

MASTER'S THESIS 2024

# Reducing Running Cost of Radio Base Station with Electrical Batteries

Using Dijkstra's Algorithm

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## **Abstract**

Ericsson, a leading global telecom equipment manufacturer, is addressing the increasing Total Cost of Ownership (TCO) of Radio Base Stations (RBS) by developing a dynamic battery management system. This research leverages historical electricity price data and advanced optimization algorithms, such as Dijkstra's, to minimize energy consumption and costs. By strategically utilizing batteries as a continuous energy storage solution, the system reduces reliance on the grid during peak pricing periods, enhancing both cost-effectiveness and environmental sustainability. This approach contributes to the industry's goal of building more efficient telecommunication networks.

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## List of Abbreviations

RBS: Radio Base Station

TCO: Total Cost of Ownership

RAN: Radio Access Network

MU: Main Unit

RF: Radio frequency

AD: Analog-to-digital ; DA: Digital-to-analog

# 1

## Introduction

Radio Base Stations (RBS) are essential components of modern telecommunication networks, providing connectivity to mobile devices across diverse geographical areas. While traditionally reliant on grid power, the integration of battery systems offers significant potential for optimizing energy consumption and reducing operational costs.

Battery systems in RBS serve a dual purpose: ensuring uninterrupted service during power outages and reducing energy costs by strategically charging during off-peak hours (when electricity prices are low) and discharging during peak hours (when prices are high). This research delves into the application of optimization techniques to minimize energy costs in RBS operations. By leveraging historical electricity price data and advanced algorithms, aim to develop a dynamic battery management system that can effectively balance energy consumption and cost, contributing to the overall sustainability of telecommunication infrastructure.

### 1.1 Problem Statement

Radio Base Stations (RBS) are critical infrastructure for modern telecommunications, but their energy consumption can significantly impact operational costs. To mitigate these costs, this research aims to optimize battery usage in RBS by developing a dynamic charging and discharging strategy based on electricity pricing. By strategically leveraging battery storage, we seek to reduce energy consumption and enhance the overall sustainability of RBS operations.

### 1.2 Objectives

This thesis aims to

- Demonstrate the feasibility of using RBS batteries for electricity price arbitrage.
- Quantify the potential cost savings and sustainability benefits of this approach.

- Develop algorithms and control systems to optimize RBS battery charging and discharging.

### **1.3 Scope of the Thesis**

This thesis investigates the optimization of battery management for Radio Base Stations (RBS) to reduce energy costs. By leveraging Dijkstra's algorithm, we aim to dynamically optimize battery usage based on fluctuating electricity prices and RBS power consumption patterns. The research will explore both short-term and long-term optimization strategies, considering factors like initial battery levels, penalty costs, and target battery levels.

# 2

## **Related Work**

The increasing energy consumption of Radio Base Stations (RBS) has prompted significant research efforts to optimize their energy efficiency. Several studies have explored various techniques to reduce energy consumption, including power-saving modes, adaptive transmission power control, and energy-efficient hardware design.

One relevant study [14] focuses on reducing RBS energy consumption by optimizing power-saving features and exploring hybrid energy solutions, such as solar and diesel. While this research offers valuable insights into hardware-level optimizations, it does not explicitly address the dynamic nature of electricity pricing.

Another related work [8] investigates the application of reinforcement learning techniques to optimize the operation of community energy storage systems, considering factors like solar generation, electricity consumption, and price volatility. This study highlights the potential of AI-driven approaches for energy management, but it focuses on a different scale and application domain.

This thesis, however, specifically focuses on the dynamic optimization of battery usage in RBS to reduce energy costs. By leveraging Dijkstra's algorithm, we aim to develop a control strategy that can adapt to fluctuating electricity prices and RBS power consumption patterns. This approach complements existing research by providing a practical and effective solution for optimizing RBS energy consumption.

By combining the insights from these previous studies and applying them to the specific context of RBS energy management, this research aims to make a significant contribution to the field of sustainable telecommunications.

# 3

## System Design

The design model comprises four key components: the grid, Radio Base Station (RBS), battery, and power control system. As illustrated in Figure 1, the overall system integrates the RBS with the grid and the power control system. In this configuration, the grid serves as the primary source of energy.

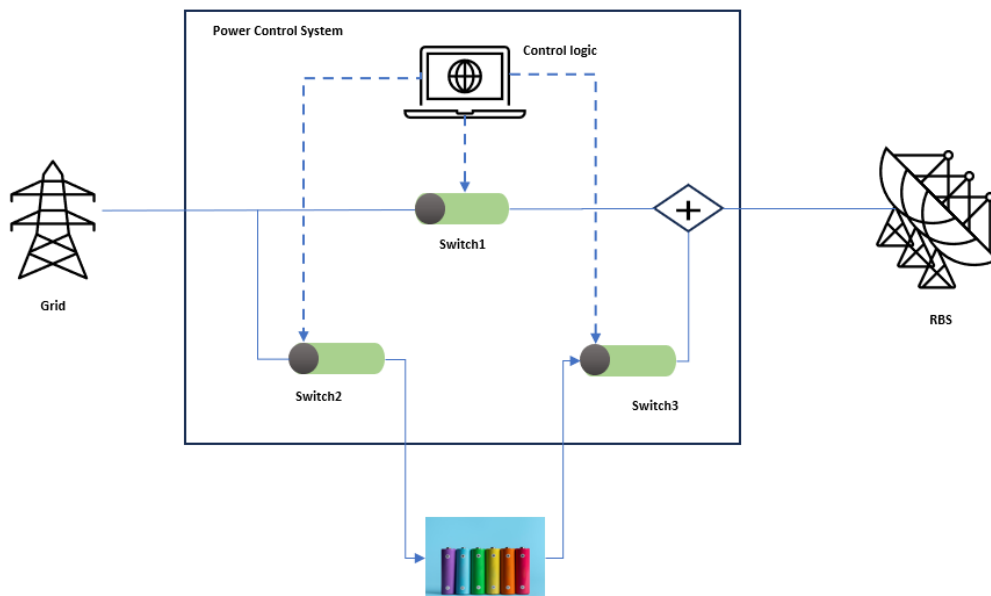


Figure 1 System with the power system in focus.

### 3.1 Grid

The grid serves as the primary energy source for both the battery and the Radio Base Station (RBS) in this system, providing a continuous and virtually unlimited supply of power. However, the cost of energy from the grid varies depending on the time of use.

### 3.2 Power control system

The power control system, as shown in the diagram above, is a logical unit with three switches that manage the energy supply to the Radio Base Station (RBS) from the grid or the battery. It operates based on three battery states: idling, charging, and discharging. And total cost is influenced by different states.

**Idling:** In this state, the RBS is powered solely by the grid. During this state, the total cost doesn't change. The control switches are set as follows: switch 1: closed, switch 2: open, switch 3: open.

**Charging:** During charging, the RBS is powered by the grid, and the battery is charged simultaneously. During this state, the total cost increases as time goes. The switch settings for this state are switch 1: closed, switch 2: closed, switch 3: open.

**Discharging:** In this state, the battery discharges to power the RBS, typically when the grid is unavailable or during high energy price periods. During this state, the total cost decreases as time goes. The control switches are configured as follows: switch 1: open, switch 2: open, switch 3: closed. The table below also shows the relationship between battery states and switch configurations.

Battery state	Switch 1	Switch 2	Switch 3
Idling	Closed	Open	Open
Charging	Closed	Closed	Open
Discharging	Open	Open	Closed

*Table 1 Relationship between battery states and switch configurations*

### 3.3 Radio Base Station

The Radio Base Station (RBS) is a key component in Radio Access Networks (RANs), enabling communication between users and network providers. The RBS includes:

**Antenna and Radio Unit:** This unit transmits and receives signals, with the antenna optimized for coverage and the radio unit handling RF processing tasks such as modulation, demodulation, frequency conversion, and signal conversion. It also manages duplexing for simultaneous transmission and reception on the same frequency.

Main Unit (MU): The Main Unit performs critical baseband processing, including channel coding, decoding, modulation, and demodulation. It ensures proper encoding and decoding of signals for effective communication.

The Main Unit and Radio Unit are connected via high-bandwidth fiber optic cables, enabling efficient and reliable data transfer, and ensuring the RBS operates effectively within the network.

## 3.4 Battery

Batteries convert chemical energy into electrical energy and come in various types, including:

**Lead-Acid Batteries:** These are the first rechargeable batteries, known for their low energy density but high surge current capability. They are commonly used in vehicles and backup power systems due to their low cost and ability to supply high currents. “[15]

**Nickel-Metal Hydride (NiMH) Batteries:** These were the first rechargeable batteries with a significant efficiency advantage. They are commonly used but have longer charging times and reduced power output with repeated use.

**Lithium-Ion Batteries:** These are widely used in modern devices like laptops and phones due to their quick charging, consistent power output, and cost-efficiency over time, despite their higher initial purchase price.

### 3.4.1 Key Properties Considered

#### 3.4.1.1 *Charging and Discharging Rate (kWh)*

**C-rate:** Measures the speed of charging or discharging. For instance, a 1C rate means a battery charges from 0-100% in one hour. Faster rates (e.g., 3C) charge the battery more quickly but can reduce battery life. Lithium-Ion batteries typically charge at a rate of C/2, or 50% per hour.” [9] As charging and discharging rates can vary in real-world applications, this thesis considers variable charging and maximum discharging rates.

### 3.4.1.2 **Capacity (kWh)**

This indicates the maximum energy storage. Battery capacity is commonly measured in watt-hours (Wh), kilowatt-hours (kWh), or ampere-hours (Ah). For this thesis, battery capacity is expressed in kWh.” [10]

### 3.4.1.3 **Battery Range**

Refers to the portion of battery capacity utilized in the study. The linear portion of the charging curve (0-80%, blue dashed line) is used for simplicity, as the curve becomes nonlinear beyond this range.

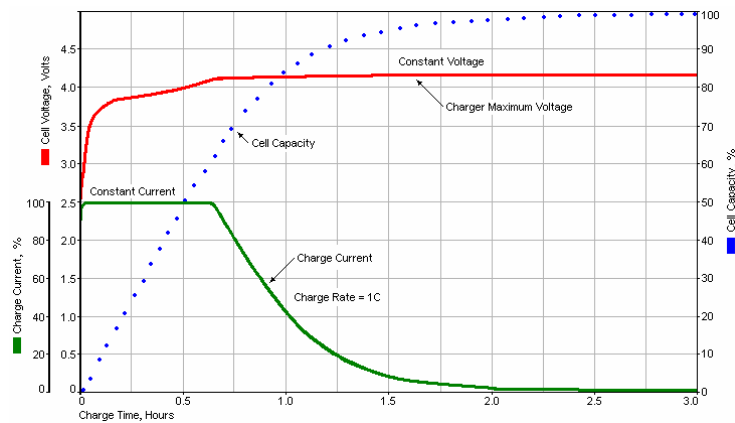


Figure 2 Rapid charging characteristics of battery [2]

### 3.4.1.4 **Lower Charge Limit**

Represents the minimum charge level to avoid damaging the battery. For instance, if the lower charge limit is 25%, only 25% of the battery's total capacity is usable.” [10]

### 3.4.1.5 **Upper Charge Limit**

Represents the maximum charge level to avoid damaging the battery. For example, in Figure 2 is 80% of the battery's total capacity.

### 3.4.1.6 **Initial Battery Level**

Refers to the battery’s starting capacity at the beginning of an operational cycle or simulation. This starting value is crucial for optimizing system performance and is set for the purposes of this thesis.



# 4

## Mathematical System model for Battery Management

The system model provides a framework for dynamic battery management optimization, aimed at minimizing costs by efficiently controlling battery charging, idle, and discharge cycles. The goal is to charge the battery during low-cost periods and discharge during high-cost periods to achieve maximum cost savings.

### 4.1 Constants

**Time ( $t$ ):** Discrete time variable, representing each hour in a 24-hour period where  $t \in \{0, 1, \dots, T\}$  with  $T$  being the terminal hour of the day (i.e., 23).

**Charging Rate (CR):** Is shown more specifically in chapter 3.4. The battery gains energy at a rate of 2 kW per hour when charging in this thesis.

**Discharging Rate (DR):** Is shown more specifically in chapter 3.4. The battery can discharge up to 5 kW of energy per hour in this thesis.

**Upper Charge Limit ( $C_u$ ):** The maximum allowable charge in the battery.

**Lower Charge Limit ( $C_l$ ):** The minimum allowable charge in the battery.

**Battery Capacity  $W$ :** The total capacity of the battery. Is shown more specifically in chapter 3.4.

### 4.2 Parameters

**Electricity price  $R(t)$ :** This denotes the price of electricity at time  $t$ , The electricity price fluctuates throughout the day.

**Power Consumption  $P(t)$ :** Power demand at time  $t$ .

**Current Battery level:  $B(t)$**  represents current battery level at time  $t$ , is constrained between  $C_l \leq B(t) \leq C_u$  for all times  $t$ .  **$B(0)$**  is initial battery level at  $t=0$ . The changes in battery level between consecutive time step is denoted by  $\Delta B(t)$ .

$$\Delta B(t) = B(t) - B(t - 1) \quad 4.1$$

### **State (policy) $X(t)$**

$X(t)$  denotes the operational status of the battery at hour  $t$ .

- $X(t)=1$ : when the state is charging.
- $X(t)=0$ : when the state is idle.
- $X(t)=-1$ : The battery is discharging.

**Total cost  $C_{\text{cost}}(t)$**  : Total Cost for a specific time  $t$ .

## **4.3 Battery Level Dynamics (distinguished by different states)**

### **4.3.1 Charging**

$\Delta B(t)$  represents the amount of energy battery gets when charging in 1 hour.

$$\Delta B(t) \mathbb{I}_{X(t)=1} = \min(CR, C_u - B(t-1)) \quad 4.2$$

The amount of energy the battery gains in the given hour  $t$  is the minimum of the charging rate and the remaining battery level to charge.

### **4.3.2 Idling**

Since the battery gains no energy while idling,  $\Delta B(t)$  is used to represent the amount of energy it receives during this state, which is 0.

$$\Delta B(t) \mathbb{I}_{X(t)=0} = 0 \quad 4.3$$

### **4.3.3 Discharging**

The maximum amount of energy battery can provide in the given hour  $t$  can be represented as  $-\Delta B(t)$ .

$$\Delta B(t) \mathbb{I}_{X(t)=-1} = -\min(DR, P(t), (B(t-1) - C_l)) \quad 4.4$$

The amount of energy the battery can provide in the given hour  $t$  is the minimum of the discharging rate, power consumption, and the remaining battery level for given time  $t$ .

#### 4.3.4 Battery Charge dynamics

Let  $\mathbb{I}$  represents the indicator function which is distinguished by different states. The charge of the battery in the given hour  $t$ , depends on the initial charge  $B(0)$  and the sum of energy charged or discharged over time. This relationship can be expressed as follows:

$$B(t) = B(0) + \sum_{i=1}^t (\Delta B(i) \mathbb{I}_{X(i)=1} + \Delta B(i) \mathbb{I}_{X(i)=-1}) \quad 4.5$$

### 4.4 Cost and loss function

The objective of this optimization problem is to minimize the Loss function  $L$  over a time horizon of  $T$  time steps. The Loss function  $L$ , is the sum of the accumulated cost and penalty over period  $T$ . The  $C_{cost}(i, X(i))$  for any given hour  $i$  is the product of the electricity price  $R(i)$  and the change in battery level  $\Delta B(i)$  :

$$C_{cost}(i, X(i)) = R(i) * \Delta B(i) \quad 4.6$$

The cost function over the period  $T$  is given by:

$$C_{cost}(T) = C_{cost}(B(0), X) = \sum_{i=1}^T C_{cost}(i, X(i)) \quad 4.7$$

The loss function over  $T$  period is calculated as:

$$L(B(0), X) = C_{cost}(T) + \text{penalty}(T) \quad 4.8$$

The exact form of the penalty term is discussed below in Sec. 4.7 and 4.8 . This model is designed to optimize over  $X(t)$  (whether to charge, idle, or discharge), under constraint that  $C_l \leq B(t) \leq C_u$  for all  $t=1 \dots, T$ . To maximize cost savings, it's essential to make the correct decision on when to charge, discharge, or idle the battery. Dijkstra's algorithm is effective in determining these optimal actions, whether for a short planning horizon like one day or a long-term horizon such as one year. By evaluating electricity prices and consumption patterns, Dijkstra's algorithm ensures that the battery management strategy minimizes costs over the chosen period.

## 4.5 Optimization of the cost function

To optimize the cost of the battery management system, Dijkstra's algorithm is employed to determine the shortest-cost path of actions (charge, idle, discharge) over a graph consisting of  $T+1$  levels of nodes, where  $T$  is the planning horizon, e.g. 24 hours. Dijkstra's algorithm efficiently handles non-negative transition costs, such as electricity prices and energy transitions, making it ideal for this optimization problem.

## 4.6 Graph representation of battery management

The system is modelled as a directed weighted graph  $G(N, E)$ :

**Nodes  $N$ :** Each level  $t$  (ranging from 0 to  $T$ ), is consisted of  $N$  nodes describing all  $N$  possible charges of the battery at time  $t$ . To calculate  $N$ , here is an example, if  $C_u$  is 19 and  $C_l$  is 4, then  $N$  should be  $19-4+1=16$ .

**Weights:** The cost associated with transition between states is the total electricity cost for charging or discharging.

**Edges  $E$ :** Edges are drawn from the nodes of each layer  $t=0\dots T$ , to the consecutive level nodes  $t+1$ , describing the possible transition of the battery level from time  $T$  to time  $T+1$  when following the switching state and is assigned a weight which equals the cost of the corresponding transition. The cost of a concrete evolution of the battery levels:  $B(0), B(1), \dots B(T)$  is then the sum of the weights on the corresponding edges of the graph.

### Initialization

- Let  $B(0)$  represent the initial battery level at time  $t=0$
- The cost to reach the initial state is zero:  $C_{\text{cost}}(0) = 0$
- For all other nodes, the initial cost is set to infinity (or a very large number) to ensure that the first transition is always optimal:  $C_{\text{cost}}(0) = \infty$ , for all other initial states.

### 4.6.1 Transition cost calculation

For each transition from  $B(t)$  to  $B(t+1)$ , the transition cost depends on the action and electricity price  $R(t)$ , and is assigned to the corresponding edge in the graph.

the transition cost is assigned to be the hourly cost defined in equation 4.6.

## Cost Update

Update the total cost from one state  $B(t)$  to another state  $B(t+1)$ .

The total cost to reach (Weight) a given battery state  $B(t+1)$  at time  $t+1$  is the sum of the previous total cost  $C_{cost}(t)$  to reach the current state  $B(t)$ , and the transition cost from  $B(t)$  to  $B(t+1)$ .

$$C_{cost}(t+1) = C_{cost}(t) + \text{Transition Cost}(t, t+1) \quad 4.9$$

4.6.2

## Optimization and Path Update

For each possible transition from  $B(t)$  to  $B(t+1)$ , the algorithm checks if the new total cost is less than the current known cost for that state. If the new cost is lower, update the graph and path with this minimum cost and action.

$$C_{cost}(t+1) = \min(C_{cost}(t+1), C_{cost}(t) + \text{Transition Cost}(t, t+1)) \quad 4.10$$

The path with the lowest cost represents the most efficient schedule of actions  $X(t)$  (charging, idling, or discharging) that minimizes the total cost while ensuring the battery operates within its limits and meets any penalty conditions.

Dijkstra's algorithm is demonstrated through manual calculations on simplified scenarios, showcasing its core functionality. By varying parameters such as network size and edge weights, the algorithm's performance is evaluated across different conditions. A detailed explanation of Dijkstra's implementation is provided, including the simulation techniques and tools used to achieve efficient results.

4.7

## Penalty calculation

The penalty function is designed to disadvantage scenarios where the final battery level  $B(T)$  falls below the target level  $G$  the algorithm applies penalty, if  $B(T) < G$  The penalty is calculated as:

$$\text{Penalty}(T) = \text{coef} * (B(T) - G) \quad 4.11$$

The penalty depends on  $\text{Penalty}(T)$  and  $B(T)$  and  $G$ . but both are functions of electricity price, power consumption and policy.

Where,

*coef* represents the type of penalty applied, based on the chosen strategy (e.g., average, high, max, or zero) that are explained in chapter 4.9.

$B(T)$  – Final Battery level.

$G$  - Target Battery level.

## **4.8 Determining Target Battery Levels $G$ for Cost Optimization from One-year optimization on historical data**

### **4.8.1 Time Horizons and Input Data**

The study considers one year that is ( $T=24*365$ ), is preferred for optimization using Dijkstra's algorithm.

For the input data, we use historical electricity prices (vector  $R$  of length  $T$ ) and power consumption during this period (vector  $P$  of length  $T$ ). These inputs remain consistent throughout the study.

### **4.8.2 Ideal Policy Using Dijkstra's Algorithm**

First, we determine the optimal battery management policy by running Dijkstra's algorithm over the year. For this long-term optimization, the initial battery charge and penalty settings are not crucial, as they mainly impact the beginning and end of the period. We assume a starting charge of 10 kWh and a zero penalty for deviations in battery levels.

This result represents the best possible policy—the ideal scenario—assuming perfect knowledge of future prices. While it is not practically feasible to know future prices beyond one day ahead, this policy provides a benchmark for the maximum possible cost savings.

### **4.8.3 Analyzing Battery Levels from the Ideal Policy**

Next, we analyze the battery levels at the end of each day from the one-year optimization using Dijkstra's with historical prices. A histogram of these end-of-day battery levels is drawn to understand typical battery usage. The mean, median, and mode values of the end-of-day battery levels are calculated to provide insights into what the target battery level ( $G$ ) should be at the end of each day.

#### 4.8.4 Day-by-Day Dijkstra with Target Battery Levels and Penalties

In this step, we simulate day-by-day battery management using Dijkstra's algorithm, but with access to only one day of price data at a time. We introduce a target battery level ( $G$ ) that the system must aim to maintain. Penalties are applied for deviations from this target, using the formula:  $Penalty(T) = coef * (B(T) - G)$

Penalty coefficients: mean, max, high, zero.

#### 4.8.5 Comparing Savings

Finally, we compare the cost savings obtained from each day-by-day scenario to the one-year Dijkstra's savings with historical prices. This allows us to assess which combination of penalty coefficient and target battery level comes closest to the one-year Dijkstra's savings with historical prices solution, providing the best practical battery management strategy under real-world constraints (where only one-day-ahead price data is available).

### 4.9 Types of Penalties

Dijkstra's optimization includes four different types of penalties to determine which target battery level and which type of penalty results in high-cost savings. Penalty is applied to every single day before selecting path which saves most costs.

**Average Electricity Price Penalty:** This imposes an average penalty; this method calculates the average electricity price over the Planning Horizon  $T$  and multiply with difference between initial and target battery levels. It calculates the average electricity price ( $R_{avg}(T)$ ) across the entire planning horizon  $T$ . (e.g., one day).

$$R_{avg}(T) = \frac{1}{T} \sum_{t=1}^T R(t) \quad 4.12$$

$$Penalty(T) = R_{avg}(T) * (B(T) - G) \quad \text{if } B(T) < G \quad 4.13$$

**High Penalty:** This method applies a fixed, high penalty whenever the final battery level falls below the target battery level. This imposes a high penalty of 10 SEK, significantly affecting the total cost. Here high penalty considered as 10 SEK.

$$Penalty(T) = 10 \text{ SEK} * (B(T) - G) \quad \text{if } B(T) < G \quad 4.14$$

**Max Penalty:** This imposes a maximum penalty; this approach uses the maximum electricity price  $R_{max}$  from the time (that is highest price for the planning horizon) and multiply with difference between initial and target battery levels.

$$\text{Penalty}(T) = R_{max} * (B(T) - G) \quad \text{if } B(T) < G \quad 4.15$$

**Zero Penalty:** This method applies no penalties when determining the shortest path.

$$\text{Penalty}(T) = 0 \quad 4.16$$

**Note:** If the final battery level exceeds or equal to the target battery level, no penalty is applied.

Incorporating penalties into Dijkstra's algorithm:

The penalty depends on the type of penalty applied and the difference between the final battery level  $B(T)$  and the target battery level  $G$ . The penalty term plays a key role in discouraging scenarios where the battery's final charge  $B(T)$  falls significantly below its target level  $G$ . While the penalty does not guarantee that the battery will always reach or exceed the target level, it penalizes situations where the battery ends up below the desired charge level.

## 4.10 Hand calculation

This chapter demonstrates the effectiveness of Dijkstra's algorithm in managing battery operations over both short and long time periods. The examples provided show how the algorithm performs across different battery levels, consistently charging when prices are low and discharging when prices are high. These results confirm that Dijkstra's algorithm makes optimal decisions for battery management, regardless of the time horizon. In this hand calculation section, all the values taken as integer only.

### 4.10.1 Example 1

The example illustrates the Dijkstra's algorithm to determine the optimal path for minimizing cost associated with battery charging, discharging and idling states over 4-hour period. Below figures show the results for path selection for different initial battery levels.

#### Parameters:

- Charging rate = 5kW
- Discharging Rate = 5kW
- Capacity = 20kWh



- Initial Battery Level= 5,10,15,20kWh
- Electricity price = [1, 3, 5, 2]
- Average price = 2.75

The following section will present a hand-calculation figure with same parameters as mentioned above with various initial battery levels. By keeping other parameters constant, we can evaluate which starting battery charge is most effective in minimizing energy costs. Moreover, we assume battery level changes in integer numbers, more specifically, getting 5kWh when charging, providing 5kWh when discharging and remaining the same when idling.

#### **Explanation of Figure 4 (same for all the other 3 figures):**

**Initial State:** At the start of hour 1, the initial battery level is 10 kWh, and the accumulated cost(weight) is 0.

#### **Transition Costs:**

From 10 kWh to 15 kWh: Charging 5 kWh costs 5 SEK.

From 10 kWh to 10 kWh: Idling has a cost of 0 SEK.

From 10 kWh to 5 kWh: Discharging 5 kWh when the price is 1 SEK/kWh, so the overall gains 5 SEK.

**Calculating Minimal Accumulated Cost:** For each node, calculate the minimal accumulated cost by considering all possible incoming edges and selecting the one with the lowest cost.

**Example Calculation:** For the green edge (10 kWh after the first hour), the minimal accumulated cost is the minimum of: Cost to 15 kWh: 5 SEK, Cost to 10 kWh: 0 SEK, Cost from 5 kWh: -5 SEK.

For the red or purple edge (10 kWh after the second hour), the minimal accumulated cost is the minimum of: Cost to 10 kWh from 10 kWh: 0 SEK, Cost to 10 kWh from 15kWh:  $5 + (-15) = -10$  SEK. So, Cost to 10 kWh is: -10 SEK.

#### **Optimal Strategy for Minimizing Costs**

**Considering Recharging Costs to reach full charge:** End of 4 hour accumulated cost won't consider cost to reach full battery so let's take one more layer cost to reach full battery. To fully recharge the battery, calculate the cost based on the average price of electricity. Example: Ending at 10 kWh will cost (full battery level - ending battery level) times (average price) =  $(20-10) \times 2.75=27.5$ SEK to recharge, which is the cost to full charge.

**Starting with 10 kWh:** The Figure 4 would show different transitions based on the new initial charge. The best strategy might involve a combination of charging, idling, and discharging, aiming to minimize costs based on the given price sequence.

**Shortest path explanation:** For the prices 1,3,5,2, and an average price of 2.75 SEK/kWh. Figure 3 to Figure 6 show the different states of the battery charge across the 4 hours, with edges representing the path between charge levels and representing the transitions between these states due to charging, discharging, or idling. The purple, red, green, and amber color in denoted all four possible paths. Each node cannot have more than one incoming edges. Four tables below show paths of different final battery level and total accumulated cost with different initial battery levels. Moreover, C represents Charge, D represents Discharge and I represent Idle in the tables.

4.10.1.1 **Hand Calculation for Initial Battery level 5kWh**

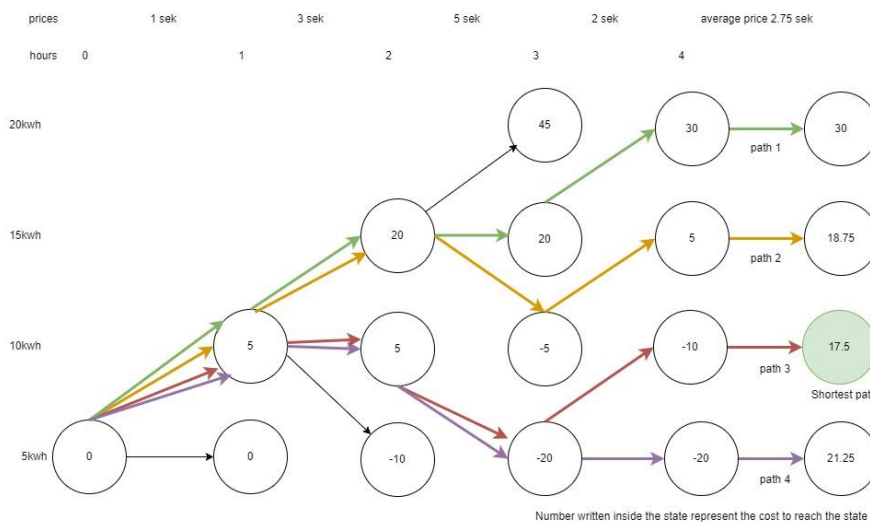


Figure 3 Hand calculation of 4 hours with different electricity price with initial battery level 5kWh.

Path	Start battery level (kWh)	Final battery level (kWh)	Action at 1 <sup>st</sup> hour	Action at 2 <sup>nd</sup> hour	Action at 3 <sup>rd</sup> hour	Action at 4 <sup>th</sup> hour	Total accumulated cost (SEK)
1	5	20	C	C	I	C	30
2	5	15	C	C	D	C	18.75
3	5	10	C	I	D	C	17.5
4	5	5	C	I	D	I	21.25

*Table 2* The path of different final battery level and total accumulated cost when initial battery level is 5 kWh.

Path 3 has the smallest value among all the four paths. So that path 3 is chosen as the shortest path. Path 3 was chosen as the optimal solution because it results in the lowest total accumulated cost among all the evaluated paths, with a final cost of 17.5 SEK. This path strategically leverages the variation in electricity prices by charging when prices are low, idling during moderate prices, and discharging when prices are high. To be more specifically,

Charging during low-price periods: In the first hour, where the electricity price is 1 SEK/kWh, charging the battery is cost-effective. This minimizes the cost of increasing the battery level early in the time frame. Idling when prices are moderate: In the second hour, with a price of 3 SEK/kWh, no action is taken, preventing unnecessary costs. Idling at this point allows the system to avoid paying a higher price for charging. Discharging when prices are high: In the third hour, where the price peaks at 5 SEK/kWh, discharging the battery maximizes profits, taking advantage of the high selling price for stored electricity. Final charging during a moderate price: In the fourth hour, the price drops to 2 SEK/kWh, which is relatively low, so charging again is cost-effective for the final period. These behaviors result in the optimal cost-benefit balance, making Path 3 the most efficient choice in terms of minimizing total electricity costs while maintaining an appropriate final battery level.

4.10.1.2 **Hand Calculation for Initial Battery level 10kWh**

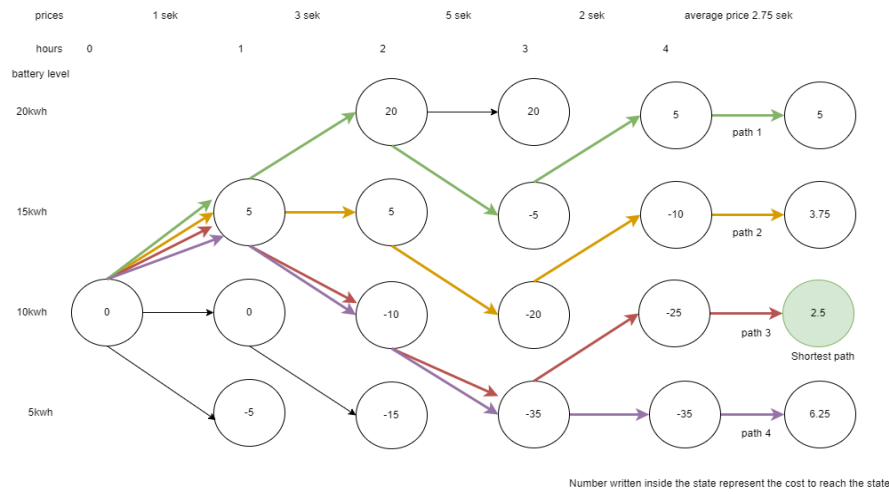


Figure 4 Hand calculation of 4 hours with different electricity price with initial battery level 10kWh.

Path	Start battery level (kWh)	Final battery level (kWh)	Action at 1 <sup>st</sup> hour	Action at 2 <sup>nd</sup> hour	Action at 3 <sup>rd</sup> hour	Action at 4 <sup>th</sup> hour	Total accumulated cost (SEK)
1	10	20	C	C	D	C	6.25
2	10	15	C	I	D	C	3.75
3	10	10	C	D	D	C	2.5
4	10	5	C	D	D	I	5

Table 3 The path of different final battery level and total accumulated cost when initial battery level is 10 kWh.

Path 3 has the smallest value among all the four paths. Same as the reason above, path 3 is chosen as the shortest path. Path 3 was chosen as the optimal solution because it results in the lowest total accumulated cost among all the evaluated paths, with a final cost of 2.5 SEK.

4.10.1.3 **Hand Calculation for Initial Battery level 15kWh**

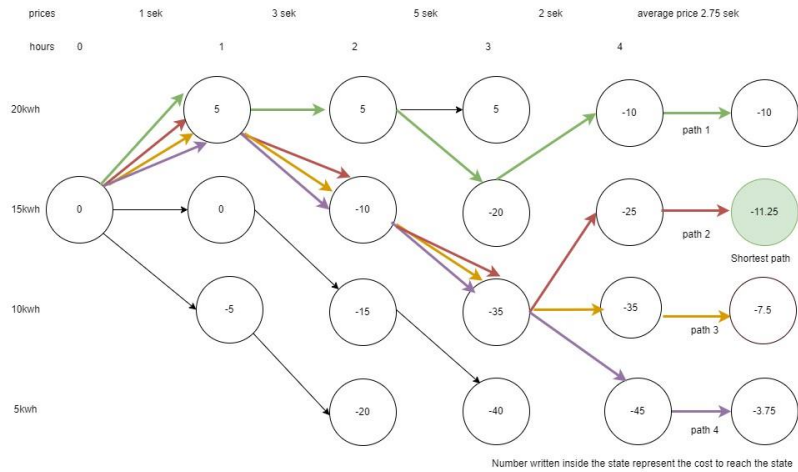


Figure 5 Hand calculation of 4 hours with different electricity price with initial battery level 15kWh.

Path	Start battery level (kWh)	Final battery level (kWh)	Action at 1 <sup>st</sup> hour	Action at 2 <sup>nd</sup> hour	Action at 3 <sup>rd</sup> hour	Action at 4 <sup>th</sup> hour	Total accumulated cost (SEK)
1	15	20	C	I	D	C	-10
2	15	15	C	D	D	C	-11.25
3	15	10	C	D	D	I	-7.5
4	15	5	C	D	D	D	-3.75

Table 4 The path of different final battery level and total accumulated cost when initial battery level is 15 kWh.

Path 2 was chosen as the shortest path because it results in the lowest total accumulated cost among all the evaluated paths, with a final cost of -11.25 SEK.

4.10.1.4 **Hand Calculation for Initial Battery level 20 kWh**

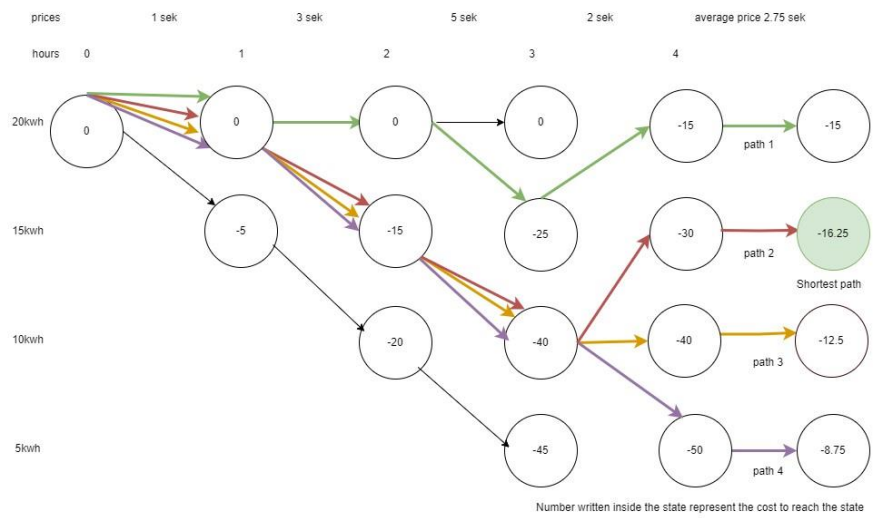


Figure 6 Hand calculation of 4 hours with different electricity price with initial battery level 20kWh.

Path	Start battery level (kWh)	Final battery level (kWh)	Action at 1 <sup>st</sup> hour	Action at 2 <sup>nd</sup> hour	Action at 3 <sup>rd</sup> hour	Action at 4 <sup>th</sup> hour	Total accumulated cost (SEK)
1	20	20	I	I	D	C	-15
2	20	15	I	D	D	C	-16.25
3	20	10	I	D	D	I	-12.5
4	20	5	I	D	D	D	-8.75

Table 5 The path of different final battery level and total accumulated cost when initial battery level is 20 kWh.

Path 2 has the smallest value among all the four paths. Path 2 was chosen as the shortest path because it results in the lowest total accumulated cost among all the evaluated paths, with a final cost of -16.25 SEK.

From all the figures and tables, we can observe a consistent pattern regardless of the starting battery level, whether it begins at 10kWh, 5 kWh, 15 kWh, or 20 kWh. The behavior of battery

above aligns with the objective of minimizing operational costs by capitalizing on price fluctuations.

#### 4.10.2 Example 2

This example illustrates the Dijkstra’s algorithm to determine the optimal path for minimizing cost associated with battery charging, discharging and idling states over 6-hour period for different charging and discharging rate.

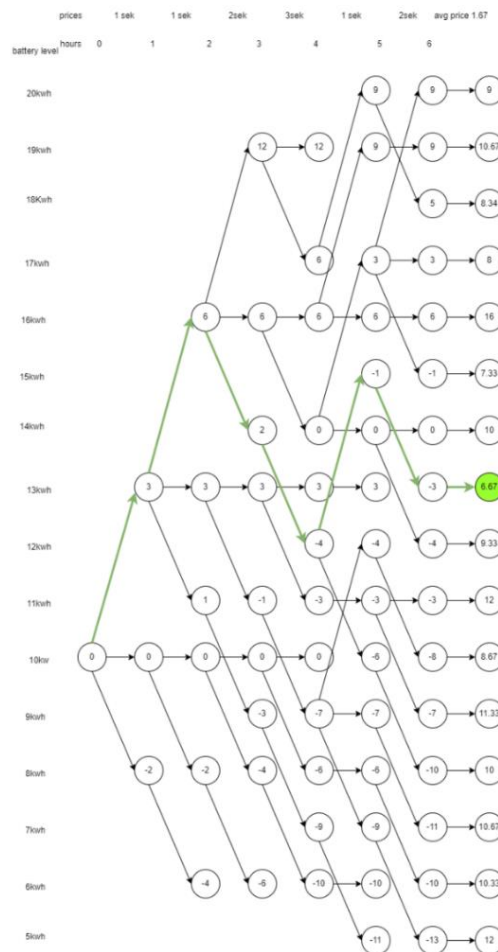


Figure 7 Hand calculation of 6 hours with different electricity price.

To show the complexity of Dijkstra’s this example considered. The complexity increases the run time also increases.

Parameter:

- Charging rate = 3kW
- Discharging rate = 2kW
- Capacity = 20kWh
- Initial Battery Level= 10kWh
- Electricity price = [1, 1, 2, 3, 1, 2] SEK
- Average price = 1.67

The shows the different states of the battery charge across the 6 hours, with edges representing the charge levels and edges representing the transitions between these states due to charging, discharging, or idling. The green color denotes the shortest path. from these examples it is confirmed Dijkstra's correctly charge and discharge based on the prices for any scenarios.

## 4.11 Implementation of Dijkstra's algorithm

To illustrates the implementation of Dijkstra's algorithm. The flow chart explanation provided below.

### 4.11.1 Dijkstra's flow chart

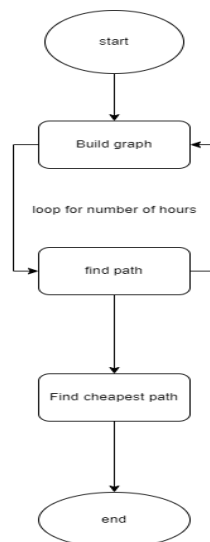


Figure 8 Dijkstra's Flow chart.



Graph and path are updated for each hour. Once all the hours are looped shortest path shall be found.

#### 4.11.2 Build Graph and find path

The detailed explanation of Build graph and finding the paths shows in below flow chart.

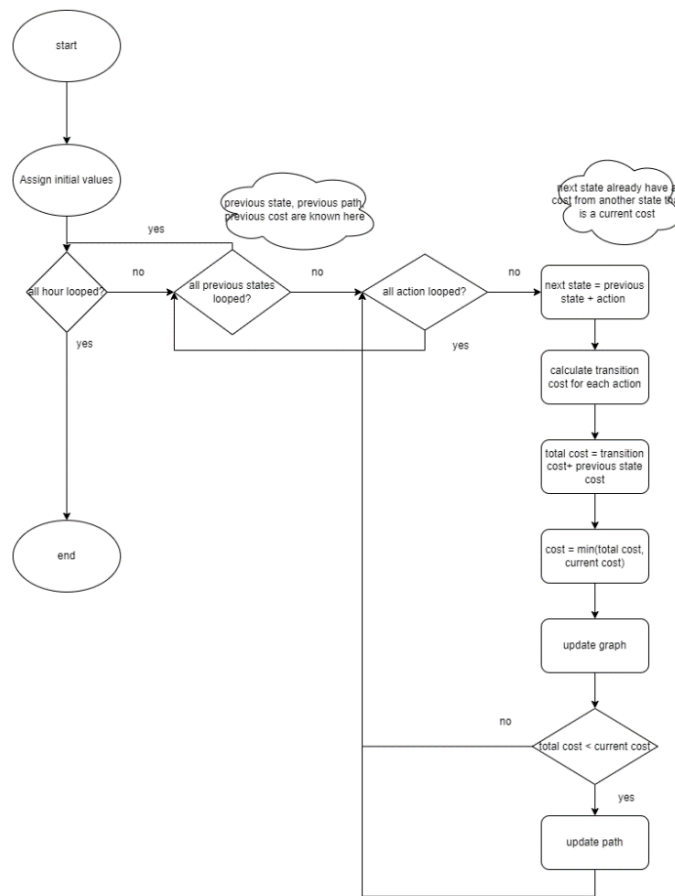


Figure 9 flow chart for build graph and finding path.

Below the explanation for the flow chart for build graph and finding path.

**Assign initial values:** Here the battery level at the initial hour and the cost to reach that level are assigned (that is zero).

**Loop Hourly:** Checks if all hours have been looped.

No: continuous the loop for the next hour.

Yes: End the process.

**Loop through previous states:** Checks if all previous states have been looped. When looping the previous states, previous path and the previous cost are known here.

Yes: continuous looping through the previous states.

No: proceeds to check if all the actions have been looped.

**Loop actions:** Checks if all possible actions from the previous state have been looped.

No: continuous to goes to next state.

Yes: continuous loop through actions.

**Next state:** Next state is the battery level from the previous state for every action. Next state might already have a cost from another state that is current cost.

**Transition cost calculation:** Transition cost calculates based on the action, rates, and the electricity prices. Actions are denoted as charging, discharging and idle.

**Total cost calculation:** Total cost is the combination of transition cost and the previous hour cost.

**Compare cost:** Minimum of total cost and the current cost.

**Update graph:** Update the graph with minimum cost.

**Check cost:** Checks if the total cost is less than the current cost.

No: Continues the loop without updating the path.

Yes: Updates the path with the new cost and action.

**Update path:** Updates the path for the next state with the new cost and action taken.

**End:** The process ends here after all hours and states have been looped through.

## 4.12 Power Consumption for Radio Base Stations

To get the proportion of time each radio is on for different time of the day, a uniform distribution is employed. The RBS workload is then constant during each hour but different constant for each hour of the day. Below is a detailed explanation of the process.

#### 4.12.1 Theory for power consumption

The theory for power consumption of a general radio is shown in this paragraph. Moreover, one radio has 2 basic states, which can be called “on” and “off”.

OFF: In this case the power consumption is equal to 0.

ON: The power consumption of the radio depends on the actual traffic load which can change very rapidly. However, when calculating the power consumption for one hour, a simple model can be to assume that the power consumption is uniformly distributed between  $P_{min}$  and  $P_{max}$  when the radio state is “on”.

Let  $u_1$  be a random variable that follows a uniform distribution on the interval  $[P_{min}, P_{max}]$ . Assume that the actual traffic load of one radio varies between full load and no load,

The power consumption at full load is denoted by  $P_{max}$ . The power consumption at no load is denoted by  $P_{min}$ . Since  $u_1$  is uniformly distributed between  $P_{min}$  and  $P_{max}$ , it can represent the proportion of both the full traffic load. Then the total power consumption for one radio can be written as (unit is kW):

$$P_{total} = (P_{max} - P_{min}) * u_1 + P_{min} \quad 4.17$$

#### 4.12.2 Example for calculation

The following types of radios are used in this thesis, where the whole RBS includes 9 different radios, which can be divided into 3 different types. The table below outlines the radio types and their respective power consumption values.

Radio number	Radio type number	Working time	Power consumption at full load ( $P_{max}$ ) [kW]	Power consumption at no load ( $P_{min}$ ) [kW]
1-3	1	0-23 hours	0.25	0.2
4-6	2	6-22 hours	0.5	0.4
7-9	3	8-11,16-19 hours	0.8	0.6
baseband	-	0-23 hours	0.3	-

*Table 6 Specific power consumption of different radios*

Different types of radios have different states during every hour in a day. They use during different hours because the amount of traffic served by the RBS varies between different hours. Hence, at low traffic times few radios are needed and therefore some of them are in state OFF to save energy. During all the 24 hours, three radios of radio type 1 are used. Three radios of radio type 2 are used from 6 to 22. Last three radios of radio type 3 are used only from both 8 to 11 and 16 to 19, which are all the busiest traffic hours in a day. Based on the information above, a time scheduled optimization algorithm is designed to handle different amount of traffic.

For radio type 1, assume 3 parameters  $u_1^{(1)}, u_2^{(1)}, u_3^{(1)}$  which all obeys a uniform distribution from 0.2 to 0.25, then the total power consumption for radio type 1 working one hour  $V^{(1)}$  can be written based on 12.16 as:

$$\begin{aligned}
 V^{(1)} &= 0.25*(u_1^{(1)}+u_2^{(1)}+u_3^{(1)})+0.2*[3-(u_1^{(1)}+u_2^{(1)}+u_3^{(1)})] \\
 &= 0.6+0.05*(u_1^{(1)}+u_2^{(1)}+u_3^{(1)})
 \end{aligned} \tag{4.18}$$

For radio type 2, assume 3 other parameters  $u_1^{(2)}, u_2^{(2)}, u_3^{(2)}$  which all obeys a uniform distribution from 0.4 to 0.5, then the total power consumption for radio type 2 working one hour  $V^{(2)}$  can be written as:

$$V^{(2)} = 1.2 + 0.1*(u_1^{(2)}+u_2^{(2)}+u_3^{(2)}) \tag{4.19}$$

For radio type 3, assume 3 other parameters  $u_1^{(3)}, u_2^{(3)}, u_3^{(3)}$  which all obeys a uniform distribution from 0.6 to 0.8, then the total power consumption for radio type 3 working one hour  $V^{(3)}$  can be written as:

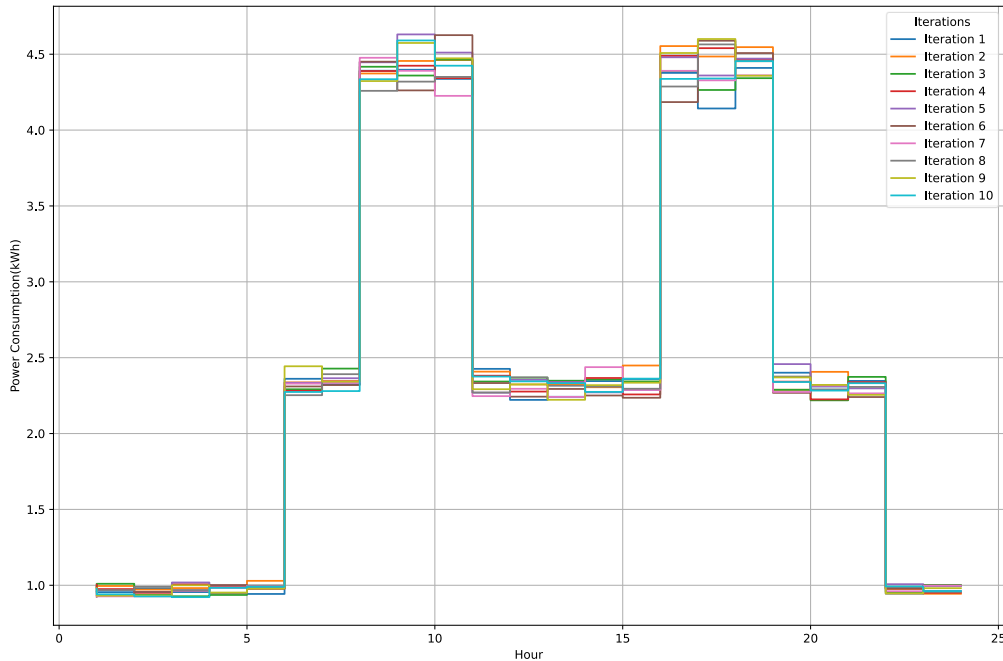
$$V^{(3)} = 1.8 + 0.2 * (u_1^{(3)} + u_2^{(3)} + u_3^{(3)}) \quad 4.20$$

To calculate the power consumption for one day, just use 3 parameters  $V^{(1)}(i), V^{(2)}(i), V^{(3)}(i)$  to represent the total power consumption for radio type 1 to 3 of different hours in a day, which are Independent and identically distributed with  $u_1^{(j)}, u_2^{(j)}, u_3^{(j)}$ .  $V^{(4)}(i)$  represents the power consumption of baseband unit for an hour. After that, sum all the power consumption of every radio during the time they are working,  $P_{one\ day}$ , which can be written as:

$$P_{one\ day} = \sum_{i=1}^{24} V^{(1)}(i) + V^{(2)}(i) + V^{(3)}(i) + V^{(4)}(i) \quad 4.21$$

### Comparison of Power Consumption Over Hours for Multiple Iterations

Power consumption fluctuates each hour and varies from day to day. The figure below illustrates the power consumption across 10 different runs.



*Figure 10* Comparison of Power Consumption Over Hours for Multiple Iterations

Figure 10 clearly shows that the power consumption varies slightly across iterations, leading to noticeable differences in daily energy consumption. It is shown clearly that the power consumption for 10 iterations is same when the electricity prices are low (0-5, 23), similar when electricity prices are moderate (6-7, 12-15, 20-22), and changes dramatically when the electricity prices are high (8-11, 16-19).

Hour	Iteration	Power Consumption(kWh)
1.0	1.0	1.0011732575253958
2.0	1.0	0.9411138245822313
3.0	1.0	0.9789660685264822
4.0	1.0	0.9457909351680774
5.0	1.0	0.9950722559288445
6.0	1.0	0.9851559115325222
7.0	1.0	2.219727933227266
8.0	1.0	2.2859337679614473
9.0	1.0	4.316357058694886
10.0	1.0	4.490776999092901
11.0	1.0	4.506764106053039
12.0	1.0	2.2974591026476077
13.0	1.0	2.3069990487212095
14.0	1.0	2.311295529826298
15.0	1.0	2.331272011814031
16.0	1.0	2.2358918535810637
17.0	1.0	4.54360833907324
18.0	1.0	4.513547096312015
19.0	1.0	4.670055062308989
20.0	1.0	2.367111064219769
21.0	1.0	2.307526639263112
22.0	1.0	2.348711324452653
23.0	1.0	0.9683237990040697
24.0	1.0	0.9292420996394061

*Table 7* Power Consumption for one Iteration

Table 7 presents the raw power consumption values for a single iteration, which cannot be used directly. If we consider 10 initial battery levels with reductions exceeding 10 floating-point values, the number of nodes would increase significantly, making the execution more complex and time-consuming. To simplify this, a rounding factor of 1 is applied to the power consumption values. For example, in the table, the values for hour 1, hour 2, and hour 3 are rounded to 1.0, 0.9, and 0.9, respectively, and so on.

### 4.13 Tools and Libraries Utilized for Simulation and Visualization

While manual calculations for Dijkstra algorithm were effective for cost optimization in our study, using this approach in a real system would be computationally overwhelming. Specialized software is needed to handle the complexities like vast network models, dynamic load variations, and real-time data integration for accurate cost optimization. In the next section will discuss the specific software used for our power system simulation.

The simulations were conducted using Python, a versatile and widely used programming language known for its robust libraries and ease of use in scientific computing. Specifically, the following libraries and tools were employed:

- OS: used for file and dictionary management.
- matplotlib. Pyplot: creating high-quality visualization and graphs.
- NumPy: used for numerical operations.

- Copy: import the copy module and make use of the copy function
- Enum: is a set of symbolic names (members) bound to unique values.
- matplotlib. Gridspec: A grid layout to place subplots within a figure.
- pandas: Pandas is a used for working with data sets.
- time: used to import the time library.
- random: used to create random numbers.

These libraries and methods collectively support various functionalities within the codebase, including data manipulation, visualization, pathfinding algorithms, and time measurement.

For the block diagram and the hand calculations diagram draw.io is used.

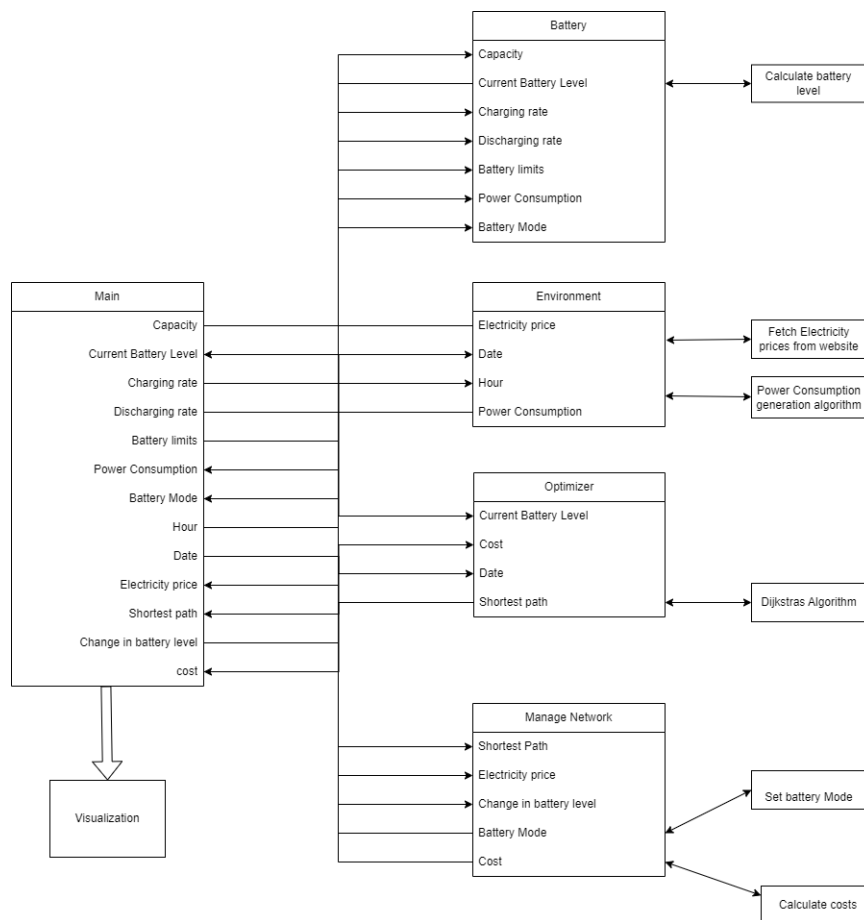


Figure 11 Diagram for software design.



# 5

## Simulation – One Day

In this section, we present and analyze the simulation results. The analysis begins with a simple one-day scenario, designed to illustrate how Dijkstra's algorithm functions, providing a fundamental understanding of its charging, and discharging decisions based on electricity prices. This initial focus allows us to examine key parameters such as battery discharge, electricity prices, and power consumption.

This chapter presents four cases that demonstrate how Dijkstra's algorithm performs under different conditions. Dijkstra's optimization determines battery states based on the optimal path identified by the algorithm, allowing us to observe how varying electricity prices and consumption patterns influence state transitions.

Each of the following subchapters examines distinct scenarios, each with unique conditions:

chapter 5.1.1 Constant power consumption and a price which changes a few times.

chapter 5.1.2 Maximum discharging rate with changing prices and power consumption.

chapter 5.1.3 Low discharge rate changing power consumption with price that are changing.

chapter 5.1.4 Real prices with high discharging rate and changing prices and power consumption.

These comprehensive test shows how it works in finding optimal paths under such conditions. Graphs will illustrate changes in battery levels, accumulated costs, the state of the battery system, and power consumption, highlighting how battery levels evolve over time.

### 5.1.1 **Case 1: Constant power consumption and a price which changes a few times**

In this scenario, electricity prices fluctuate slightly and change several times throughout the day, while power consumption remains constant. The table below displays the electricity

prices for specific time intervals, which are consistent across all cases. The fourth case utilizes real electricity prices. The following parameters are used:

<i>Hour</i>	<i>Electricity Price</i>
<i>0-9</i>	<i>1 SEK/kWh</i>
<i>9-17</i>	<i>2 SEK/kWh</i>
<i>17-24</i>	<i>3 SEK/kWh</i>

*Table 8 Electricity price table*

- Power consumption: 1 kW (constant for all 24 hours)
- Battery parameters:
- Capacity: 20 kWh
- Initial Battery Level: 10 kWh
- Charging rate: 3 kW
- Maximum discharging rate: 2 kW
- Upper charge limit 18 kWh
- Lower charge limit 5 kWh

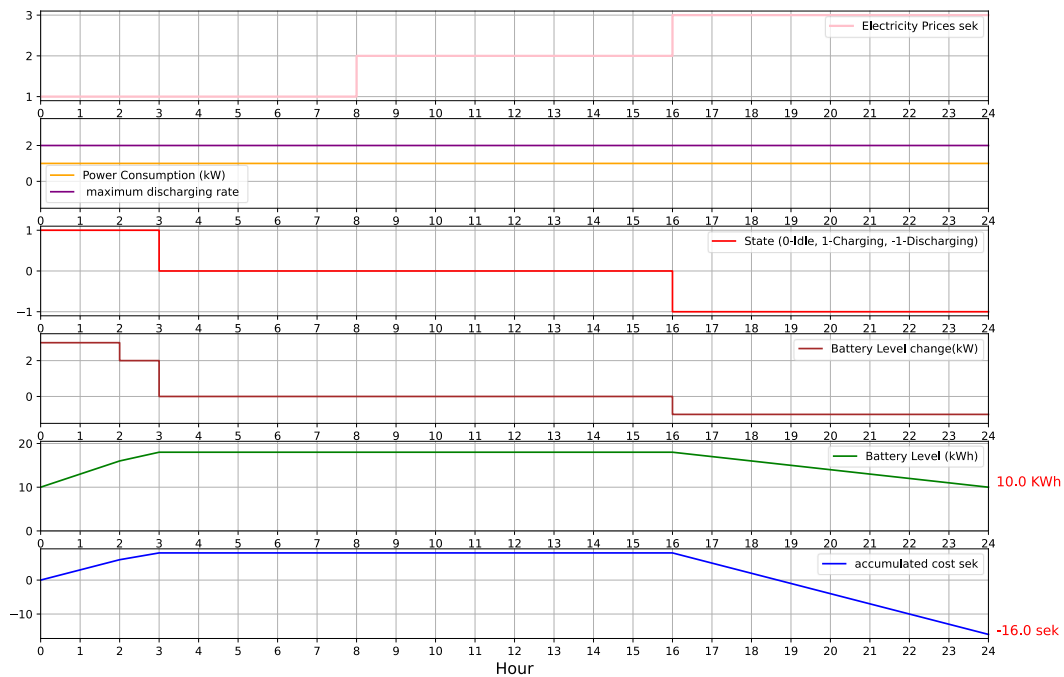


Figure 12 Dijkstra’s Algorithm with constant power consumption and Fluctuating Prices.

The result from the simulation can be described in the following way:

**State changes:** These state changes chosen by Dijkstra’s algorithm based on paths. Below is the explanation of the path chosen.

**Charging and Battery Level Increase (Hour 0-3):** Dijkstra's algorithm prioritizes charging during the first three hours to take advantage of the lower electricity prices as shown in above Figure 12 Plot 1. This strategy allows for an efficient increase in the battery level.

**Idle State and Battery Level Maintenance (Hour 3-16):** From hour 3 to hour 16, Dijkstra operates in "idle" mode because the battery has reached its upper charge limit.

**Discharging and Cost Efficiency (Hour 16-24):** From Hour 16 to 24, Dijkstra's algorithm determined that the optimal action is to discharge the battery. This decision is primarily driven by the increasing electricity prices and the fact that the battery has reached its limit. Dijkstra’s algorithm effectively identifies optimal times to charge and discharge the battery, taking advantage of periods when electricity prices are low for charging and high for discharging. This strategic approach leads to significant cost savings, highlighting the algorithm's potential in dynamic pricing environments.

The accompanying graph clearly illustrates that by prioritizing discharging during high-price periods and charging during low-price periods, battery operation costs are minimized. As a result, the accumulated cost using Dijkstra's algorithm is -16 SEK.

5.1.2

**Case 2: Introduce a maximum discharging rate with changing power consumption and price changes few times**

In Case 2, a maximum discharging rate is introduced along with changes in electricity prices. This scenario allows for a detailed analysis of how the system strategically balances cost-efficiency.

Parameters:

- Power consumption changes as per chapter 4.12
- Battery parameters:
- Capacity: 20 kWh
- Initial Battery Level: 10 kWh
- Charging rate: 3 kW
- Maximum discharging rate: 5 kW
- Upper charge limit 18 kWh
- Lower charge limit 5 kWh

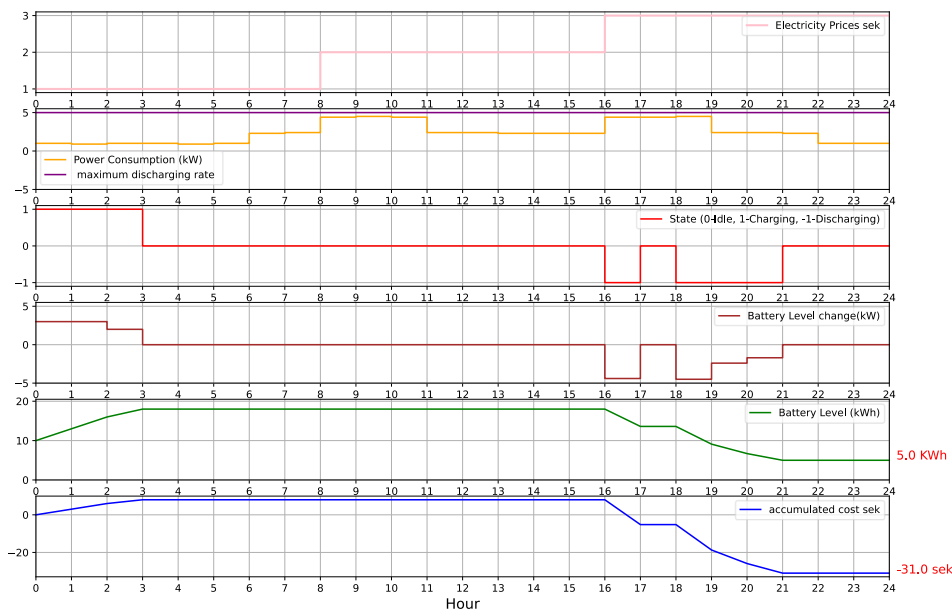


Figure 13 Dijkstra's Algorithm with maximum discharging rate, constant power consumption and fluctuating prices.

The result from the simulation can be described in the following way:

### **State Changes**

These states changes chosen by Dijkstra algorithm based on paths. Below is the explanation of the path chosen.

Dijkstra's algorithm prioritizes charging during the first three hours to take advantage of the lower electricity prices as shown in Plot 1. This strategy allows for an efficient increase in the battery level. From hour 3 to 16 Dijkstra's decided to be idle because battery reach upper charge limit. From 16 to 17 Dijkstra's decided to be discharge because electricity price start increasing at this stage. From hour 17 to 18 it decides to be idled because power consumption high here. Then hour 18 to 21 decide to be discharge because of high price. From hour 21 to 24 decide to be idle because battery reaches lower charge limit.

This scenario demonstrates the effectiveness of Dijkstra's algorithm in battery management. The algorithm discharges the battery when electricity prices and power consumption are high, and charges it during periods of low prices. To prevent excessive depletion of the battery, the system avoids discharging at a high rate. As a result, with Dijkstra's algorithm, the accumulated cost amounts to -31 SEK, reflecting significant cost savings.

5.1.3

### **Case 3: Low discharge rate changing power consumption with price that are changing.**

In Case 3, a low discharging rate is introduced along with changes in electricity prices.

Parameters:

- Electricity price same as case 5.1.1
- Power consumption changes as per chapter 4.12
- Battery parameters:
- Capacity: 20 kWh
- Initial Battery Level: 10 kWh
- Charging rate: 3 kW
- Maximum discharging rate: 2 kW
- Upper charge limit 18 kWh

- Lower charge limit 5 kWh

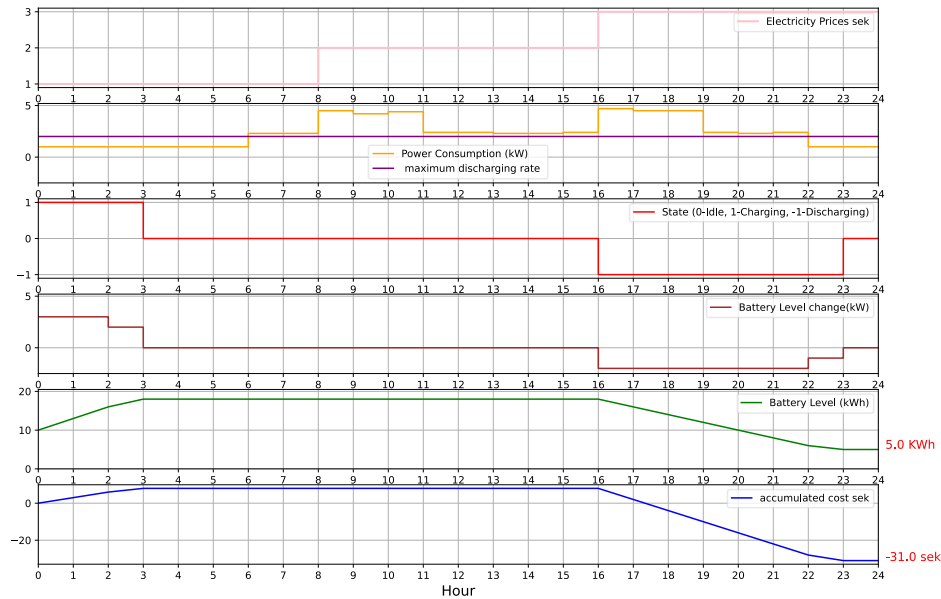


Figure 14 Dijkstra's Algorithm with Low discharge rate, variable power consumption, and fluctuating prices.

The result from the simulation can be described in the following way:

### State changes

The state changes are determined by Dijkstra's algorithm based on optimal paths. Below is an explanation of the selected path.

Dijkstra's algorithm prioritizes charging during the first 3 hours to take advantage of lower electricity prices, as shown in Plot 1. This strategy allows for efficient battery level increase. From hours 3 to 16, the algorithm chooses to remain in "idle" mode since the battery reaches upper charge limit by hour 3, and electricity prices are not very high enough to justify discharging. From hours 16 to 23 Dijkstra's algorithm opts to discharge as prices begin to rise and power consumption also high. In hour 24 decide to be idle even though it has high price, battery reach lower charge limit. So, it can't discharge after that. So, it remains idle.

The changing prices present a unique opportunity for the algorithm to minimize energy costs by strategically managing charging and discharging cycles. By effectively choosing when to charge, idle, or discharge, the system can optimize battery usage and significantly reduce overall energy costs. Figure clearly illustrates this strategy, showing that discharging is prioritized during high-price periods and high-power consumption, while charging occurs

during low-price periods. With Dijkstra's algorithm, the accumulated cost amounts to -31 SEK, indicating significant cost saving.

#### 5.1.4 **Case 4: Introduce real price**

Case 4 delves into a more practical scenario with a maximum discharging rate. It incorporates real-world electricity price data throughout the day. This allows us to analyze how the system minimize costs while considering the maximum discharging rate and the variations present in real electricity prices, all aiming to achieve optimal battery management in a realistic setting. This is same as case 2 only difference is the real electricity prices. From this point forward in this report, real electricity price data collected from the Vattenfall website is utilized. Electricity prices for 24 hours given below.

Parameters:

- Electricity price is shown in Table 9.
- Power consumption changes as per chapter 4.12
- Battery parameters:
  - Capacity: 20 kWh
  - Initial Battery Level: 10 kWh
  - Charging rate: 3 kW
  - Maximum discharging rate: 5 kW
  - Upper charge limit 18 kWh
  - Lower charge limit 5 kWh

The below table shows the prices for one day. These prices are taken from Vattenfall website.

Hour	1	2	3	4	5	6
Prices(SEK/kWh)	0.74	0.08	0.06	0.02	0.10	3.37
Hour	7	8	9	10	11	12
Prices(SEK)	8.84	28.22	46.74	46.95	43.72	39.58
Hour	13	14	15	16	17	18
Prices(SEK)	35.33	33.92	39.54	44.91	49.27	56.46
Hour	19	20	21	22	23	24
Prices(SEK)	59.75	62.07	54.03	50.89	47.38	41.12

Table 9 Real electricity price for one day (1.10.2024)

### With Dijkstra's algorithm

The final case uses real-world electricity prices, presenting the most realistic and challenging scenario. Dijkstra's algorithm shows a huge benefit in charging during low-price periods and discharging during high-price periods.

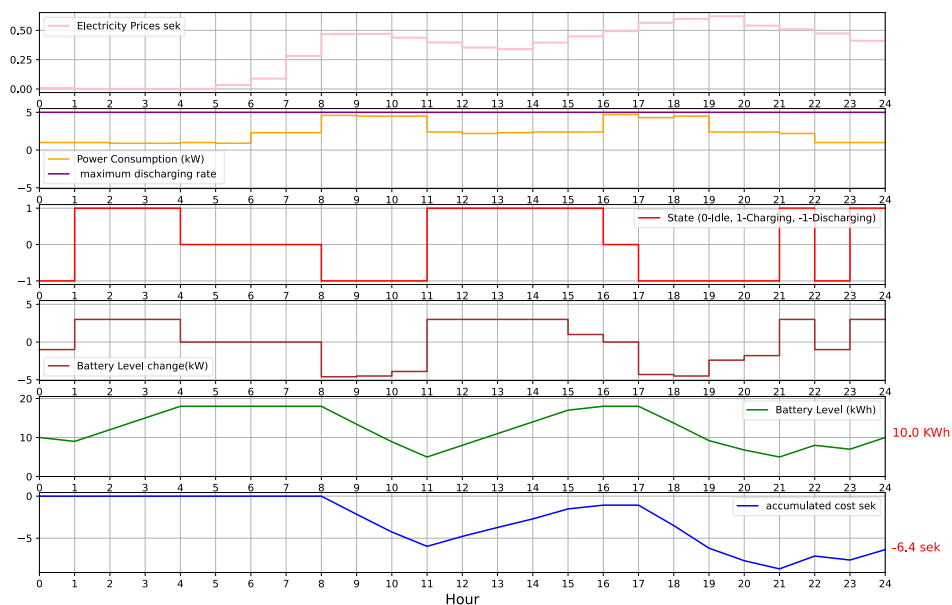


Figure 15 Dijkstra's Algorithm for Real electricity prices.



The result from the simulation can be described in the following way, which is called state changes:

The state changes are selected by Dijkstra's algorithm based on optimal paths. Above figure clearly shows that Dijkstra efficiently adjusts states in response to fluctuating electricity prices. It charges when the prices are low and discharge when the prices are high, same as previous cases. The provided graph clearly demonstrates that discharging is prioritized during high-price periods while charging occurs during low-price periods. This strategic approach effectively minimizes battery operation costs, resulting in an accumulated cost of 6.4 SEK.

### **Explanation of one day results**

Across all one-day scenarios, Dijkstra's algorithm demonstrates its effectiveness in optimizing battery usage based on electricity price fluctuations and power consumption. However, when price fluctuations occur, Dijkstra's algorithm strategically adapts by prioritizing charging during low-price periods and discharging during high-price periods, leading to cost savings. When Dijkstra's select the path to saves high cost it mostly ends up with lower charge limit because it saves more cost when discharging. As a result, battery reaches lower charge limit. This adaptive behavior is clearly shown in the various figures.

# 6

## **Simulation – One Year Optimization on Historical data**

In this one-year optimization with historical data, we aim to optimize battery operation costs over a one-year period using historical electricity prices (vector  $R$  of length  $T$ ) and simulated power consumption during this period (vector  $P$  of length  $T$ ). These inputs remain consistent throughout this chapter. To identify the best possible cost-saving strategy, we apply Dijkstra's algorithm to calculate the optimal battery management policy over the entire period  $T$ . For such a long optimization period, the initial battery charge and penalty settings have minimal impact, as they mostly influence only the first and last few days of the period. Therefore, we assume a starting charge of 10 kWh and set the penalty to zero. This represents the maximum potential savings from one-year optimization with historical data, assuming future prices were fully known. However, in practical applications, future prices are only known one day in advance, making this unattainable. The goal of the study is to determine how close a practical, day-by-day approach can come to this one-year Dijkstra's savings with historical prices solution using only daily price information. When optimizing costs day by day, there is a risk of battery depletion by the end of each day. To address this, we establish a target battery level ( $G$ ) for the end of each day and penalize deviations below this target, ensuring that the battery remains sufficiently charged for future needs. To inform the choice of an appropriate end-of-day target level ( $G$ ), we analyze the battery levels at the end of each day in the one-year Dijkstra's savings with historical prices. By plotting a histogram of these end-of-day levels, we can observe common values such as the mean, median, or mode. These metrics help identify a reasonable target level for the day-by-day optimization that balances cost savings with battery sustainability. After determining an appropriate target level ( $G$ ) based on the one-year Dijkstra's savings with historical prices solution, we run Dijkstra's algorithm on a day-by-day basis. In this approach, penalties are applied to deviations from the target level at the end of each day. We evaluate the algorithm across various scenarios, like initial battery level 5, 10, 15 kWh, using different penalty coefficients (mean, max, high, zero) and for different target levels ( $G$ ). By comparing the total cost savings achieved in each day-by-day scenario with those from the one-year

Dijkstra’s savings with historical prices solution, we identify the combination that yields the closest performance to the one-year Dijkstra’s savings with historical prices cost savings.

The one-year simulation was conducted using historical prices and simulated loads for the period from June 30, 2023, to June 22, 2024.

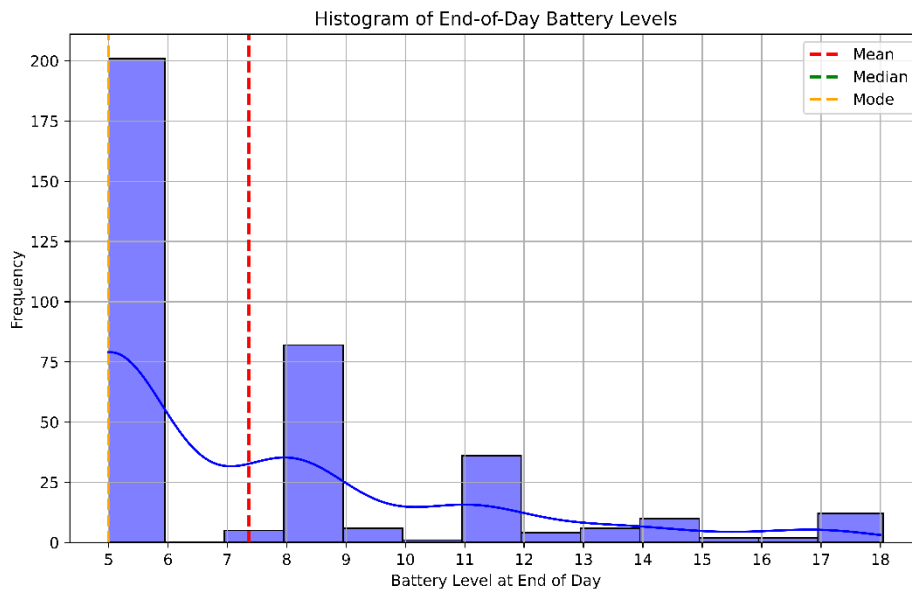


Figure 16 Histogram for end of the day battery level for the one-year Dijkstra’s savings with historical prices.

**Battery Levels:** The x-axis represents the end-of-day battery levels in kWh, with each bar covering a specific range. The battery level in range between 5 kWh to 18 kWh.

**Frequency:** The y-axis shows how often the battery reached each end-of-day level. Taller bars indicate levels that were more common throughout the year, reflecting the frequency of those levels. The blue bars represent the frequency of each end-of-day battery level, while the smooth density curve overlaid on the histogram illustrates the overall distribution.

Key indicators marked on the histogram include.

**Mean (Red dashed line):** This line represents the average battery level at the end of the day across all days in the year.

**Median (Green dashed line):** The median shows the midpoint of the distribution, meaning that half of the days ended with a battery level above this value and half below.

Mode (Yellow dashed line): The mode indicates the most frequently occurring end-of-day battery level, which lies toward the lower end of the scale. This suggests that, under one-year Dijkstra's savings with historical prices optimization, the battery often ends the day with a lower charge.

Mean Battery Level at End of Day: 7.36

Median Battery Level at End of Day: 5.00

Mode Battery Level at End of Day: 5.00

The histogram provides a detailed view of typical battery levels achieved at the end of each day when electricity prices are fully known in advance. The chart captures insights into battery behavior over the course of one-year Dijkstra's savings with historical prices optimization conditions. The data indicates that the battery level frequently falls between 5kWh and 6kWh at the end of the day, with both the median and mode values being 5kWh. This suggests that 5kWh is the most common battery level. While the mean value for the less frequent higher battery levels (between 8kWh and 9kWh) is 7.36kWh, prioritizing the more frequent lower level of 5kWh as a target battery level is reasonable. As a result, from this trend we know the target battery levels that are mean, median and mode. Using these as a target battery level are along with different penalty types. And compare the one-year Dijkstra's savings with historical prices results with day-by-day Dijkstra's.

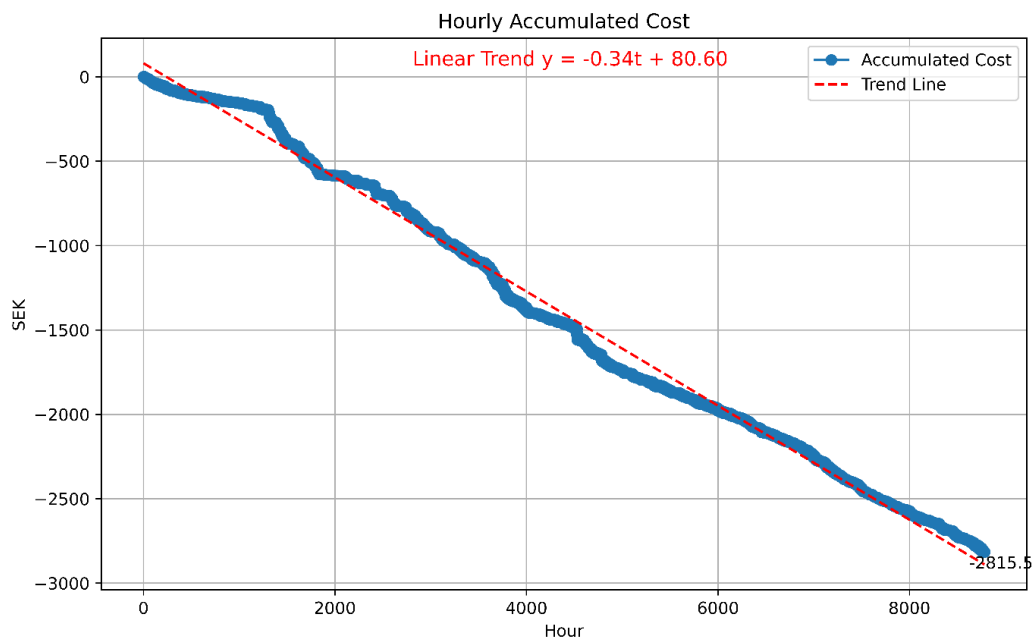


Figure 17 Annual accumulated cost with linear trend line.

The Figure 17 illustrates the Hourly Accumulated Cost over one year and the linear trend of accumulated cost.

Blue Line: Actual Accumulated Cost, the blue line represents the actual accumulated cost at each hour, fluctuating up and down because of the changes in electricity prices over time.

Red Dashed Line: Linear Trend Line, the red dashed line is a linear trend line that approximates the overall direction of the accumulated cost throughout the year.

The equation for this linear trend line is  $y = -0.34t + 80.60$

y represents the accumulated cost in SEK, x represents time in hours.

The slope of  $-0.34$  indicates that, on average, the accumulated cost decreases by 0.34 SEK per hour. The y-intercept of 80.60 is the starting point of the trend line when  $t=0$ , setting an initial reference point for the cost trend. The trend line that is regression equation says that we would save on average  $0.34 * 24 = 8.16$  SEK per day when using the optimal strategy, knowing the whole year's prices ahead. This trend line provides a simplified view of the general cost-saving trend, highlighting how the algorithm drives down operational costs over time. The accumulated cost at the end of one year reaches 2,815.5 SEK. This final value represents the maximum achievable savings over the year.

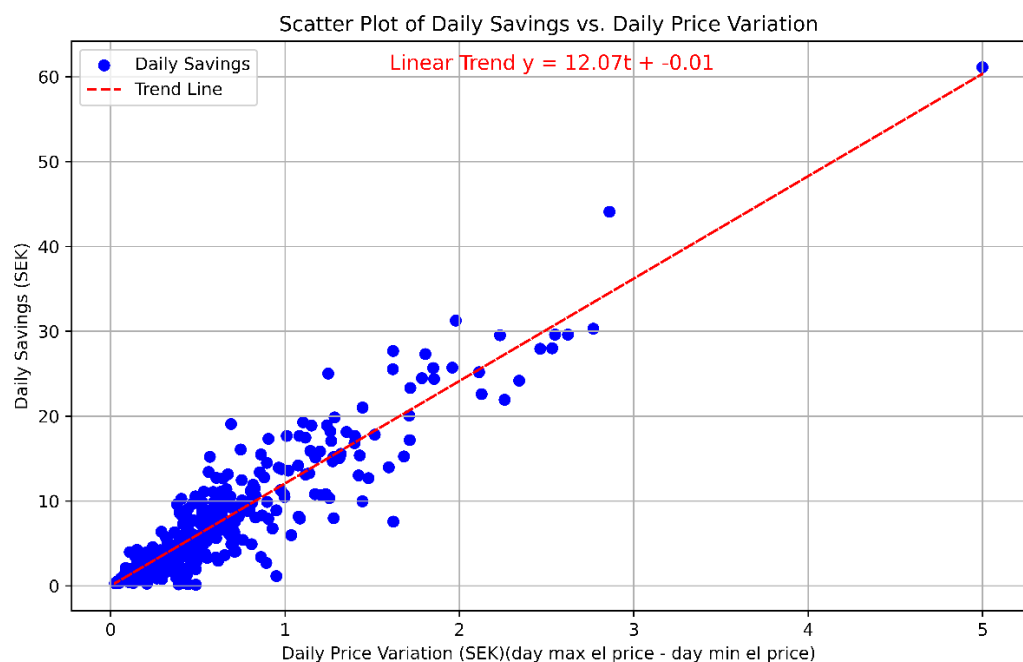


Figure 18 Linear Trend Between Daily Savings and Daily Price Variation

The figure above shows the relationship between daily price variation and daily savings along with it shows the linear trend of daily savings. daily price variation is subtraction of maximum daily price and minimum of daily price. Most of the days prices varies between 0 SEK to 1 SEK, leading to daily savings that typically vary from 0 SEK to 10 SEK.

Red Dashed Line: The red dashed line represents a linear trend line that approximates the overall relationship between daily price variation and daily savings.

The equation for this line is  $12.07 t + (-0.01)$

y denotes the daily savings (in SEK), x represents the daily price variation (in SEK).

The slope of 12.07 indicates that for each 1 SEK increase in daily price variation, the daily savings increase by approximately 12.07 SEK. The y-intercept of (-0.01) suggests that when there is no price variation (i.e.,  $t=0$ )

For example, regression equation says yesterday the electricity price variation is 2.26 kronor. Potentially, one could save about  $12.07 * 2.26 = 27.3$  kronor. Today, it is 53 ore. So, the savings would be only about  $12.07 * 0.53 = 6.4$  kr. This trend line provides an overall view of how daily price variations affect daily savings, illustrating that greater price fluctuations generally result in increased cost savings. The below table shows the number of hours a battery spent in different states over a certain period.

### Hours Spent in Different Battery States

Battery State	Number of Hours	Relative Frequency (%)
Charging	3142	35.8
Discharging	2903	33.0
Idling	2740	31.2

*Table 10 Hours spent in different battery states.*

# 7

## Simulation – Day by Day optimization

In our day-by-day optimization, we are extending the time frame to one year. we analyzed three methods: time-scheduled optimization, Dijkstra's optimization, and a no-optimization. By comparing the total cost savings achieved in each day-by-day scenario with those from the one-year Dijkstra's savings with historical prices solution, we identify the combination that yields the closest performance to the one-year Dijkstra's savings with historical prices cost savings.

**No Optimization:** This approach involves no active battery management. Energy is directly sourced from the grid without any consideration for price fluctuations or battery storage.

**Day-by-Day Time-Based Optimization:** This traditional method relies on fixed time windows for charging, discharging, and idling. By analyzing historical electricity price trends (typically higher during the day and lower at night), we pre-determine optimal time ranges for each operation like charging, discharging, and idling. Simulations are conducted with various initial battery levels (5kWh, 10kWh, and 15kWh) to assess the impact of starting conditions. No penalties are applied.

**Day-by-Day Dijkstra's Optimization:** This method utilizes Dijkstra's algorithm to determine the optimal sequence of charging, discharging, and idling states. Different initial battery levels are tested, along with various penalty types and target battery levels. By incorporating these factors, the optimization aims to minimize energy costs while maintaining desired battery levels. The table below presents the accumulated cost over one year using Dijkstra's algorithm with various penalty types and target battery levels. The results are also compared with a time-scheduled approach and with no optimization applied.

Optimization Name	Initial Battery Level at day 1 (kWh)	PenaltyType	Target battery level type	Accumulated Cost (SEK)
dijkstras_optimization	15	zero	none	-2785.83
dijkstras_optimization	5	zero	none	-2783.47
dijkstras_optimization	5	max	mode	-2783.3
dijkstras_optimization	10	zero	none	-2783.0
dijkstras_optimization	5	high	mean	-2782.86
dijkstras_optimization	5	avg	mean	-2782.76
dijkstras_optimization	5	max	median	-2782.74
dijkstras_optimization	5	max	mean	-2782.69
dijkstras_optimization	5	avg	mode	-2782.48
dijkstras_optimization	5	high	median	-2782.19
dijkstras_optimization	5	avg	median	-2781.57
dijkstras_optimization	5	high	mode	-2780.96
dijkstras_optimization	10	avg	mean	-2642.2
dijkstras_optimization	10	high	mean	-2642.16
dijkstras_optimization	10	max	mean	-2642.14
dijkstras_optimization	10	avg	median	-2641.73
dijkstras_optimization	10	high	mode	-2641.15
dijkstras_optimization	10	avg	mode	-2640.46
dijkstras_optimization	10	high	median	-2640.18
dijkstras_optimization	10	max	median	-2640.07
dijkstras_optimization	10	max	mode	-2639.9
dijkstras_optimization	15	max	mean	-2267.38
dijkstras_optimization	15	max	mode	-2265.88
dijkstras_optimization	15	high	median	-2265.5
dijkstras_optimization	15	avg	median	-2265.39
dijkstras_optimization	15	max	median	-2264.74
dijkstras_optimization	15	avg	mode	-2264.65
dijkstras_optimization	15	avg	mean	-2264.36
dijkstras_optimization	15	high	mode	-2262.0
dijkstras_optimization	15	high	mean	-2258.9
time_scheduled_optimization	15	none	none	-1363.44
time_scheduled_optimization	10	none	none	-1361.57
time_scheduled_optimization	5	none	none	-1360.3

*Table 11 Accumulated cost table for one year.*

**Cost Without Optimization:** Without any battery optimization, the total cost would be 11,912 SEK.

**Time-Scheduled Optimization:** In comparison, a time-scheduled approach, which does not adapt to daily price variations, saves 1,363 SEK, or 11.43% of the total expense. While beneficial, this method offers substantially lower savings than Dijkstra-based strategies.



**One-year Dijkstra Optimization on historical electricity prices:** If electricity prices and loads were fully known for the year, the optimal switching policy using Dijkstra's algorithm could save 2,814 SEK, equivalent to a 23.63% reduction in total expense. This represents the maximum potential cost reduction achievable by optimizing battery usage.

**Day-by-Day Optimization:** Using a day-by-day version of Dijkstra's algorithm, results in savings close to the one-year Dijkstra's savings with historical prices. This approach yields 2,785 SEK (23.38% cost reduction), only 28.82 SEK or 0.25% less than the one-year Dijkstra's with historical prices. The small discrepancy indicates that day-by-day optimization closely approximates the one-year Dijkstra's savings with historical prices. An initial battery level of 15 kWh with a zero penalty type results in the highest cost savings, with a cost saved of 2,785 SEK. By applying Dijkstra's day-by-day optimization, the expense reduces to 9,127 SEK, achieving significant savings of 2,785 SEK from total expense.

# 8

## Discussion

**Adaptability and Scalability:** The algorithm's ability to adapt to varying constraints, such as maximum discharge rates and fluctuating power consumption, highlights its versatility in real-world battery management systems. The consistent performance of Dijkstra's algorithm demonstrates its scalability.

**Real-world Implications and Performance Comparison:** By leveraging historical electricity price data and real-world constraints, the simulations provide valuable insights into the practical application of optimization algorithms in battery management systems. These findings can inform the development of future smart grid technologies and energy management solutions.

A key finding of this research is that even without perfect knowledge of future electricity prices, Dijkstra's algorithm can still yield significant cost savings.

# 9

## Future Enhancements

- **Algorithm Execution Location:** Determine the optimal location for executing the algorithm, considering whether it should run directly on battery management system hardware or be offloaded to an external server. This decision will affect the efficiency, cost, and latency of the system.
- **Advanced Optimization Techniques:** Explore further research into advanced optimization techniques to enhance overall performance. Specifically, integrating machine learning models with methods like Dijkstra's algorithm could improve system effectiveness by using ML for forecasting power consumption and price patterns, while Dijkstra's algorithm optimizes resource allocation based on these predictions.
- **Hybrid Algorithms:** Develop hybrid algorithms that combine Dijkstra's algorithm with other optimization techniques, such as genetic algorithms or dynamic programming, to improve performance in complex scenarios and enhance the robustness of the system.
- **Real-time Data Integration:** Enhance the system's responsiveness and adaptability by incorporating real-time data inputs from smart grids, weather forecasts, and other relevant sources. This integration would improve the system's ability to adapt to changing conditions and optimize battery usage more effectively.
- To further enhance cost savings, future research could explore the development of advanced models to predict future electricity prices. By integrating these models with Dijkstra's algorithm, the system can make more accurate decisions, leading to even greater cost reductions.

# 10

## Conclusion

Dijkstra's algorithm has proven to be highly effective in reducing costs. When price fluctuations occur, Dijkstra's algorithm strategically adapts by prioritizing charging during low-price periods and discharging during high-price periods, leading to cost savings.

We conducted a series of simulations to evaluate the performance of Dijkstra's algorithm under various conditions. Four one-day case studies were performed to analyze its behavior in different scenarios. Additionally, a one-year Dijkstra's savings with historical prices was simulated to establish a benchmark for optimal performance. Finally, a day-by-day comparison was conducted between time-based optimization, Dijkstra's optimization for different initial battery levels, penalty types, target battery levels from one-year Dijkstra's savings with historical prices and a no-optimization.

Dijkstra's algorithm achieves significant cost savings, reducing expenses by 23.38% compared to no optimization. Time-scheduled optimization offers a more modest 11.43% reduction. The one-year Dijkstra's savings with historical prices, with perfect foresight, yields a 23.63% reduction, while day-by-day Dijkstra's optimization closely approximates this with a 23.38% reduction.

In conclusion, Dijkstra's algorithm emerges as an efficient tool for optimizing energy consumption and reducing costs in RBS operations. By effectively responding to dynamic electricity prices, this algorithm offers a significant advantage over traditional approaches.

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