to determine the latest possible manoeuvre start time for each of the models and analyze if there exists a significant difference between them.

Once the above-mentioned simulation model is set in the Simulink environment, the parameters that need to be fixed based on constraints of lateral accelerations and maximum possible lateral displacement of the vehicle are decided.

The simulation process is carried out first for all the three models having non-linear tire model. A suitable comparison is made among the three models to suggest which is the best model in terms of the last point of steering in seconds by satisfying all the criteria mentioned before. Then the same process is carried out for all the three models having linear tire model. Similarly, the last point of steering in seconds is calculated by varying cornering stiffness. Based on the outputs, the best model is selected for avoiding the obstacle.

## 3.10 Model Validation for different steering input

### 3.10.1 Model validation for maximum and minimum criteria

By providing the inputs to the model and satisfying the decided maximum criteria, the models are simulated for three different steering inputs in Radians($rad$), a low steering input of 0.03 $rad$, a high steering input of 0.07 rad and an average steering input of these two values are selected to analyze how the vehicle models perform to avoid the obstacle and which model stands out in terms of the last point of no-return, the model which manages to avoid the obstacle as late as possible by satisfying the maximum criteria can be picked for testing and evaluation of active safety systems during evasive manoeuvring. A similar approach was carried out for minimum criteria Refer A.1 and A.2 .

### 3.10.2 Vehicle Specifications

The table 3.1 shows the vehicle specifications for Saab 9-3, The specifications for the vehicle models were used from the vehicle dynamics lecture handout[22]:

**Table 3.1:** Saab 9-3 Specifications.

| **Vehicle Specifications** | **Value** | **Unit** |
|---|---|---|
| Mass of the vehicle | 1915 | Kilogram |
| Moment of inertia about Z-axis | 2617 | $kgm^2$ |
| Wheelbase | 2.675 | meter |
| Track-width | 1.7 | meter |
| Distance from CoG to front axle | 1.07 | Meter |
| Distance from CoG to rear axle | 1.605 | Meter |
| Front axle cornering stiffness at origin | 147,600 | $N/rad$ |
| Rear axle cornering stiffness at origin | 123,470 | $N/rad$ |
| Sprung mass | 1515 | Kilogram |
| Vehicle inertia about X axis | 1000 | $Kgm^2$ |
| Vehicle total roll stiffness | 60000 | $Nm/rad$ |
| Vehicle total roll damping | 5000 | $Ns/m$ |
| Vehicle roll stiffness distribution | 0.55 | - |
| Vehicle roll damping distribution | 0.55 | - |
| Steering ratio | 15.9 | - |
| Height of CoG | 0.543 | Meter |
| Front roll center height | 0.045 | Meter |
| Height of CoG above roll axis | 0.4762 | Meter |
| Magic formula parameter C | 1.5 | - |
| Magic formula parameter E | -1 | - |
| Tyre load based non-linearity parameter for friction | 0.00006 | - |
| Tyre stiffness parameter | 10.65 | - |
| Tyre load based non-linearity parameter for stiffness | 0.000111 | - |
| Rated load for the tyre | 4000 | Newton |

# 4
# Results

The procedure mentioned in the methodology section has been adopted to find the last point of steering in time to successfully avoid a collision for three different vehicle models representing planar dynamics. The evasive manoeuvre designed only depends on steering and does not have a braking component to avoid the obstacle; therefore, the emphasis is much more on lateral dynamics of the vehicle and the last point of steering.

The following section consists of tables and scatter plots depicting the results from the simulation. The results are discussed for two different limits on lateral acceleration which are $5m/s^2$ and $9.81m/s^2$(1g lateral acceleration) and is followed by results obtained by varying the cornering stiffness of the tire model .

The cornering stiffness of the front and rear tires are reduced from 147600 $N/rad$ and 123470 $N/rad$ to 90,000 $N/rad$ and 70,000 $N/rad$ respectively for linear-tire models.

The first set of results are analyzed for the vehicle travelling on an urban motorway, where the speed limit is set to 50 km/hr as mentioned earlier in the methodology section and the second set of results are analyzed for the vehicle travelling on a rural motorway, where the speed limit is set to 100 km/hr.

## 4.1 Dataset1 for comparing the last point of steering

The following table contains results from simulations performed for three different vehicle models of varying complexities, for a vehicle velocity of 50 km/hr.
The tables used to represent the results consist of delta, which is the maximum front-wheel steering angle achieved in *rad*, manoeuvre start time in *s* and manoeuvre duration in *s* for different conditions. By performing iterations to satisfy the lateral displacement with an additional safety tolerance and lateral acceleration limit for the vehicle, the steering input, delta is determined. Once the steering input delta is determined the manoeuvre start time or the last point of steering is determined through iterative simulations.

**Table 4.1:** Results for Vehicle models with linear tire model at 50 $km/hr$

| **Vehicle models** | **Steering input (rad)** | **Maneuver start time (s)** | **Maneuver duration (s)** | **Refer Appendix for plots** |
|---|---|---|---|---|
| 1. Lateral acceleration limit 5 $m/s^2$ | | | | |
| Two-track without load transfer | 0.0799 | 13.02 | 2.539 | A.7 |
| Single-track model | 0.0809 | 13.03 | 2.524 | A.8 |
| Two-track with load transfer | 0.0825 | 13.04 | 2.533 | A.9 |
| 2. Lateral acceleration limit 9.81 $m/s^2$ | | | | |
| Two-track without load transfer | 0.1612 | 13.406 | 1.797 | A.7 |
| Single-track model | 0.1645 | 13.419 | 1.78 | A.8 |
| Two-track with load transfer | 0.1688 | 13.437 | 1.794 | A.9 |
| 3. Reduced cornering stiffness 5 $m/s^2$ | | | | |
| Two-track without load transfer | 0.0893 | 12.975 | 2.421 | A.7 |
| Single-track model | 0.091 | 12.992 | 2.40 | A.8 |
| Two-track with load transfer | 0.0933 | 12.999 | 2.423 | A.9 |

Further, for analyzing the determined simulation results, the vehicle model with the highest complexity which is, the two-track model with load transfer is considered to be the benchmark, against which the results of two-track without load transfer and single-track vehicle models are compared. Such an analysis is done to examine if there any huge differences in the results when the vehicle models of relatively lower complexity are used. In this case, we are examining deviations in steering input and the last point of steering in time, based on which the impact of varying complexity of vehicle models in virtual simulations can be studied:

For simplicity purpose, the three selected vehicle models are named as follows:
Model 1- Two-track model with load transfer
Model 2- Two-track model without load transfer
Model 3- Single-track model

**Table 4.2:** Percentage deviations in steering input and the last point of steering

| **Simulation parameters** | **Model 1** | **Model 2** | **% change in parameter w.r.t model 1** | **Model 3** | **% change in parameter w.r.t model 1** |
|---|---|---|---|---|---|
| 1. Lateral acceleration limit 5 $m/s^2$ | | | | | |
| Steering input (rad) | 0.0825 | 0.0799 | 3.1 % | 0.0809 | 1.9 % |
| Last point of steering in seconds (s) | 13.04 | 13.02 | 0.1 % | 13.03 | 0.07 % |
| 2. Lateral acceleration limit 9.81 $m/s^2$ | | | | | |
| Steering input (rad) | 0.1688 | 0.1612 | 4.5 % | 0.1645 | 2.5 % |
| Last point of steering in seconds (s) | 13.437 | 13.406 | 0.2 % | 13.419 | 0.1 % |
| 3. Reduced cornering stiffness | | | | | |
| Steering input (rad) | 0.0933 | 0.0893 | 4.2 % | 0.091 | 2.4 % |
| Last point of steering in seconds (s) | 12.999 | 12.975 | 0.1 % | 12.992 | 0.05 % |

From the table, the deviations in steering input and the last point of steering for Model 2 and model 3 are compared against model 1 as it has been chosen as the benchmark for comparison. It can be noticed that Model 2 and Model 3 take less steering input for all the cases to avoid the collision successfully. However, as far as we are concerned, the deviation in the last point of steering must be taken into account specifically for comparison. The deviation in the last point of steering is substantially low and does not provide any huge differences for suggesting one specific model at 50 km/hr. The trend of the results is similar for all the three cases mentioned in the table. The increased steering input for two-track with load transfer is due to the load transfer phenomena. Vehicle dynamics theory on the influence of load transfer on the resulting axle characteristics illustrates that at load transfer, the outer tire exhibiting a larger load will generate a larger side force than the inner tire. However, the average side force will be smaller than the original value it had in the absence of load transfer; this is mainly due to the non-linear relationship between $F_y$ and $F_z$ [30]. However, when it comes to the last point of steering in seconds, a minor difference in time was observed.

**Table 4.3:** Results for Vehicle models with non-linear tire model at 50 $km/hr$

| **Vehicle models** | **Steering input (rad)** | **Maneuver start time (s)** | **Maneuver duration (s)** |
|---|---|---|---|
| 1. Lateral acceleration limit 5 $m/s^2$ | | | |
| Two-track without load transfer | 0.082 | 13.025 | 2.52 |
| Single-track model | 0.16 | 13.010 | 2.222 |
| Two-track with load transfer | 0.1004 | 13.028 | 2.384 |

The results are tabulated for all the three-vehicle models with non-linear tire models at 50 km/hr. The main motive here is to interpret the results obtained from the iterative simulations. By performing iterations as mentioned earlier, provides the latest point of steering for all the vehicle models with non-linear tire behaviour, which can be further used to analyze how the varying complexity of the vehicle models has an impact on it.

**Table 4.4:** Percentage deviations in steering input and the last point of steering

| **Simulation parameters** | **Model 1** | **Model 2** | **% deviation w.r.t model 1** | **Model 3** | **% deviation w.r.t model 1** |
|---|---|---|---|---|---|
| 1. Lateral acceleration limit 5 $m/s^2$ | | | | | |
| Steering input (rad) | 0.1004 | 0.082 | 18.3 % | 0.16 | 37.2 % |
| Last point of steering in seconds (s) | 13.028 | 13.025 | 0.02 % | 13.010 | 0.1 % |

When similar comparison is made for vehicle models with varying complexities equipped with non-linear tire model, the last point of steering in time for all the models were very close to each-other and no substantial deviations were noticed just like in the case of linear-tire models.

## 4.2 Scatter plot for 50 $km/hr$

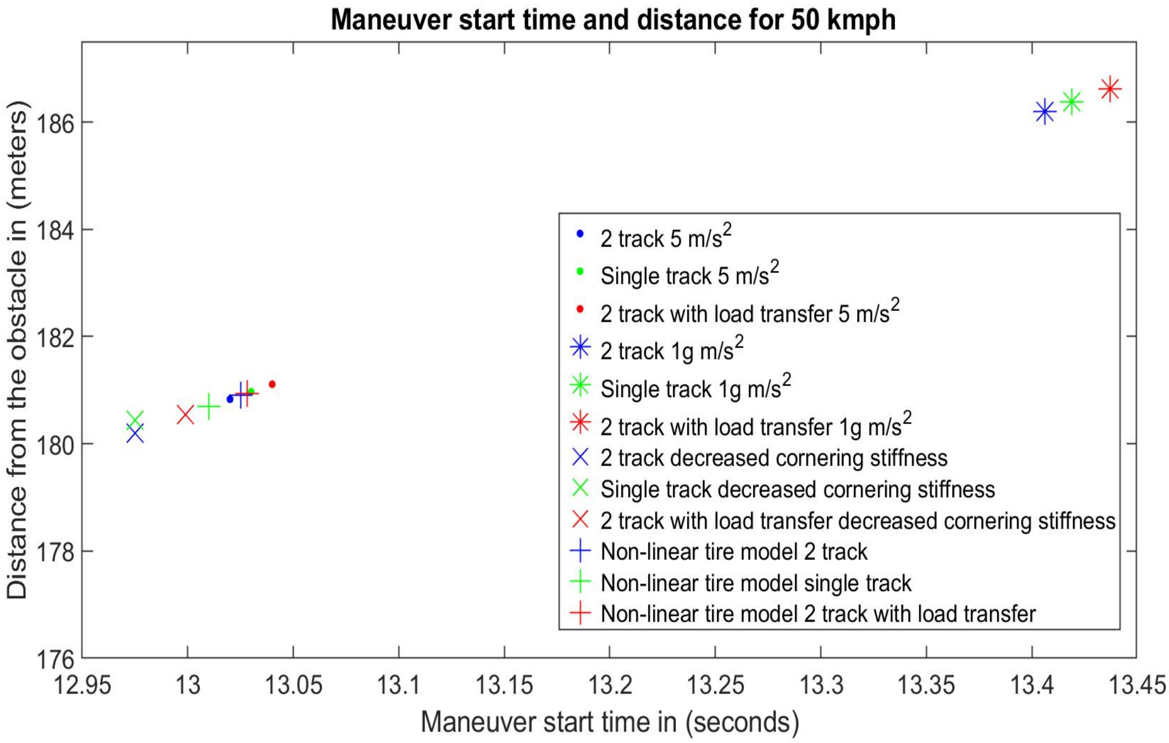


**Figure 4.1:** Scatter plot for 50 $km/hr$

The scatter plots are used to summarize the results depicted in the above tables; it includes results from simulations performed for each of the vehicle models for all the different conditions considered. The last point of steering in time is plotted against the distance from the stationary vehicle.

From the figure it can be noticed that the trend for the three vehicle models is same when the simulations were performed by limiting lateral acceleration to 5 $m/s^2$, 9.81 $m/s^2$ also when the cornering stiffness was reduced. If we compare these models, the two-track with load transfer for all the conditions gives the latest point of steering in time just by a few milliseconds. However, on a broader picture, if we consider the varying complexity of the models, it does not play a crucial part at 50 km/hr.

## 4.3 Dataset2 for comparing the last point of steering

The following table contains results from simulations performed for three different vehicle models of varying complexities, for a vehicle velocity of 100 km/hr.

The approach adopted is similar to the approach adopted for 50 km/hr to find out the latest point of steering for all the three selected models.

**Table 4.5:** Results for Vehicle models with linear tire model at 100 $km/hr$

| **Vehicle models** | **Steering input (rad)** | **Maneuver start time (s)** | **Maneuver duration (s)** | **Refer Appendix for plots** |
|---|---|---|---|---|
| 1. Lateral acceleration limit 5 $m/s^2$ | | | | |
| Two-track without load transfer | 0.0278 | 5.7025 | 2.465 | A.12 |
| Single-track model | 0.028 | 5.758 | 2.422 | A.13 |
| Two-track with load transfer | 0.0294 | 5.765 | 2.455 | A.14 |
| 2. Lateral acceleration limit 9.81 $m/s^2$ | | | | |
| Two-track without load transfer | 0.0617 | 6.15 | 1.646 | A.12 |
| Single-track model | 0.0628 | 6.165 | 1.632 | A.13 |
| Two-track with load transfer | 0.0648 | 6.172 | 1.66 | A.14 |
| 3. Reduced cornering stiffness 5 $m/s^2$ | | | | |
| Two-track without load transfer | 0.0350 | 5.70 | 2.233 | A.12 |
| Single-track model | 0.0355 | 5.711 | 2.2177 | A.13 |
| Two-track with load transfer | 0.037 | 5.721 | 2.277 | A.14 |

Further, for analyzing the determined simulation results, the vehicle model with the highest complexity which is, the two-track model with load transfer is considered to be the benchmark, against which the results of two-track without load transfer and single-track vehicle models are compared. The deviations in steering input and the last point of steering is determined and tabulated in the table.

**Table 4.6:** Percentage deviations in steering input and the last point of steering

| **Simulation parameters** | **Model 1** | **Model 2** | **% deviation w.r.t model 1** | **Model 3** | **% deviation w.r.t model 1** |
|---|---|---|---|---|---|
| 1. Lateral acceleration limit 5 $m/s^2$ | | | | | |
| Steering input (rad) | 0.0294 | 0.0278 | 5.4 % | 0.028 | 4.7 % |
| Last point of steering in seconds (s) | 5.765 | 5.7025 | 1 % | 5.758 | 0.1 % |
| 2. Lateral acceleration limit 9.81 $m/s^2$ | | | | | |
| Steering input (rad) | 0.0648 | 0.0617 | 4.7 % | 0.0628 | 3 % |
| Last point of steering in seconds (s) | 6.172 | 6.15 | 0.3 % | 6.165 | 0.1 % |
| 3. Reduced cornering stiffness | | | | | |
| Steering input (rad) | 0.037 | 0.0350 | 5.4 % | 0.0355 | 4 % |
| Last point of steering in seconds (s) | 5.721 | 5.70 | 0.3 % | 5.711 | 0.1 % |

From the table, the deviations in steering input and the last point of steering for Model 2 and model 3 are compared against model 1 as it has been chosen as the benchmark for comparison. It can be noticed that Model 2 and Model 3 take less steering input for all the cases to avoid the collision successfully. However, as far as we are concerned, the deviation in the last point of steering must be taken into account specifically for comparison. The deviation in the last point of steering is substantially low and does not provide any huge differences for suggesting one specific model at 100 km/hr. The trend of the results is similar for all the three cases mentioned in the table. The main focus is on the trend of the results between 50 and 100 km/hr. But, when the comparison was made there was no difference in the trend of the results even though the longitudinal speed was doubled. In other words, the influence of longitudinal force on the vehicle models has minimal or negligible impact. The increased steering input for two-track with load transfer is due to the load transfer phenomena as mentioned in the results for 50 km/hr.

**Table 4.7:** Results for Vehicle models with non-linear tire model at 100 $km/hr$

| **Vehicle models** | **Steering input (rad)** | **Maneuver start time (s)** | **Maneuver duration (s)** |
|---|---|---|---|
| 1. Lateral acceleration limit 5 $m/s^2$ | | | |
| Two-track without load transfer | 0.0291 | 5.752 | 2.415 |
| Single-track model | 0.0590 | 5.74 | 1.942 |
| Two-track with load transfer | 0.0412 | 5.772 | 2.25 |

**Table 4.8:** Percentage deviations in steering input and the last point of steering

| **Simulation parameters** | **Model 1** | **Model 2** | **% deviation w.r.t model 1** | **Model 3** | **% deviation w.r.t model 1** |
|---|---|---|---|---|---|
| 1. Lateral acceleration limit 5 $m/s^2$ | | | | | |
| Steering input (rad) | 0.0412 | 0.0291 | 29 % | 0.0590 | 30 % |
| Last point of steering in seconds (s) | 5.772 | 5.752 | 0.3 % | 5.74 | 0.5 % |

When similar comparison is made for vehicle models with varying complexities equipped with non-linear tire model, the last point of steering in time for all the models were very close to each-other and no substantial deviations were noticed just like in the case of linear-tire models. Also, the trend of the results for 100 km/hr did not deviate from the results for 50 km/hr.

## 4.4 Scatter plot for 100 $km/hr$

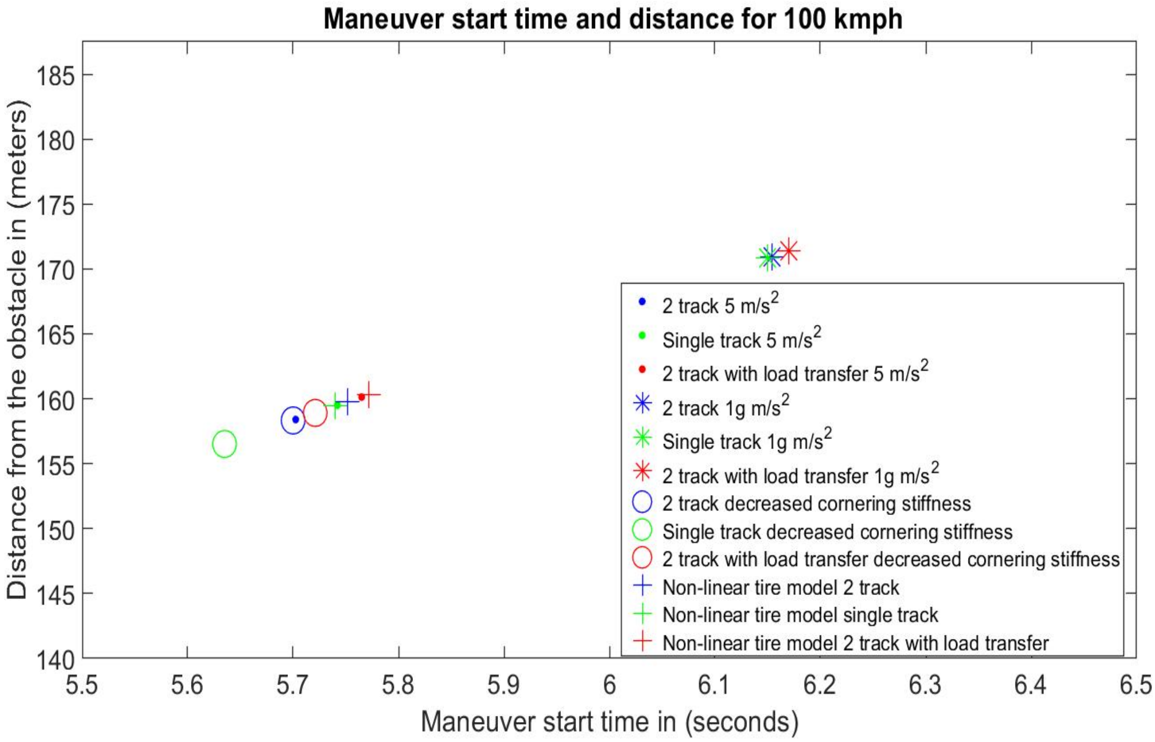


**Figure 4.2:** Scatter plot for 100 $km/hr$

From the figure it can be noticed that the trend for the three vehicle models is same when the simulations were performed by limiting lateral acceleration to 5 $m/s^2$, 9.81 $m/s^2$ also when the cornering stiffness was reduced. If we compare these models, the two-track with load transfer for all the conditions gives the latest point of steering in time just by a few milliseconds. However, on a broader picture, if we consider the varying complexity of the models, it does not play a crucial part at 100 km/hr.

## 4.5 Comparison between the results at 50 and 100 $km/hr$

On comparing the results about the last point in time to start steering between 50 and 100 $km/hr$. We can observe a similar trend for the two different vehicle velocities considered. The results corresponding to linear tire models are of similar pattern for the two different speeds, with the two-track model with load transfer requiring the highest steering input to perform the manoeuvre and with no considerable difference in terms of the last point in time to start steering.

## 4.6 Computation time

**Table 4.9:** Computation time for vehicle models with different complexity levels

| Vehicle Model | Time in seconds |
|---|---|
| 2 track without load transfer | 4.21 |
| Single track | 3.491 |
| 2 track with load transfer | 6.78 |

The difference in simulation time for the three-vehicle models with different complexities is tabulated for one iteration in the table 4.9. Previous research carried out on simulation run-time for modelling and simulation of vehicle kinematics and dynamics [34], suggests that computation time is an important factor for the feasibility of the models. From table 4.9, it can be observed that by considering a more complex model, the computation time increase substantially.

### 4.6.1 Percentage change in computation time

**Table 4.10:** Computation time in hours for 10,000 iterations

| Vehicle Model | Time in hours |
|---|---|
| Model 1 | 11.69 |
| Model 2 | 18.83 |
| Model 3 | 9.69 |

Since in our thesis iterative method has been adopted to find the last point of steering, therefore for example when the simulation run-time is considered for 10,000 iterations, the difference in total time (hours) taken by three different models to perform 10,000 iterations can be seen. Model 2 takes around 19 hours whereas, Model 1 and Model 3 takes 12 and 10 hours respectively. Since there is a minimal impact of varying complexities of models on the last point of steering in time, we argue that using the more simple model (i.e., Model 3), would be sufficient for many safety assessment tasks. Model 3 takes almost 50 and 20 percent less computation time when compared to Model 2 and Model 1, respectively.

# 5
# Conclusion

The analysis of the results shows that there were no substantial difference in the timing of the last point of steering to avoid a collision, across the different models considered. The results followed a similar pattern when the last point in time for steering was examined for different velocities, i.e. is 50 km/hr and 100 km/hr and different lateral acceleration limits such as 5 $m/s^2$ and 9.81 $m/s^2$. Even when the vehicle models were simulated with two different tire models as inputs, namely a linear tire model and a non - linear (magic tire) model, there were no prominent differences amongst the models in terms of the last point in time to start steering to avoid a collision with a stationary vehicle.

From the results, it can be inferred that the most complex model is not always necessarily the best model that can be used in virtual simulations. Previous work [32] aimed at studying desired simulation models for vehicle development arrived at a similar conclusion where comparison was made between a simple bicycle model and a full vehicle simulation model with flexible components which compares analytical capability of these models. Another study conducted on this topic concluded that vehicle models involving lateral roll transfer and body roll as essential parameters to be included in the models while simulating high g manoeuvres, along with an appropriate non-linear tire model [31]. The models considered for the current thesis evaluated the same with non-linear tire models at lateral acceleration limit of 5 $m/s^2$, and results do not imply a substantial difference.

The complexity of the vehicle models is also important in terms of simulation time and cost as there is an ever-increasing pressure on automotive manufacturers to reduce cost and time involved in the vehicle development stage. Based on our simulation framework built on MATLAB, different vehicle models of varying complexities were compared based on their computation time, the time taken for 10,000 iterations were noted down for all the considered vehicle models, We suggest that using a simpler model with lower run time for simulations can be effectively used for many cases as an alternative to models of higher complexity, as results in terms of timing of the point of no return are similar. However further work can be done on the analysis of the required tire model in predicting tire forces, slip and lag need, but it was not in the scope of the thesis.

Evaluation of vehicle safety systems on a virtual platform considers different scenarios. Consequently, vehicle models used for virtual simulations will be needed to represent the vehicle behaviour for a given scenario with reasonable accuracy. Previous work carried out on how model complexity affects the performance of a vehicle to design active safety systems for autonomous vehicles [35] suggests that using simple single-track model yields very accurate results. However, when a more complex kinematic vehicle model was considered, the kinematic model showed slightly more accurate results when compared to dynamical vehicle model. Therefore, according to their simulations, using a less complex model was sufficient as the impact of the kinematical (complex) model had a minimal impact on the results. However, the complexity of the vehicle models required might be of varying significance based on the scenario and safety system being evaluated. Previous work carried out on vehicle dynamic models for virtual testing of autonomous trucks [36] aims at evaluating different vehicle models such as single-track model(STM), one track model with linear tyre slip(OTM), and a complex model termed as Volvo transport model(VTM), to be used in a simulator for different scenarios imitating common situations in traffic and to evaluate the difference between vehicle models. Sinusoidal manoeuvring test adopted in [36] is of particular relevance to our thesis as it deals with assessing the vehicle's capability to evade a suddenly appearing obstacle by analysing yaw rate response. The models used in the previously mentioned study implement open-loop steering input for implementing sinusoidal manoeuvring and compare the models based on yaw rate response exhibited by different models. Results from [36] suggest that for the case of sinusoidal manoeuvring at low steering amplitudes and constant frequency there is not much difference between the chosen vehicle models and conclude a simple model such as single-track model can be used over complex vehicle model. However, when sinusoidal manoeuvring was performed at varying steering frequencies there exists a noticeable difference in terms of vehicle behaviour between a simple model and complex model [36]. This difference can be attributed to the increased complexity with which the VTM or the complex model describes the interaction between the road pavement and tyre by considering the combined effect of both longitudinal and lateral forces [36]. Hence models of higher complexity might prove important when relatively unstable manoeuvres such as high-frequency sinusoidal manoeuvring are being simulated. The work carried out as part of the thesis considers a collision avoidance manoeuvre in the form of a single lane change at relatively lower steering amplitude with an open-loop steering profile given by a single period sinusoidal function. This sort of manoeuvring can be considered a scenario depictive of more stable vehicle behaviour, and lateral acceleration limitations considered in the thesis eliminated the possibility of analysing model behaviour at higher steering amplitudes and varying steering frequencies. Hence with regard to the scenario considered as part of this thesis the absence of a noticeable difference between the models is a result that seems to be consistent with those suggested in the above-mentioned work.[36]

Hence from our results out of three dynamical models considered, the bicycle model being the simplest vehicle model can be picked for development and assessment of active safety systems virtually, as the difference in the last point of steering is of the order of few milliseconds among other considered vehicle models. By taking into account the advantage offered by the simplest vehicle model in terms of computation time, it can be suggested that for developing and evaluating active safety systems the most complex model is not always necessarily the best model that can be used in such simulations. Further work will need to focus on assessing the benefits gained by using vehicle models of higher complexity for a host of other scenarios and safety systems of varying functionalities.

# 6
# Limitations and Future Work

- The thesis mainly focuses on finding the last point of no return in terms of time and distance by taking constant longitudinal velocity and acceleration. The model can be improved by considering varying longitudinal velocities and accelerations.

- It also mainly focuses only on the aspects of lateral dynamics of the vehicle.

- It is tough to emulate the same nature in virtual simulations as that of the real world, for example, different contact patches with different coefficient of friction is not taken into account.

- In our virtual simulations, incorporation of ADAS systems such as Electronic Stability control is absent. It would be interesting to know the dynamic behaviour of the vehicle when such driver assistance systems are present; also, the simulation does not consider any interaction between the vehicles and the surroundings.

- The simulations are carried out without taking driver behaviour as input. Further, it can be improved by considering different available driver models.

- Our steering model is only a basic open-loop steering model which can be improved by considering a closed-loop feedback system to have better control over trajectories. Previous research on the continuous curvatures called VDT's [21] where closed-loop steering system with a PID controller was used to study the trajectories. The research also suggests for making use of controllers which can be implemented with the steering system, one such research on different controllers by[20], suggests different types of controllers to see the effect of lane-change manoeuvre in automated highway systems, where it involves evaluation of controllers based on linear-quadratic optimal control, frequency shaped linear quadratic and sliding mode control during lane change manoeuvre.

- During the initial phase of the project, we tried to acquire HighD data-set from the concerned institution, but unfortunately, our plea was rejected. Therefore, in the future, it would be better to identify lane change manoeuvre in real-world traffic scenarios such as ones provided by HighD and compare trajectories and average distance at which the human drivers tend to start overtaking.

This, in turn, the data sets can be used to obtain a statistical distribution of the distance for overtaking and compare them to those obtained from our simulations.

- The simulations can be extended to analyze more complex scenarios; one such scenario would be to include a moving obstacle or a target vehicle. where the speeds of the ego-vehicle and the target vehicle can be altered and the last point of steering in time and distance can be inferred for such scenarios.

- In our case, the evasive manoeuvre was solely based on steering but in future, the dynamic behaviour and last point of steering to avoid the collision of the vehicle in time and distance from the obstacle can be studied, by including combined effect of braking and steering, where a complex braking system can also be incorporated such as Anti-lock Braking Systems.

- Along with the current models, few more complex models with Multi-body dynamics can be considered, different vehicle modelling software packages such as IPG-Carmaker, Modelica and VI-grade can be considered in future to study what is the impact on the last point of steering.

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# A

# Appendix 1

Before performing the actual counterfactual simulations to find the last point of steering under different conditions, Model validation is done in order to check how the model behaves under different criteria, By comparing plots for trajectories lateral acceleration in $m/s^2$ and heading angle in *rad*.

The process mentioned in the methodology section is implemented for maximum and minimum condition.

## A.1 Maximum Condition

By providing the inputs to the model and satisfying the decided maximum criteria, the models are simulated for three different steering inputs in *rad*, A low steering input of 0.03 *rad*, a high steering input of 0.07 rad and an average steering input of these two values are selected to analyze how the vehicle models perform to avoid the obstacle and which model stands out in terms of last point no return, the model which manages to avoid the obstacle as late as possible by satisfying the maximum criteria can be picked for testing and evaluation of active safety systems during evasive maneuvering.

**Table A.1:** 0.03 rad steering input

| Model | *delta* | LC1 | LCD |
|---|---|---|---|
| 2 track | 0.03 | 12.2 | 4.13 |
| 1 track | 0.03 | 11.89 | 4.58 |
| 2 track with load transfer | 0.03 | 12.18 | 4.15 |

**Table A.2:** 0.05 rad steering input

| Model | *delta* | LC1 | LCD |
|---|---|---|---|
| 2 track | 0.05 | 12.67 | 3.2 |
| 1 track | 0.05 | 12.40 | 3.55 |
| 2 track with load transfer | 0.05 | 12.66 | 3.2 |

**Table A.3:** 0.07 rad steering input

| Model | *delta* | LC1 | LCD |
|---|---|---|---|
| 2 track | 0.07 | 12.2 | 4.13 |
| 1 track | 0.07 | 11.89 | 4.58 |
| 2 track with load transfer | 0.07 | 12.18 | 4.15 |

The table shows for same input, how 3 different models behaves in terms of, LC1 which is the maneuver starting time and LCD which is the lane change duration. The process mentioned in the methodology is carried out to validate the models for different open loop steering input. Figure below shows the trajectories, lateral accelerations and heading angle for the steering inputs tabulated above.

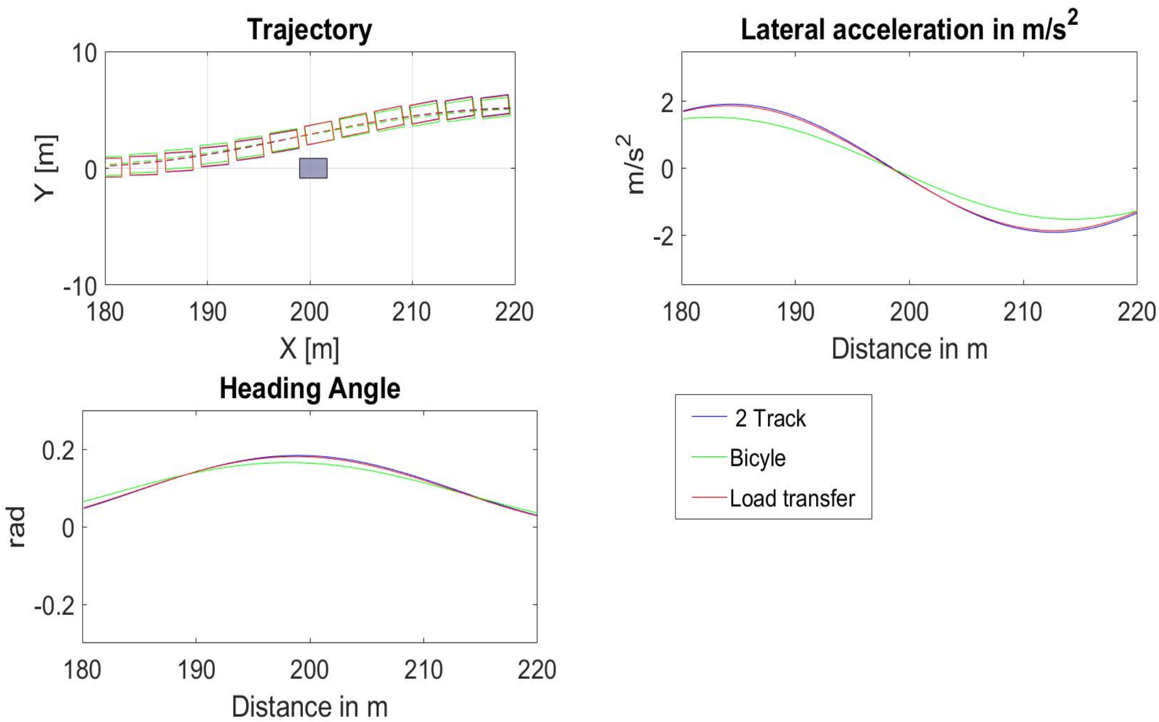


**Figure A.1:** Comparison of three models for 0.03 rad steering input

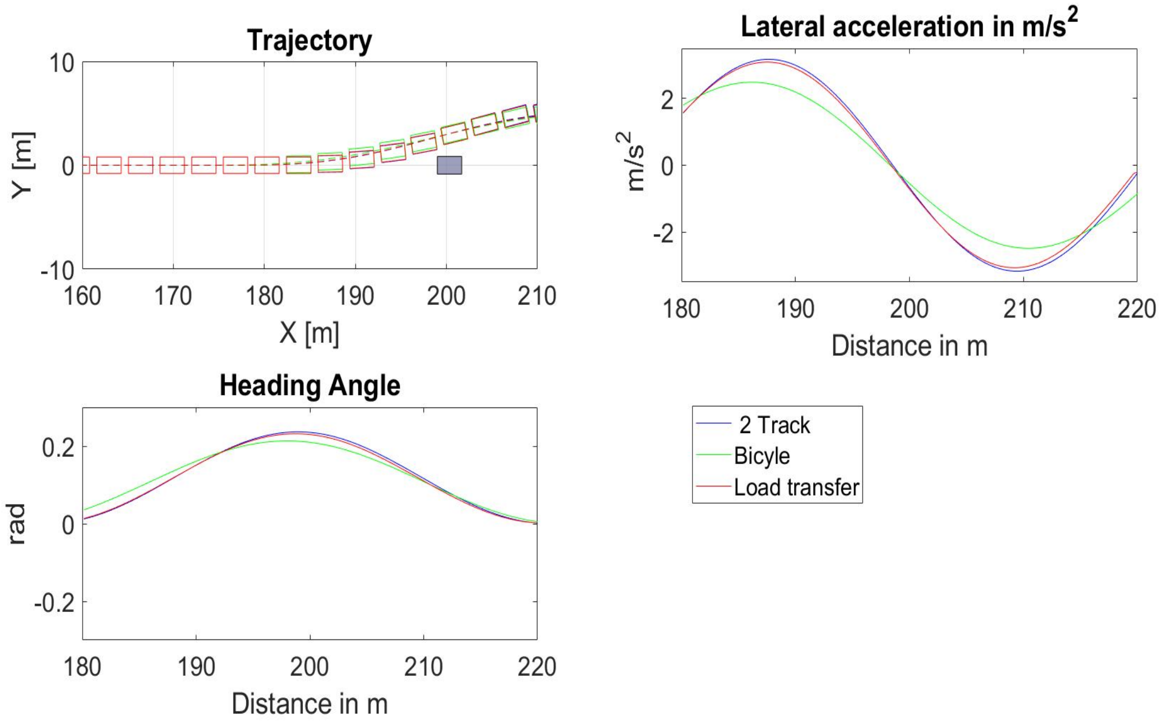


**Figure A.2:** Comparison of three models for 0.05 rad steering input

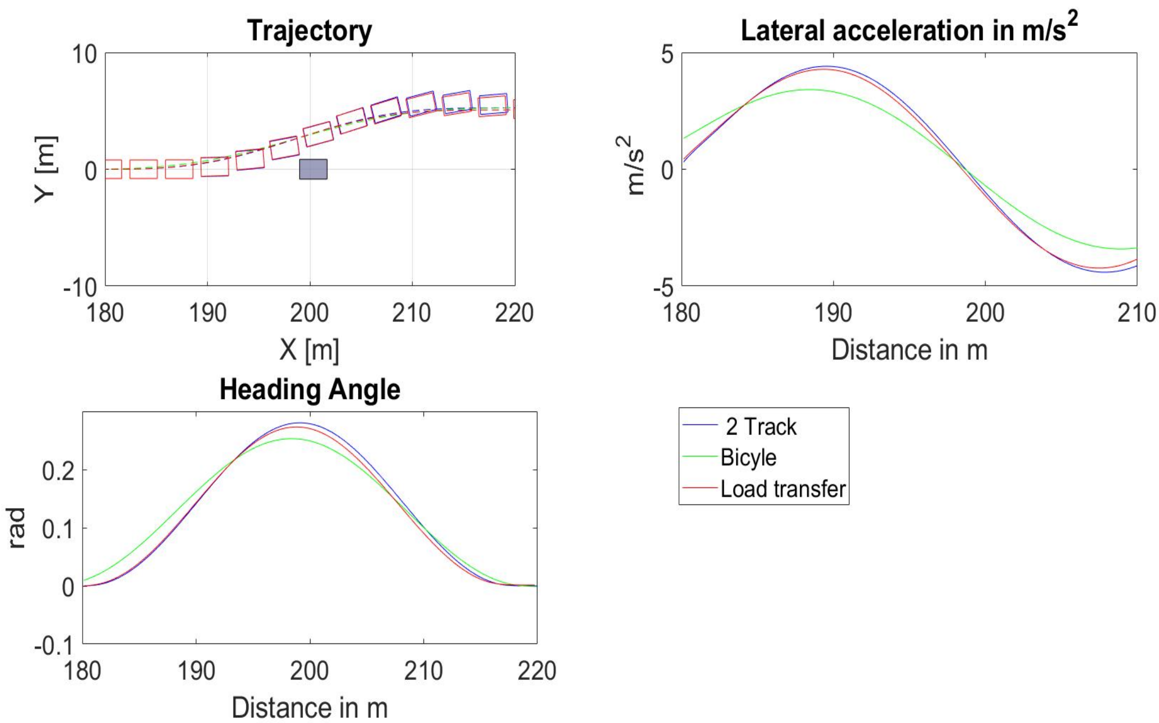


**Figure A.3:** Comparison of three models for 0.07 rad steering input

## A.2 Minimum Condition

Here to validate the models with non-linear tire model we start by providing the inputs to the model and satisfying the decided minimum criteria mentioned in the methodology section, the models are simulated for three different steering inputs in *rad*, A low steering input of 0.03 *rad*, a high steering input of 0.07 rad and an average steering input of these two values are selected to analyze how the vehicle models perform to avoid the obstacle and which model stands out in terms of last point no return, the model which manages to avoid the obstacle as late as possible by satisfying the minimum criteria can be picked for testing and evaluation of active safety systems during evasive maneuvering.

**Table A.4:** 0.03 rad steering input

| Model | *delta* | LC1 | LCD |
|---|---|---|---|
| 2 track | 0.03 | 11.5 | 2.98 |
| 1 track | 0.03 | 10.86 | 3.05 |
| 2 track with load transfer | 0.03 | 10.87 | 3.11 |

**Table A.5:** 0.05 rad steering input

| Model | *delta* | LC1 | LCD |
|---|---|---|---|
| 2 track | 0.05 | 12.18 | 2.3 |
| 1 track | 0.05 | 11.83 | 2.43 |
| 2 track with load transfer | 0.05 | 11.9 | 2.38 |

**Table A.6:** 0.07 rad steering input

| Model | *delta* | LC1 | LCD |
|---|---|---|---|
| 2 track | 0.07 | 12.38 | 1.95 |
| 1 track | 0.07 | 12.12 | 2.08 |
| 2 track with load transfer | 0.07 | 11.3 | 2.01 |

The table shows for same input, how 3 different models behaves in terms of, LC1 which is the maneuver starting time and LCD which is the lane change duration.
The process mentioned in the methodology is carried out to validate the models for different open loop steering input. Figure below shows the trajectories, lateral accelerations and heading angle for the steering inputs tabulated above.

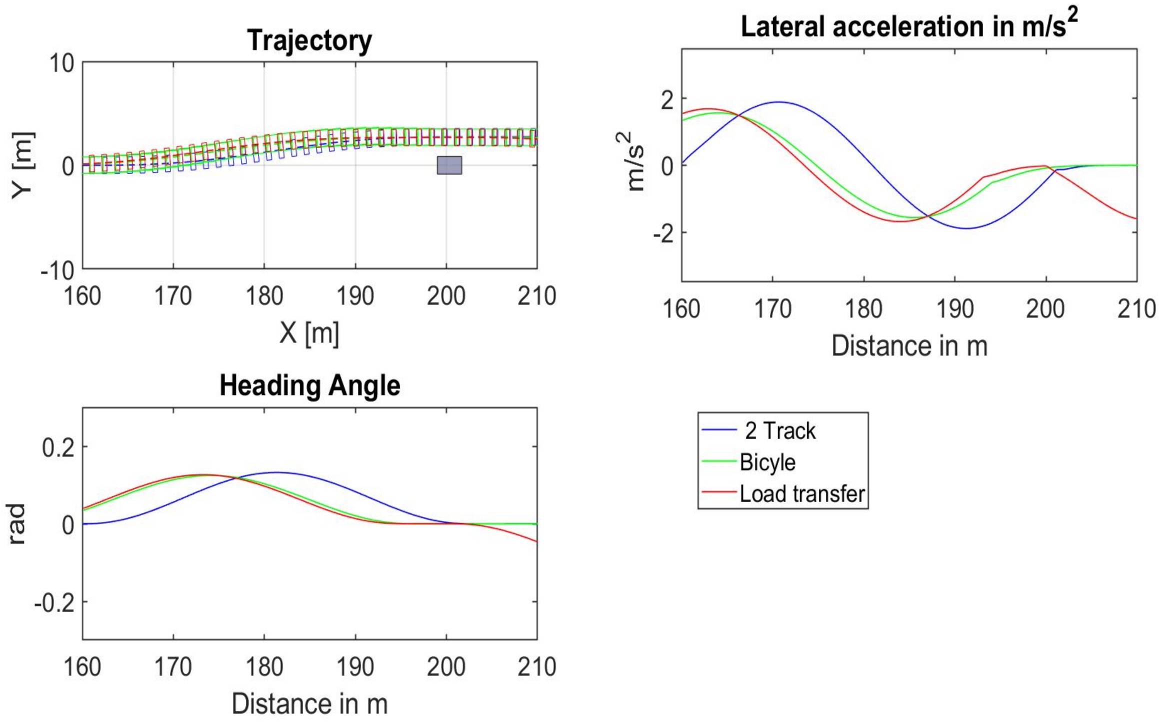


**Figure A.4:** Comparison of three models for 0.03 rad steering input

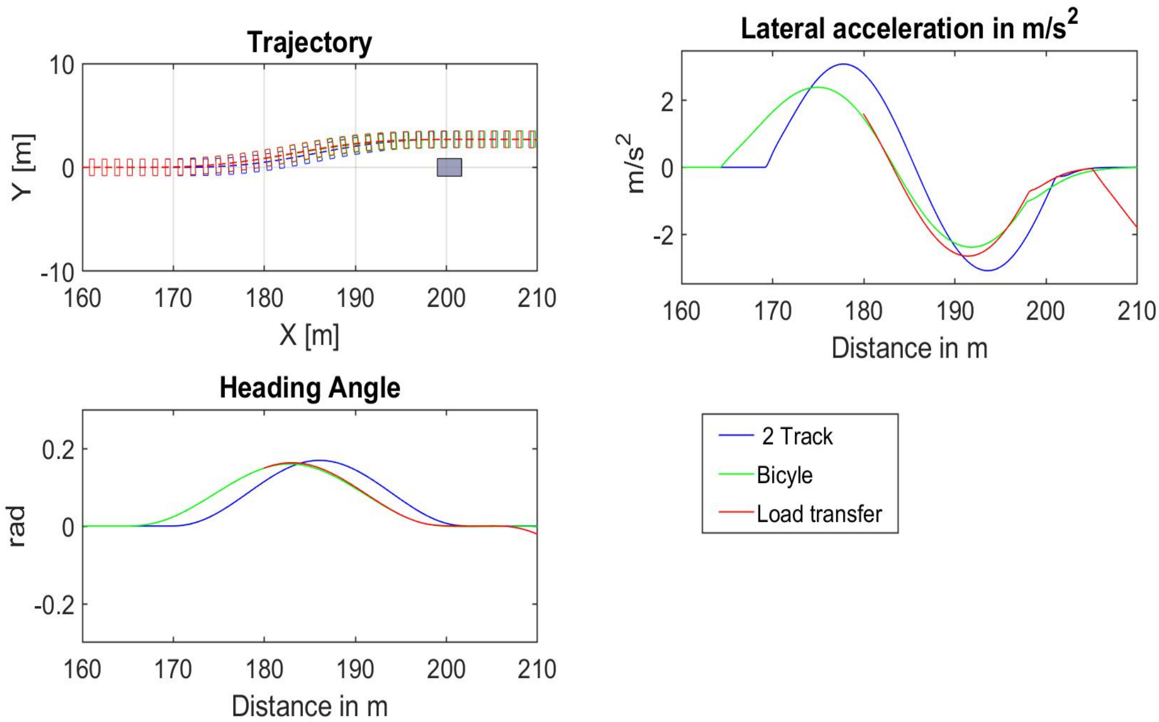


**Figure A.5:** Comparison of three models for 0.05 rad steering input

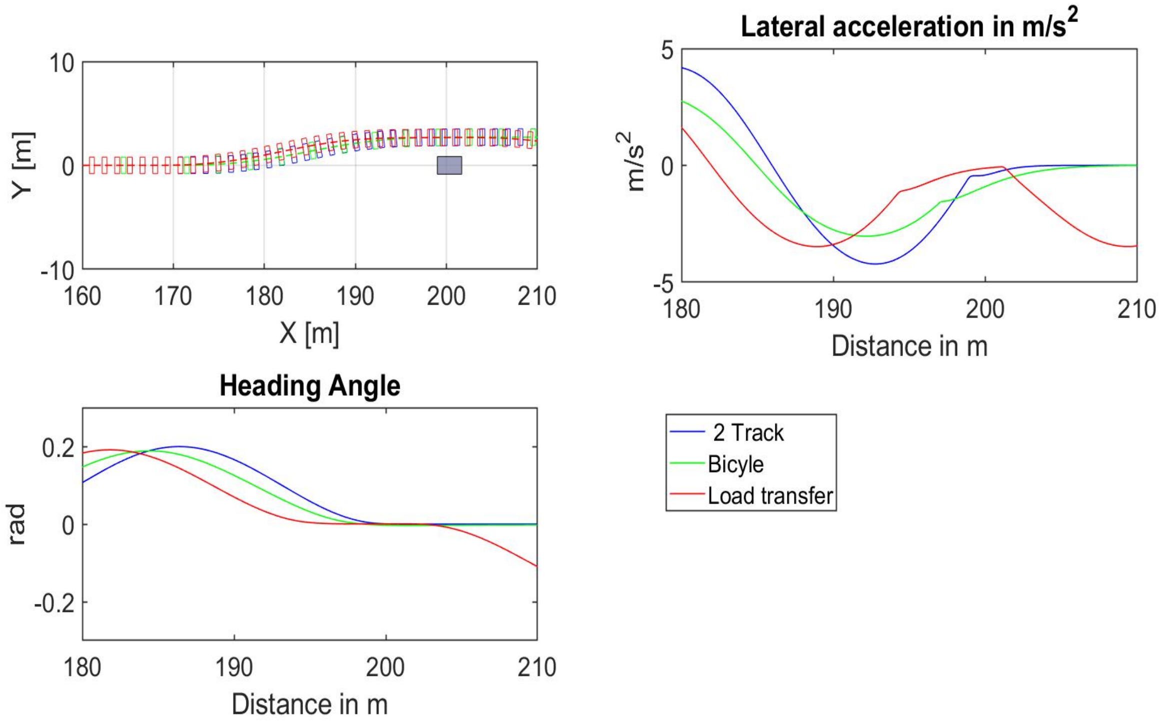


**Figure A.6:** Comparison of three models for 0.07 rad steering input

## A.3 Plots for 50 km/hr

The figure A.7 shows the vehicle trajectory, heading angle and lateral acceleration for two track without load transfer vehicle models limited to 5 $m/s^2$ and 9.81 $m/s^2$ lateral accelerations during the maneuver.

The figure A.8 shows the vehicle trajectory, heading angle and lateral acceleration for single track vehicle models limited to 5 $m/s^2$ and 9.81 $m/s^2$ lateral accelerations during the maneuver.

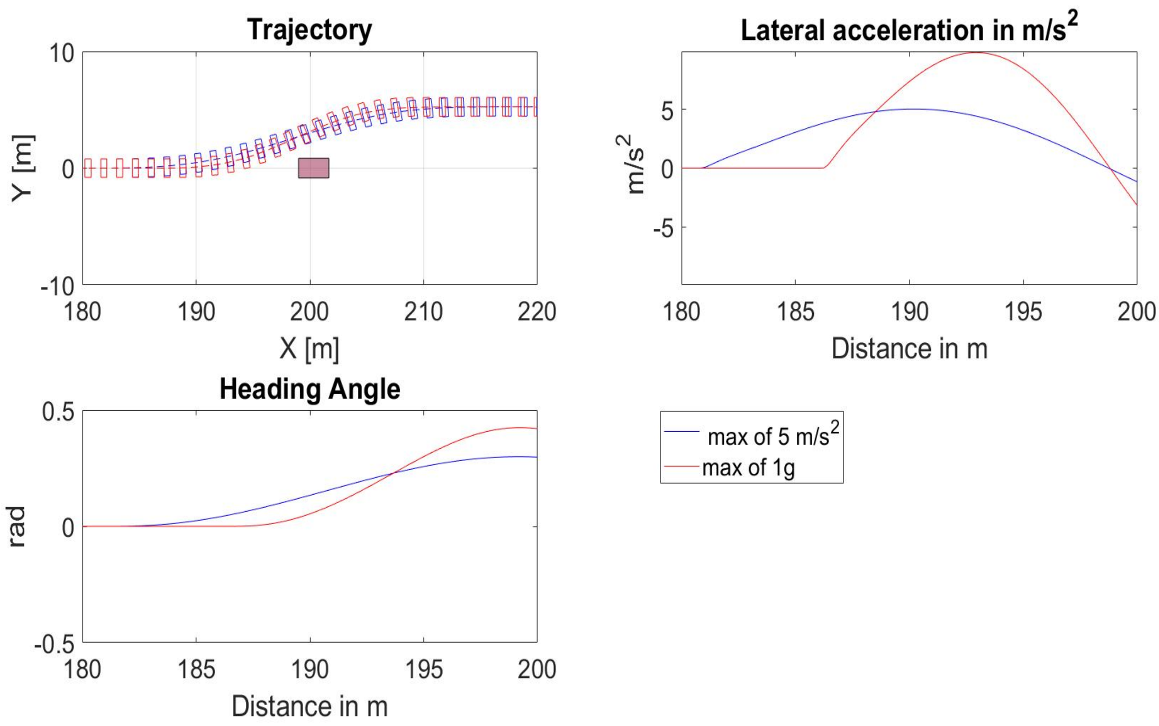


**Figure A.7:** 2 track model without load transfer

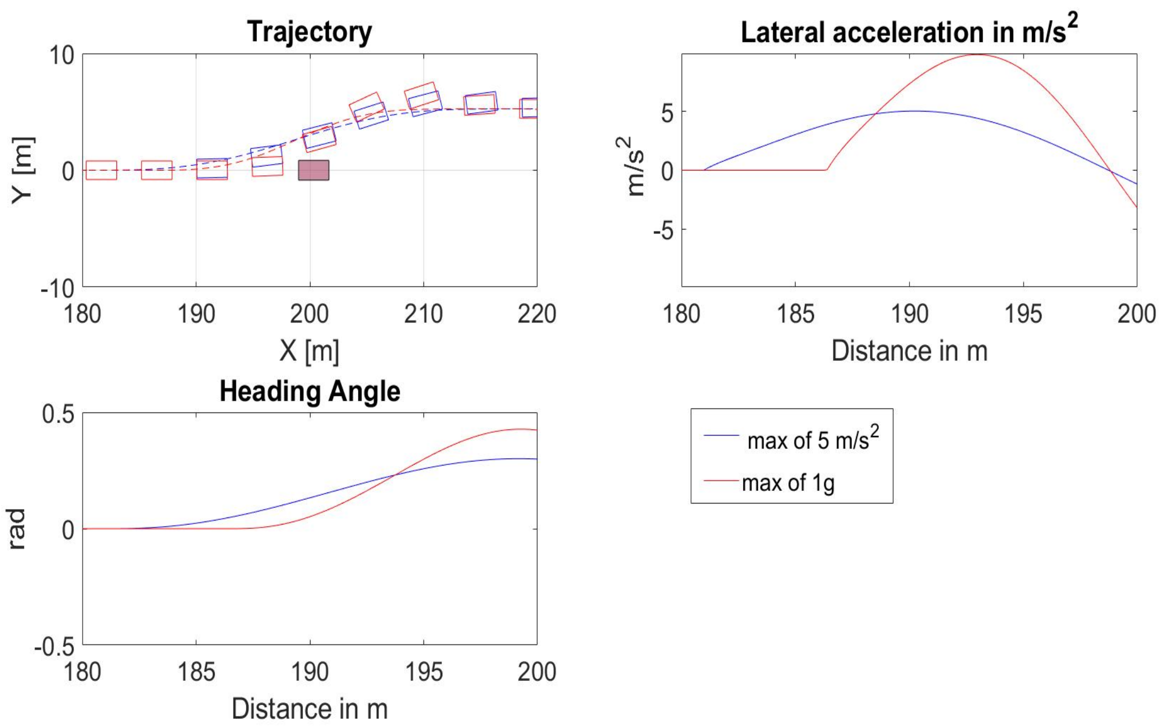


**Figure A.8:** Single track model

The figure A.9 shows the vehicle trajectory, heading angle and lateral acceleration

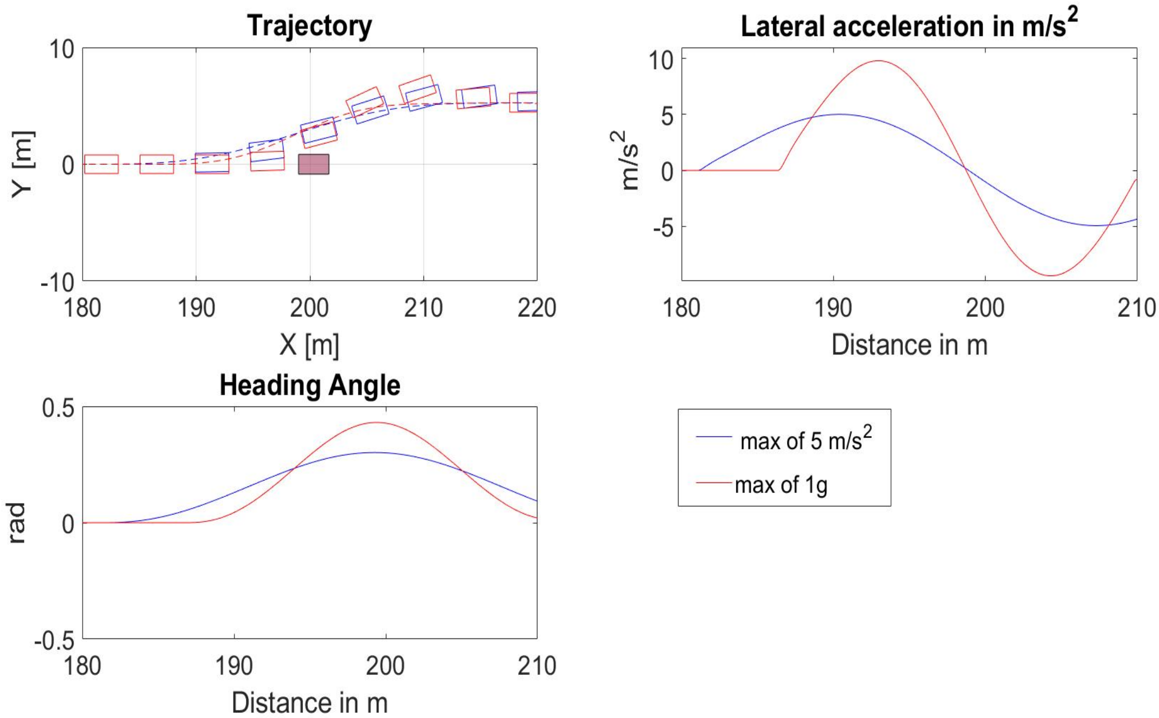


**Figure A.9:** 2 track model with load transfer

for two track with load transfer vehicle models limited to 5 $m/s^2$ and 9.81 $m/s^2$ lateral accelerations during the maneuver.

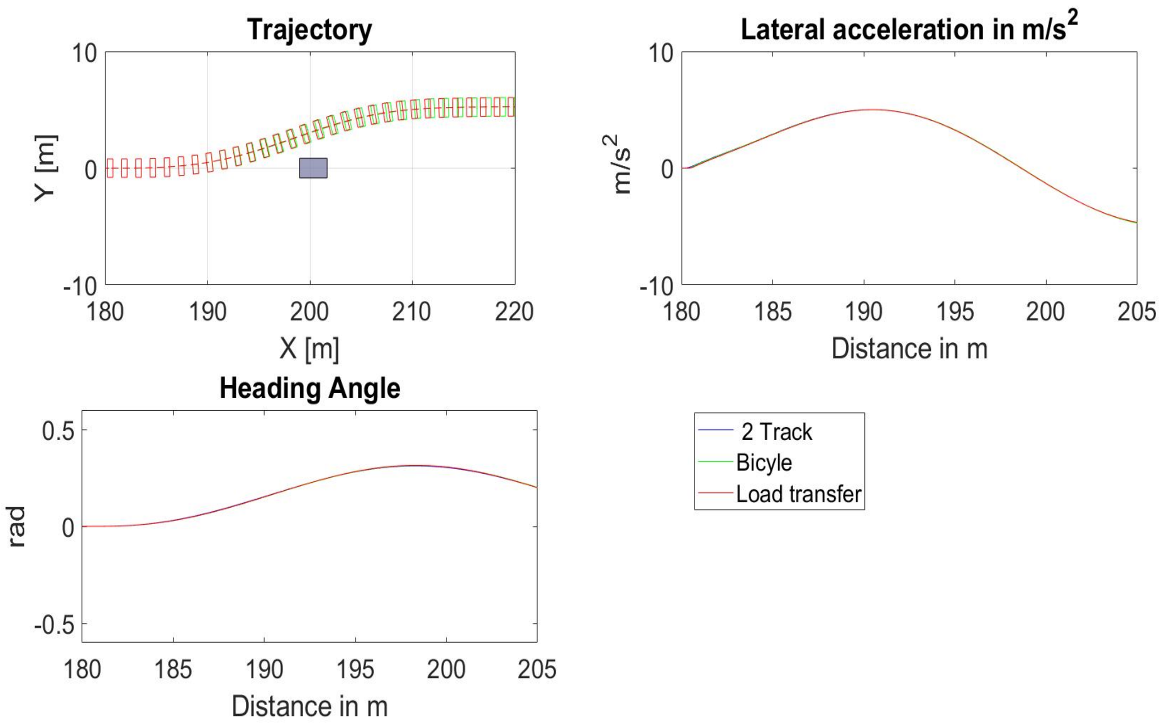


**Figure A.10:** Comparison between three different models at 50 km/hr for reduced cornering stiffness

The figure A.10 shows the vehicle trajectory, heading angle and lateral acceleration for all the three selected vehicle models with decreased cornering stiffness.

## A.4 Plots for 100 km/hr

The figure A.11 shows the vehicle trajectory, heading angle and lateral acceleration for two track without load transfer vehicle models limited to 5 $m/s^2$ and 9.81 $m/s^2$ lateral accelerations during the maneuver.
The figure A.12 shows the vehicle trajectory, heading angle and lateral acceleration for single track vehicle models limited to 5 $m/s^2$ and 9.81 $m/s^2$ lateral accelerations during the maneuver.

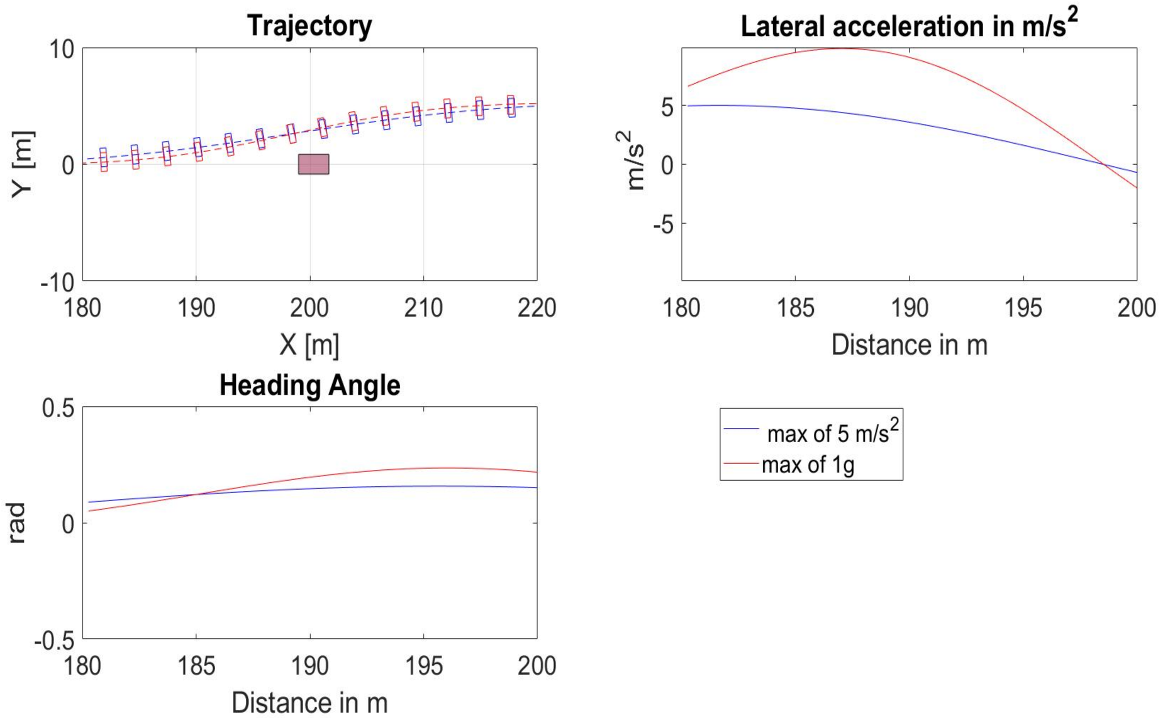


**Figure A.11:** 2 track model without load transfer

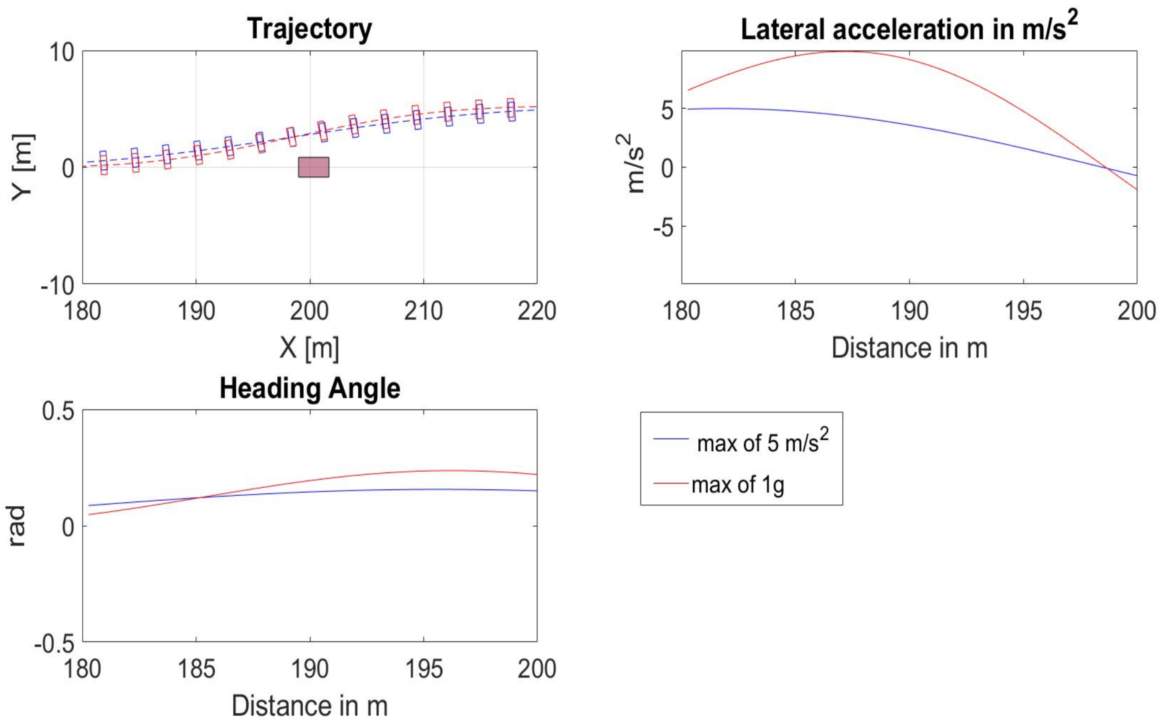


**Figure A.12:** Single track model

The figure A.13 shows the vehicle trajectory, heading angle and lateral acceleration

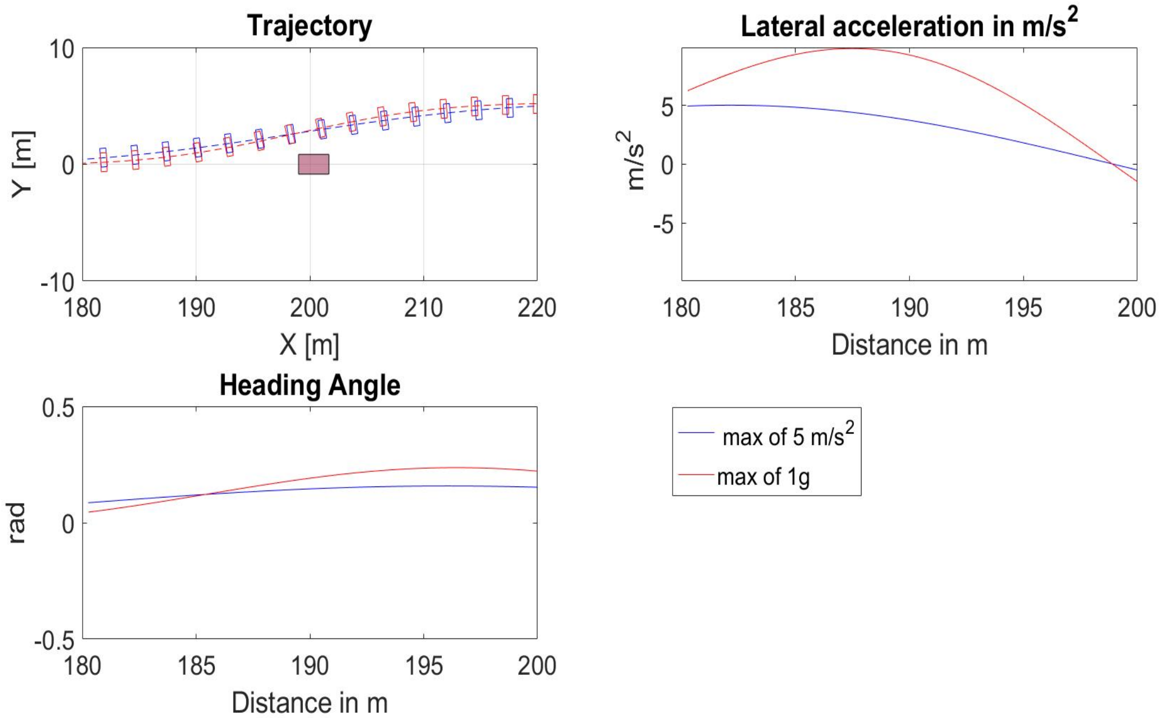


**Figure A.13:** 2 track model with load transfer

for two track with load transfer vehicle models limited to 5 $m/s^2$ and 9.81 $m/s^2$ lateral accelerations during the maneuver.

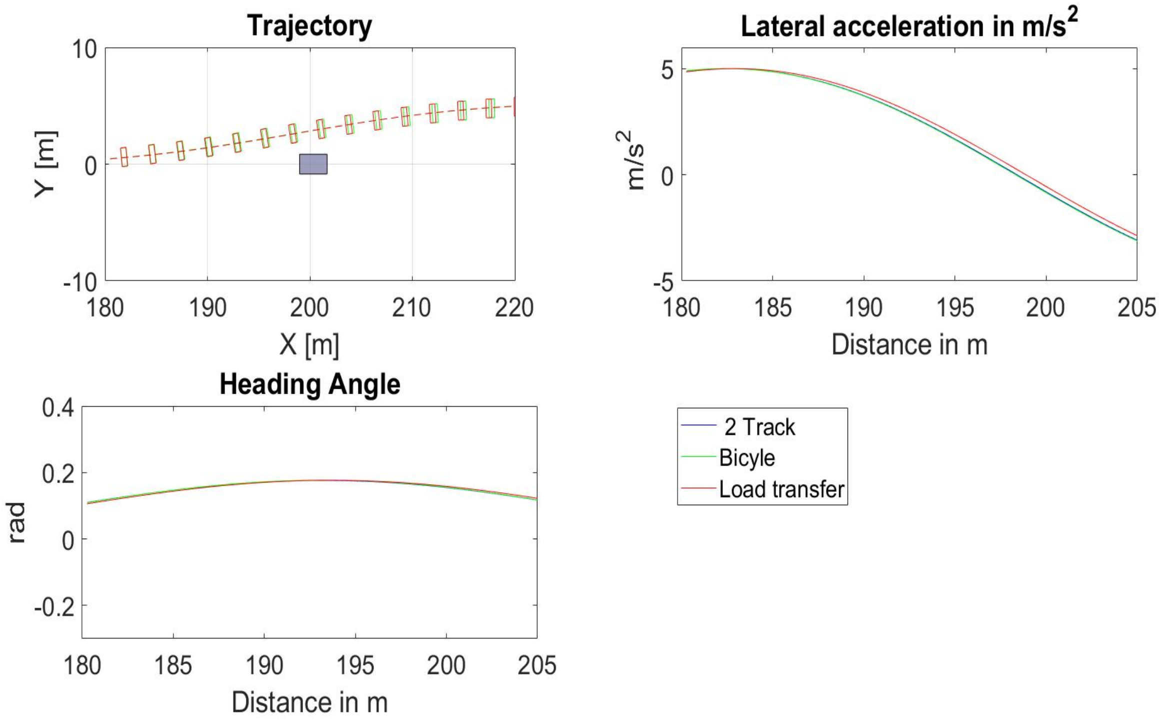


**Figure A.14:** Comparison between three different models at 100 km/hr for reduced cornering stiffness

The figure A.14 shows the vehicle trajectory, heading angle and lateral acceleration for all the three models during the maneuver for reduced cornering stiffness.