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Can we reach the UN Paris Agreement's climate goals?

A robust model predictive control approach

Master's thesis in Systems, Control and Mechatronics

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Reaching the UN Paris Agreement goals through robust model predictive control
A study on the feasibility of achieving the climate goals set by the United Nations using model predictive control and considering the equilibrium climate sensitivity in a robust way
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Abstract

Climate change is arguably one of the most critical challenges of our time. For this reason, countries have committed, under the UN Paris Agreement, to limit global warming well below 2°C (aiming for 1.5°C) by 2050 [12]. One of the main models cited in the literature whose goal is to predict climate change is the DICE Model, developed by William Nordhaus. An important issue regarding this model arises from the fact that it contains a critical parameter whose estimation can lead to highly varying values and which has a huge impact on the model's outputs: the climate sensitivity. The value of this parameter determines whether or not the above mentioned commitment is feasible. The goal of this master's thesis work is that of expanding the DICE model to add robustness to it with respect to the climate sensitivity, by considering a whole set of values instead of a single one. This robust model, combined with previous results aimed at making said model more realistic, will then be used in a model-based predictive control setting, to devise optimal control strategies aimed at reaching the goals stated in the UN Paris Agreement. In order to consider the climate sensitivity in a robust way, we will solve the original optimization problem behind the DICE model in a worst-case scenario, where the worst case comes from an "adversary agent" who tries to maximize the climate sensitivity while we try to keep the atmospheric temperature as low as possible. In this study, we will show that the objectives of the UN Paris Agreement are feasible under some conditions but also that reaching said objectives requires a strong and fast abatement effort. The impact that the value of the equilibrium climate sensitivity has on the results will also be analyzed, in order to determine how important it is to add robustness to the model when trying to comply with the UN Paris Agreement's goals.

Keywords: climate change, model predictive control, robustness, uncertainty, climate sensitivity, UN Paris Agreement

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Lorenzo Montalto, Gothenburg, May, 2022

List of acronyms and symbols

Below is the list of acronyms and symbols that have been used throughout this thesis listed in alphabetical order (except for the constants, as they can be found in the Parameters section 6):

A_{TFP}	Total factor production
A_C	Abatement costs
C	Consumption
D	Damages function
E	Total emissions
ECS	Equilibrium climate sensitivity
ECS_{rate}	Rate of ECS at a time instant
E_{LAND}	Emissions due to land use change
F_{EX}	External forcings
GtC	Gigatonne of carbon
J	Social welfare
K	Capital
L	Population
M_{AT}	Mass of carbon in the atmosphere
M_{LO}	Mass of carbon in the lower ocean
M_{UP}	Mass of carbon in the upper ocean
Q	Net economic output
R_F	Radiative forcing
s	Savings rate
t	Time index
T_{AT}	Atmospheric temperature
T_{LO}	Lower ocean temperature
Y	Gross economic output
θ_1	Cost of mitigation efforts
μ	Abatement rate
μ_{growth}	Growth of μ at a time instant
μ_{rate}	Rate of μ at a time instant
σ	Carbon emission intensity due to economic activity
Φ_M	Carbon mass transition matrix
Φ_T	Temperature transition matrix

Contents

1

Introduction

1.1 The DICE model

The DICE model is a model developed by William Nordhaus, who was awarded the Nobel Prize precisely thanks to this work. The name "DICE" stands for "Dynamic-Integrated Climate Economy" and, as the name suggests, it's a model that links

- society (e.g., population, population growth)
- economy (e.g., capital, investments)
- environment (e.g., temperatures, carbon masses)

Its ability to link these three big boxes together in such a way that they are all connected and dependent with each other is the reason why it is one of the most used models when it comes to climate change prediction. This master's thesis not only uses this model but it updates it to make it more robust and more accurate, so that the predictions obtained from it are more reliable and more useful. The need of updating the DICE model comes from the fact that it is one of the most used models among modellers, policy makers and economists [3]. It is therefore important to have it as accurate as possible.

The DICE model is quite complex, as it contains many state variables and numerous parameters. However, one parameter is of particular importance: the climate sensitivity.

1.2 Climate sensitivity

The climate sensitivity refers to the amount of global surface warming that will occur in response to a doubling of atmospheric CO_2 concentrations compared to pre-industrial levels. In other words, it is a measure of how much the environment reacts to our emissions: if it is high, it means that our emissions will have a larger impact on the environment and this, of course, is a bad thing. This is one of the problematic parameters of the DICE model, since we do not know its exact value. What we have is a range of possible values that remains stubbornly wide despite the efforts in reducing it [6]. This issue calls for either a better estimation or, from a control perspective, a robust control strategy that is able to consider all the possible values of this parameter. The DICE model is formulated as a nonlinear dynamic model subject to many constraints. Considering this and also the fact that we want to act quickly, it seems natural to employ a nonlinear model predictive control strategy to obtain optimal control policies [1]. The reason why this parameter is so important is that, in its range of possible values, there exist values for which the

UN Paris Agreement's goals are feasible and other for which they are not. More precisely, if the climate sensitivity is too high, we will not be able to comply with the UN Paris Agreement but if it is low enough then it is possible.

1.3 Aim

This master's thesis will develop a robust version of the DICE Model such that it is able to properly handle the uncertainty behind the climate sensitivity. This robust model will then be used to employ a nonlinear robust model predictive control strategy whose aim is that of reaching the goals set by the UN Paris Agreement. More precisely,

- Expand the DICE Model to properly take into account climate sensitivity in a robust way.
- Modify the model, according to the literature, in order to make it more realistic.
- Use the obtained model to analyse the possibility of achieving the goals of the UN Paris Agreement.
- Design a robust model predictive controller to develop control strategies to achieve said goals.
- Analyse the obtained results and their feasibility.
- Analyse the impact of considering the climate sensitivity in a robust manner.

1.4 Limitations

In the context of the DICE model, the following limitations apply to this thesis:

- The DICE Model aims at modeling a very complex system covering both environmental and economic aspects. Therefore, it is important to notice that the model makes some important assumptions in the way it models both the climate and the economy. Given that this thesis uses this model, such assumptions also apply to this work.
- It's also important to notice that the DICE model is global, which means that all the "local" properties are approximated and assimilated into single global ones.
- The impossibility to actually test in reality such a scenario means that everything that will be shown here is the result of simulations.

1.5 Main research questions

With this thesis work we intend to answer the following research questions:

- How to expand the model such that the equilibrium climate sensitivity is treated in a robust way?
- How to update the model according to recent literature in order to make it more realistic?

- How will the choice of such a model affect the possibility of reaching the goals stated in the UN Paris Agreement?
- Is it possible to obtain feasible control strategies by employing robust model predictive control?
- Under what circumstances can the goals of the UN Paris Agreement be achieved?
- Is it meaningful to consider the equilibrium climate sensitivity in a robust way?
- Do we have time to wait before starting to act against climate change?
- Can the fight against climate change still come second to short-term economical profit?

2
The DICE model

The model used in this thesis is an extension of [1], which is, in turn, a reformulation of Nordhaus' DICE2016R model [11]. We also integrate in the model some updates taken from [4] and some additions original to this thesis. The model starts from the year 2015 and has a time step Δ of 5 years. It is a highly nonlinear model consisting of 21 state variables (the model in [1] only has 17, the additional 4 state variables were added in this thesis and they are the last 4 variables in 2.1), most of which having a corresponding dynamic equation. Two of these 21 states are the control inputs: μ (the abatement rate) and s (the savings rate); while one is, instead, the equilibrium climate sensitivity mentioned in the introduction (chapter 1). The state vector is defined as follows:

$$x := \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \\ x_{16} \\ x_{17} \\ x_{18} \\ x_{19} \\ x_{20} \\ x_{21} \end{bmatrix} := \begin{bmatrix} t \\ T_{AT} \\ T_{LO} \\ M_{AT} \\ M_{UP} \\ M_{LO} \\ K \\ \sigma \\ L \\ A_{TFP} \\ E_{LAND} \\ F_{EX} \\ E \\ C \\ \mu \\ s \\ J \\ \mu_{rate} \\ \mu_{growth} \\ ECS \\ ECS_{rate} \end{bmatrix} \tag{2.1}$$

Among these states, as mentioned in the introduction, the equilibrium climate sensitivity (ECS), is particularly important:

- The improvement in robustness that this thesis aims to achieve is centered around the ECS due to its difficult estimation.

- It is treated as the only control input when solving the adversarial problem (more on that in chapter 3).

Now, we consider each state in detail, mentioning the respective dynamic equation. All the initial conditions for each state can be found in the Appendix at the end of the thesis (6.2).

2.1 Iteration index: t

The first state is simply the iteration index used in the optimization. As such, the dynamic equation is extremely simple:

$$t(i+1) = t(i) + 1 \tag{2.2}$$

Later on, in the Optimization chapter (3), it will be useful to have these state equations as functions of the states. So, we can rewrite the dynamic equation of the time index as a function of the states in the following way:

$$x_1(i+1) = x_1(i) + 1 \tag{2.3}$$

This is purely a notation matter and does not change the meaning of the state equation. Its only purpose is to explicitly specify which states are involved in the equation, since the state vector is quite large.

2.2 Temperature: T

The model for the temperature is divided into two components: the atmospheric temperature (T_{AT}, which is actually the combination of three contributions: atmosphere, land surface and upper ocean) and the lower ocean temperature (T_{LO}). The zero reference for such temperatures is taken as the temperature in the year 1750 (considered as the end of the pre-industrial era). The combined dynamic equation, in matrix form, for both the components, is as follows:

$$T(i+1) := \begin{bmatrix} T_{AT}(i+1) \\ T_{LO}(i+1) \end{bmatrix} = \phi_T T(i) + \begin{bmatrix} R_F(i) \\ 0 \end{bmatrix} \tag{2.4}$$

Where ϕ_T and R_F are, respectively, the temperature transition matrix and the radiative forcing at the top of atmosphere due to the enhanced greenhouse effect (both analyzed more in details below). It's interesting to notice that the atmospheric temperature depends on the radiative forcing, i.e. on the greenhouse effect.

2.2.1 Temperature transition matrix

The matrix ϕ_T dictates how the temperatures at a time step depend on the temperatures at the previous time step and is defined as follows:

$$\phi_T := \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} = \begin{bmatrix} 1 - \xi_1(\frac{\eta}{ECS(i)} + c_3) & \xi_1 c_3 \\ c_4 & 1 - c_4 \end{bmatrix} \tag{2.5}$$

An important thing to notice is that this is the equation in which the ECS appears. This state is of crucial importance for this master's thesis work and will be described thoroughly later (2.17).
Since the ECS is a state, this expression can be rewritten as a function of the states:

$$\phi_T := \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} = \begin{bmatrix} 1 - \xi_1(\frac{\eta}{x_{20}(i)} + c_3) & \xi_1 c_3 \\ c_4 & 1 - c_4 \end{bmatrix} \tag{2.6}$$

2.2.2 Radiative forcing

A satisfying definition of radiative Forcing is given by [7]:

> Radiative forcing is what happens when the amount of energy that enters the Earth's atmosphere is different from the amount of energy that leaves it. Energy travels in the form of radiation: solar radiation entering the atmosphere from the sun, and infrared radiation exiting as heat. If more radiation is entering Earth than leaving—as is happening today—then the atmosphere will warm up. This is called radiative forcing because the difference in energy can force changes in the Earth's climate.

In [1] the radiative forcing is modelled as follows:

$$R_F(i) = \xi_1 \left(\eta \frac{\ln(\frac{\zeta_{11} M_{AT}(i) + \zeta_{12} M_{UP}(i) + \xi_2 E(i)}{M_{AT,1750}})}{\ln(2)} + F_{EX}(i) \right) \tag{2.7}$$

In this equation, no control inputs appear (as stated above, the control inputs are μ and s). This means that our control action has no direct effect on the radiative forcing. As stated in [4], this is unrealistic and thus they propose the following model, which is also the one used in this thesis:

$$R_F(i) = \xi_1 \left(\eta \frac{\ln(\frac{\zeta_{11} M_{AT}(i) + \zeta_{12} M_{UP}(i) + \xi_2 E(i)}{M_{AT,1750}})}{\ln(2)} + F_{EX}(i)(1 - a_{frac}\mu(i)) \right) \tag{2.8}$$

The equation is almost the same as before but, in 2.8, F_{EX} is multiplied by an additional term that decreases the more μ increases. This means that the external forcings (F_{EX}, i.e. one of the states, expanded in details later (2.9)) get reduced as the emissions get reduced. The parameter $a_{frac} \epsilon$ [0, 1] determines how much of the external forcings can be abated by the control input μ. This of course raises a tuning problem because we need to determine the value of this parameter. In [4] a value of $a_{frac} = 0.6$ is used, which means that our control action can directly reduce by 60% the external forcings. One of the results in the Results chapter (4.4) shows the effects of changing the value of a_{frac}. In both models, the radiative forcings depend logarithmically on the mass of CO_2 in the atmosphere (M_{AT}), which means that the higher the M_{AT}, the higher the R_F (i.e. the higher the greenhouse effect). We can rewrite the radiative forcings as a function of the states:

$$R_F(i) = \xi_1 \left(\eta \frac{\ln(\frac{\zeta_{11} x_4(i) + \zeta_{12} x_5(i) + \xi_2 x_{13}(i)}{M_{AT,1750}})}{\ln(2)} + x_{12}(i)(1 - a_{frac} x_{15}(i)) \right) \tag{2.9}$$

2.3 Carbon mass: C

The model for the carbon mass is divided into three components: mass of carbon in the atmosphere (M_{AT}), mass of carbon in the upper ocean layer (M_{UP}) and mass of carbon in the lower ocean layer (M_{LO}). The dynamic equation, in matrix form, that governs all the three component is the following:

$$M(i+1) := \begin{bmatrix} M_{AT}(i+1) \\ M_{UP}(i+1) \\ M_{LO}(i+1) \end{bmatrix} = \phi_M M(i) + \begin{bmatrix} \xi_2 E(i) \\ 0 \\ 0 \end{bmatrix} \tag{2.10}$$

Where ϕ_M is the carbon mass transition matrix. As we can see from this equation, the mass of carbon in the atmosphere is driven by CO_2 emissions due to economic activity (represented in the model as the state $x_{13}(i)$=$E(i)$).

2.3.1 Carbon mass transition matrix

The matrix ϕ_M dictates how the carbon masses at a time step depend on the carbon masses at the previous time step and is defined as follows:

$$\phi_M := \begin{bmatrix} \zeta_{11} & \zeta_{12} & 0 \\ \zeta_{21} & \zeta_{22} & \zeta_{23} \\ 0 & \zeta_{32} & \zeta_{33} \end{bmatrix} \tag{2.11}$$

All these values can be found in the Appendix (6.4).
This state equation describes the carbon cycle. A satisfying definition of what the carbon cycle is can be found in [10]:

> The carbon cycle describes the process in which carbon atoms continually travel from the atmosphere to the Earth and then back into the atmosphere. Since our planet and its atmosphere form a closed environment, the amount of carbon in this system does not change. Where the carbon is located — in the atmosphere or on Earth — is constantly in flux. [...] Humans play a major role in the carbon cycle through activities such as the burning of fossil fuels or land development. As a result, the amount of carbon dioxide in the atmosphere is rapidly rising; it is already considerably greater than at any time in the last 800,000 years.

2.4 Capital: K

The DICE model assumes a single global economic "capital". The evolution of the capital is described by the following equation:

$$K(i+1) = (1-\delta_K)^{\Delta} K(i) + \Delta Q(i) s(i) \tag{2.12}$$

Where Q is the net economic output.
The model above denotes two components of the capital. The first is related to the depreciation of the capital (we talk about "depreciation" because $delta_K$ ϵ [0, 1],

which means that $1 - \delta_K$ is less than 1), while the second is about its replenishment through investments. The control input s is the savings rate and it tells us which fraction of the net economic output is reinvested in the economy, thus replenishing the total world capital.

2.4.1 Net economic output, damages and abatement costs

The net economic output Q is obtained from the gross economic output Y by removing:

- the damages to the environment caused by the rising atmospheric temperature;
- the costs that we have to sustain for our emissions abatement efforts.

More precisely, the equation is the following:

$$Q(i) = D(i)(1 - A_C(i))Y(i) \tag{2.13}$$

Where D and A_C are, respectively, the damages function and the abatement costs (both described in details below). In turn, the gross economic output Y is described as follows:

$$Y(i) = A_{TFP}(i)K(i)^{\gamma}\left(\frac{L(i)}{1000}\right)^{1-\gamma} \tag{2.14}$$

Or, as a function of the states:

$$Y(i) = x_{10}(i)x_7(i)^{\gamma}\left(\frac{x_9(i)}{1000}\right)^{1-\gamma} \tag{2.15}$$

The damages function used in the DICE model depends on the atmospheric temperature T_{AT} as follows:

$$D(i) = 1 - \frac{a_2 T_{AT}(i)^2}{1 + a_2 T_{AT}(i)^2} \tag{2.16}$$

Which can be expressed as the following function of the states:

$$D(i) = 1 - \frac{a_2 x_2(i)^2}{1 + a_2 x_2(i)^2} \tag{2.17}$$

The way damages are modelled is still cause for debate in the environmental science community. According to [1]

> This stems from the inherent difficulty of modeling in an application where experimentation is simply not possible and the fact that rising temperatures will have different local effects.

This particular choice of damages function is calibrated in a way to yield a loss of 2% at 3°C. In [1], there is a remark about this calibration:

> It is worthwhile noting that it has been vigorously argued that the above calibration of 2% loss at 3°C is unreasonably low if it is to be consistent with currently available climate science.

Which means that other choices of damage functions may be better than this one. The abatement costs in [1] only depend on the control input μ:

$$A_C(i) = \theta_1(i)\mu(i)^{\theta_2} \tag{2.18}$$

So, the costs increase as μ increases (an increase of μ translates to a higher effort in reducing emissions, so it makes sense that if μ increases, the abatement costs increase with it). However, this model lacks inertia because not only does the abatement effort μ have a cost but so does the speed at which we perform said abatement (i.e., $\dot{\mu}$). In [4] the following update to the model is proposed and this is the equation used in this thesis:

$$A_C(i) = \theta_1(i)\left((1-p)\mu(i)^{\theta_2} + p\frac{\hat{t}}{\theta_2+1}\dot{\mu}(i)\right) \tag{2.19}$$

Besides the original abatement costs, there's a new term related to $\dot{\mu}$ which adds inertia to the costs. This way, the model is more realistic and it also puts a cost to the speed at which μ changes, which can be translated as a soft constraint on $\dot{\mu}$.
We can also rewrite this one as a function of the states:

$$A_C(i) = \theta_1(i)\left((1-p)x_{15}(i)^{\theta_2} + p\frac{\hat{t}}{\theta_2+1}\dot{x}_{15}(i)\right) \tag{2.20}$$

In both models, θ_1 represents the cost of mitigation efforts and is defined as follows:

$$\theta_1(i) = \frac{p_b}{1000\theta_2}(1-\delta_{PB})^{i-1}\sigma(i) \tag{2.21}$$

Here, p_b represents the price of a backstop technology (i.e. a technology that can replace exhaustible resources with unlimited resources, such as solar power with respect to carbon) that can remove carbon dioxide from the atmosphere. It is worth noting that such a model embeds the assumption that this backstop price decreases over time (since $\delta_{PB} \epsilon [0,1]$) and increases with σ (i.e. the carbon intensity of economic activity). This means that backstop technologies are assumed to get cheaper over time but they are also assumed to be more expensive the higher the carbon intensity of economic activity is. This translates in the assumption that new technologies get cheaper as the technological progress advances but that it also costs more to replace old carbon technology when they are heavily involved in our economic activities.
Once again, this equation can also be rewritten as a function of the states:

$$\theta_1(i) = \frac{p_b}{1000\theta_2}(1-\delta_{PB})^{x_1(i)-1}x_8(i) \tag{2.22}$$

2.5 Emission intensity: σ

The evolution of the carbon intensity due to economic activity is described by the following equation:

$$\sigma(i+1) = \sigma(i)e^{-g_\sigma(1-\delta_\sigma)^\Delta(i-1)\Delta} \tag{2.23}$$

Which can be rewritten as the following function of the states:

$$x_8(i+1) = x_8(i)e^{-g_\sigma(1-\delta_\sigma)^\Delta(x_1(i)-1)\Delta} \tag{2.24}$$

This model represents a decreasing logistic curve, meaning that it is monotonically decreasing with a decreasing decrease rate, a shown in the following figure:

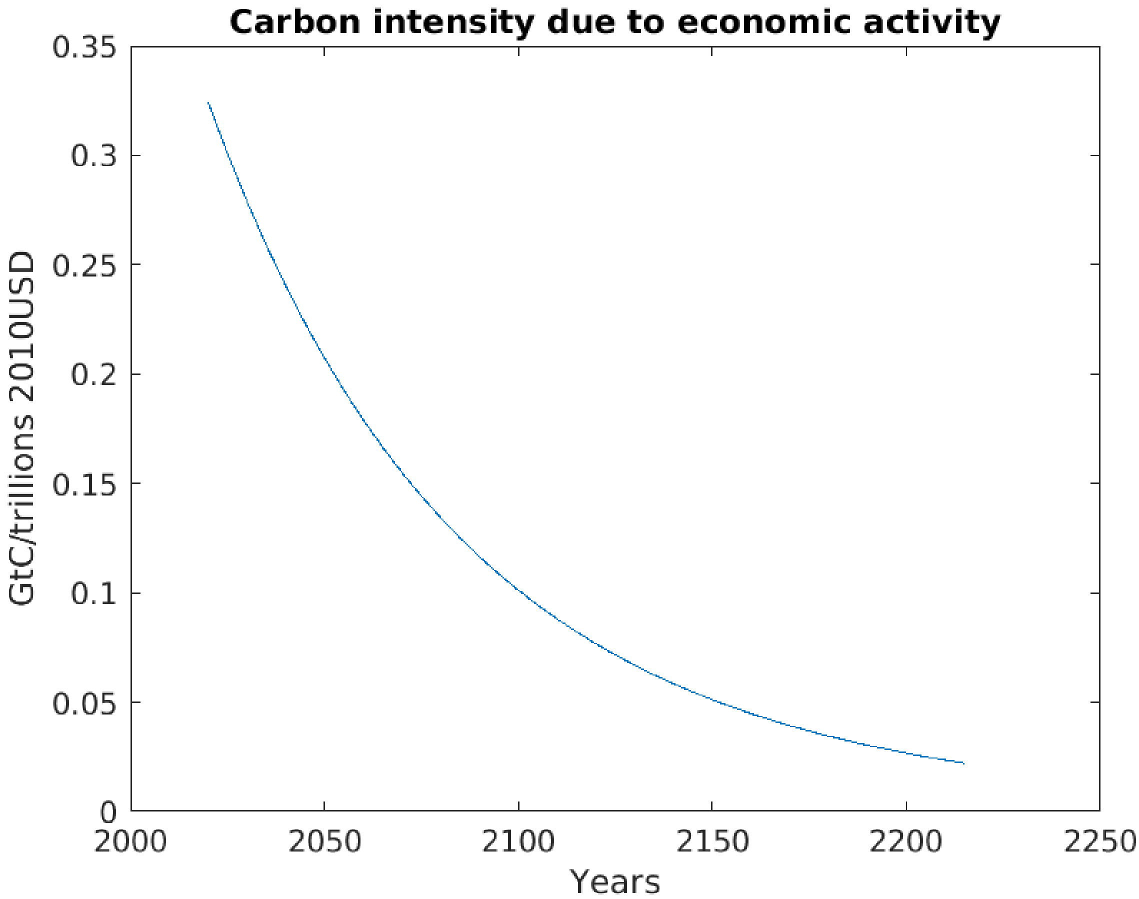

Figure 2.1: Evolution of the carbon intensity of economic activity

Regarding the initial value of the emission intensity, the following statement can be found in [1]:

> An estimate for the initial emissions intensity of economic activity $\sigma(1) = \sigma_0$ can be calculated as the ratio of global industrial emissions to global economic output. The estimate of σ_0 can be further refined by estimating the mitigation rate in the base year. In other words, with base year emissions e_0, base year economic output q_0, and an estimated base year mitigation rate μ_0, we can estimate $\sigma_0 = \frac{e_0}{q_0(1-\mu_0)} = 0.3503$ GtC/trillions 2010USD.

2.6 Population: L

The world population evolves according to the following model:

$$L(i+1) = L(i)\left(\frac{L_a}{L(i)}\right)^{l_g} \tag{2.25}$$

The model dictates an initial exponential growth, starting from $L_0 = 7403$ millions people (world population in 2015) which then saturates to the asymptotic population of $L_a = 11500$ millions people, as shown in the following figure:

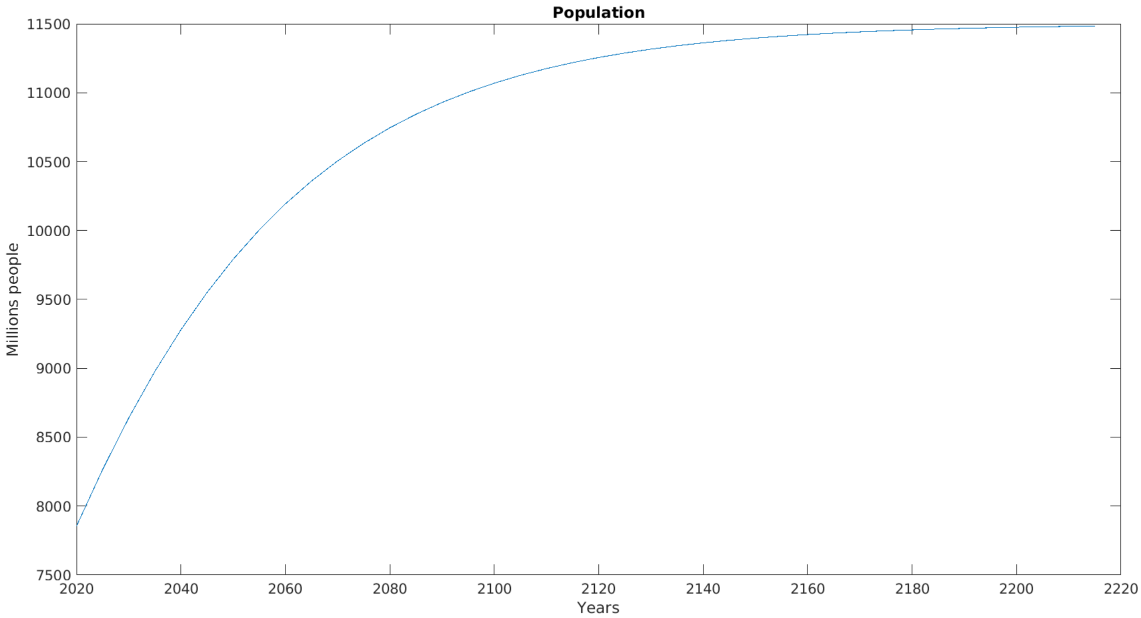

Figure 2.2: Evolution of the population

2.7 Total factor productivity: A_{TFP}

The total factor productivity is a measure of technological progress and it evolves according to the following model:

$$A_{TFP}(i+1) = \frac{A_{TFP}(i)}{1 - g_a e^{-\delta_A \Delta (i-1)}} \tag{2.26}$$

This equation describes an increasing logistic curve (i.e. a curve which is monotonically increasing with a decreasing growth rate), as shown in the following figure:

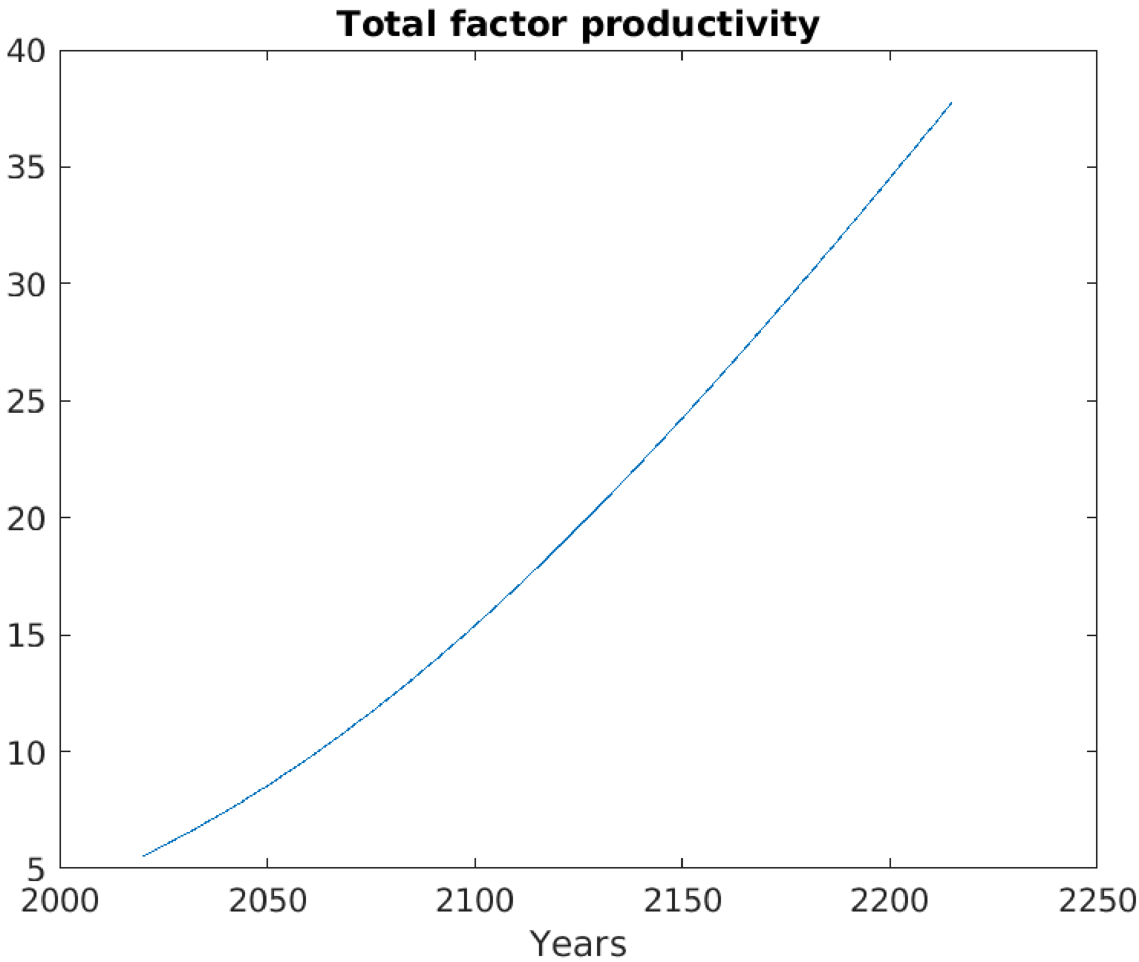

Figure 2.3: Evolution of the total factor productivity

2.8 Emissions due to land use changes: E_{LAND}

The emissions due to land use changes evolve over time according to the following model:

$$E_{LAND}(i+1) = E_{L0}(1-\delta_{EL})^{i} \tag{2.27}$$

Which can be expressed as a function of the states as follows:

$$x_{11}(i+1) = E_{L0}(1-\delta_{EL})^{x_1(i)} \tag{2.28}$$

2.9 External forcings: F_{EX}

The external forcings are the greenhouse effects caused by gases other than CO_2, such as methane, nitrous oxide, and chloroflourocarbons (the specific effects of each one of these gases is outside of this thesis' scope). As shown both in 2.7 and in 2.8, these forcings contribute to the total radiative forcing. These external forcings evolve according to the following model:

$$F_{EX}(i+1) = f_0 + \min\left(f_1 - f_0, \frac{f1 - f0}{t_{force}} i\right) \tag{2.29}$$

2.10 Emissions: E

This state represents the CO_2 emissions due to economic activity and land use. These emissions evolve according to the following model:

$$E(i+1) = \Delta(\sigma(i+1)(1-\mu(i+1))Y(i+1) + E_{LAND}(i+1)) \tag{2.30}$$

We can see that these emissions depend on two contributions. The first contribution is $\sigma(1-\mu)Y$ and it represents the emissions due to economic activity. In fact, this term grows when either σ or Y grows, which makes sense since they represent, respectively, the carbon intensity of economic activity and the gross economic product. Moreover, the term decreases as μ increases, which reflects the fact that acting on our control input μ (i.e. the abatement rate) has the direct effect of reducing emissions. The second contribution is related to the emissions due to land use changes E_{LAND} (another previously mentioned state).

2.11 Consumption: C

The consumption represents the portion of the net economic output Q that does not get reinvested in the economy. Since the savings rate s is the one dictating the portion of the net economic output Q that gets reinvested in the economy, we have that the consumption evolves according to the following equation:

$$C(i+1) = \Delta(1 - s(i+1))Q(i+1) \tag{2.31}$$

By defining the investments I as $I(i) = Q(i)s(i)$, we can define the consumption C as $C(i) = Q(i) - I(i)$, which precisely means that the consumption is whatever remains of the net economic output after we reinvest part of it.

2.12 Abatement rate: μ

The abatement rate μ is one of the two control inputs and it represents the rate at which mitigation of industrial carbon dioxide emissions occur. In other words, the higher μ, the faster the emissions get reduced.
Since μ is a decision variable of the optimization problem, it does not have a dynamic equation that describes its evolution.

2.13 Savings rate: s

The savings rate s is the second control input and it represents the fraction of the net economic output that gets reinvested in the economy. To better understand

what this means, we can imagine a scenario: let us assume we are in a very critical situation regarding the environment (for instance, we have a quite high atmospheric temperature) and we only have s as a control input (meaning that we cannot act on μ to reduce emissions). A possible solution could be to reduce s in order to reduce emissions by directly reducing the economic activity. In fact, reducing s translates to reducing the investments, which means reducing the economic activity and, therefore, reducing the emissions caused by it.
Since s is a decision variable for the optimization problem, there is not a dynamic equation that describes its evolution.

2.14 Negative social welfare: J

The negative social welfare is the objective function that the optimization problem aims to minimize, while all the other state dynamics act as constraints to the problem. It is basically just the social welfare changed by sign.
In [1] the following model for computing the social welfare is used:

$$J(i+1) = J(i) - \frac{U(i)}{(1+\rho)^{\Delta(i-1)}} = J(i) - L(i)\frac{(\frac{1000}{L(i)}C(i))^{1-\alpha} - 1}{(1-\alpha)(1+\rho)^{\Delta(i-1)}} \tag{2.32}$$

Where $U(i) = L(i)\frac{(\frac{1000}{L(i)}C(i))^{1-\alpha}-1}{(1-\alpha)}$ represents the utility function. So, the negative social welfare is the discounted sum of the negative utility ($\rho > 0$ is a prescribed discount rate).
The model for the social welfare used in this thesis is slightly different than the one mentioned above. The goal was to try to keep the atmospheric temperature below 2°C (as the UN Paris Agreement dictates) without putting an explicit upper bound on it (as it was instead done in [1]). The way this was done was by adding a sort of soft constraint in the objective function such that having $T_{AT} > 2$°C is considered unfavorable by the solver. The model used is the following:

$$J(i+1) = J(i) - L(i)\frac{(\frac{1000}{L(i)}C(i))^{1-\alpha} - 1}{(1-\alpha)(1+\rho)^{\Delta(i-1)}} + s_w(T_{AT}(i) - T_{AT,max}) \tag{2.33}$$

The equation is mostly the same with the only addition of the term $s_w(T_{AT} - T_{AT,max})$, which gets summed to the original social welfare. This additional term increases the objective function (that we want to minimize) whenever $T_{AT} > 2$°C and decreases it whenever $T_{AT} < 2$°C. This way, we avoid the feasibility problems that may arise from inserting an explicit upper bound on T_{AT} but we still embed in the problem the will to keep the temperature as low as possible.

2.15 Abatement speed: μ_{rate}

When trying to reduce emissions, it may happen that the optimization problems finds it optimal to immediately increase μ to values very close to 1 and thus reaching 1 (i.e. a total elimination of emissions) in a very short period of time. This is obviously not feasible in reality and so we need to tell the solver that it cannot change μ at whatever speed. This can be done in two ways and one of these is by adding this state, which symbolizes the speed at which μ changes, and by putting an upper bound on it. The definition comes directly from that of an incremental ratio:

$$\mu_{rate}(i+1) = |\mu(i+1) - \mu(i)| \tag{2.34}$$

This is what was also done in [1] to avoid having unrealistic trajectories for μ.

2.16 Abatement growth: μ_{growth}

The other way to prevent the control input μ to grow arbitrarily fast is to constrain its growth. This is analogous to constraining its rate but leads to slightly different results, so having also this state provides the possibility of choosing how to constrain μ. The definition is some sort of normalized incremental ratio:

$$\mu_{growth}(i+1) = \frac{\mu(i+1) - \mu(i)}{\mu(i)} \tag{2.35}$$

Exactly as μ_{rate}, also μ_{growth} was used in [1] to avoid unrealistic trajectories for μ.

2.17 Equilibrium climate sensitivity: ECS

2.17.1 Definition

A satisfying definition of the equilibrium climate sensitivity can be obtained from [5]:

> [The ECS is] the eventual increase in global annual average surface temperature in response to a doubling of atmospheric CO_2 concentration.

To explain it in an easier way, we can consider the following chain of implications:
High ECS $\Rightarrow$ The temperature increases more when we double the atmospheric CO_2 concentration $\Rightarrow$ Bad!
Another definition can be found in [1]:

> The Equilibrium Climate Sensitivity (ECS) is defined as the steady-state atmospheric temperature arising from a doubling of atmospheric carbon.

This definition includes the term "steady-state" which is the reason why it is called **equilibrium** climate sensitivity. This is due to the fact that there is not just one type of climate sensitivity but, in fact, there are more. In [6], three kinds of climate sensitivity are mentioned:

- Equilibrium climate sensitivity (ECS): defined above. It refers to the increase of temperature due to the doubling of CO_2 concentration in the atmosphere after the transient processes have reached equilibrium. So, the ECS is the warming that occurs after the Earth's climate had the time to adjust to changes in the atmospheric CO_2 concentration. This means that the ECS is more useful in the long term (since it neglects the transient processes that may be

important to consider in the short term). This climate sensitivity is the one employed in this thesis.
- Transient climate response (TCR): it refers to the increase of temperature due to the doubling of CO_2 concentration in the atmosphere at the time when CO_2 doubles. So, this measure considers the transient effects that come from doubling the CO_2 concentration and is, therefore, more useful in the short term.
- Earth system sensitivity: this measure includes very long-term Earth system feedbacks, such as changes in ice sheets or changes in the distribution of vegetative cover.

2.17.2 Estimation

The estimation of the ECS is still an open problem and the best we have so far is a range of values which still remains quite large. For many years, the values for the ECS that were considered were between 1.5°C and 4.5°C [6]. However, in [5] it is claimed that this range can be narrowed down to [2°C, 4°C] and this is the range that will be considered in this thesis. In the most important literature used as a reference for this work ([1], [2] and [4]), the value for the ECS that was used was 3.1°C (which is around the average value in the interval mentioned above). Since the goal of this thesis is to employ robust control strategies, we will consider the entire range with a worst-case scenario approach that will be described in details in the Optimization chapter (3).

2.17.3 State equation

Due to how the problem is structured, the ECS has no state equation. The worst-case scenario ECS is obtained by solving a MinMax problem in which we can distinguish two situations for the ECS:
- During the minimization, the ECS is fixed.
- During the maximization, the ECS is a treated as a decision variable which will be increased.

More details on this can be found in the Optimization chapter (3).

2.18 Equilibrium climate sensitivity rate: ECS_{rate}

During the maximization phases of the MinMax problem, the ECS is the decision variable and the "adversary player" will try to raise it so as to put us in the worst case scenario (more details on this in the Optimization chapter (3.3)). For feasibility reasons, it helps to limit the rate at which the ECS is allowed to vary. Therefore, we added this state as to put an upper bound on it, so that the "adversary player" cannot change the ECS at an arbitrary speed. The definition of this state is simply the incremental ratio of the ECS:

$$ECS_{rate}(i+1) = |ECS(i+1) - ECS(i)| \tag{2.36}$$

3

Optimization

The optimization scenario of this thesis work is a robust version of the one in [1]. The robustness is achieved by having two adversary agents:

- The direct problem (the "good guy"), whose goal is that of minimizing the negative social welfare J by acting on the control inputs μ and s.
- The adversary problem (the "bad guy"), whose goal is that of maximizing the negative social welfare J by acting on the equilibrium climate sensitivity ECS (i.e. by raising it).

The MinMax approach is achieved through these two opposing optimization problems, which are solved in an iterative manner. First, the direct problem computes the optimal control inputs μ and s (for a specific ECS) to minimize the negative social welfare. Then, the adversary problem computes the optimal ECS to maximize the negative social welfare and so on. This way, the direct problem will always find itself in the worst-case scenario and will have to try to compute the best control inputs that counteract the actions of the adversary agent. This is where the robustness comes from: we are not only considering a constant value for the ECS (as was done in the literature used for this thesis) but we change it at every iteration, making things always worse to see what kind of control actions we would need to take in a situation that is increasingly getting worse.
Before this iterative process, just like in [1], some sort of an initialization problem is solved, whose goal is that of finding feasible starting conditions which are then used as a starting point for the actual MinMax problem (more on this in section 3.2).
What follows is a detailed description of the three problems (min, max and initialization) followed by a section that describes how the full MinMax approach was implemented.

3.1 Minimization: the "direct problem"

As previously stated in the introduction to this section, the direct problem (i.e. the "good guy") is the one which, at every iteration, computes the best control inputs μ and s given the specific situation it finds itself in. For the direct problem, the ECS is constant and decided during the previous iteration.

This problem is defined as follows ([1]):

$$
\begin{aligned}
\min_{\mu,s} \quad & x_{17}(N+1) \\
\text{subject to} \quad & \\
& x(j+1) = f(x(j), u(j)), \quad j = 1, ...N \\
& x(1) = x^*(2|i-1) \\
& g(x,i) \leq u_b \\
& h(x,i) \geq l_b \\
& \mu \ \epsilon \ [0,1] \\
& s \ \epsilon \ [0,1]
\end{aligned}
\tag{3.1}
$$

The objective function is, as previously mentioned, the negative social welfare, which is the 17^{th} state, computed at time $N+1$, where N is the prediction horizon length (i.e. "how far in the future we are looking").
In this problem, $x(i) \ \epsilon \ \mathbb{R}^{21}$ is the state vector at time i, while $u(i) = [\mu(i+1), s(i+1)]$ is the shifted input vector.

3.1.1 Equality constraints for the state dynamics

The first set of equality constraints are the state dynamics described in the Model chapter (2). So, in other words, these are the equations that describe how the states evolve. This means that there is an equality constraint for every state. These equality constraints are, thus, the followings:

- Time index (2.3): $f_1(x(j), u(j)) = x_1 + 1$
- Temperature: the expression would be too long to write it down entirely but it can be obtained by plugging 2.6 and 2.9 into 2.4.
- Carbon mass (2.10): $\begin{bmatrix} f_4(x(j), u(j)) \\ f_5(x(j), u(j)) \\ f_6(x(j), u(j)) \end{bmatrix} = \phi_M \begin{bmatrix} x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} \xi_2 x_{13} \\ 0 \\ 0 \end{bmatrix}$
- Capital: the expression would be too long to write down but it can be obtained by plugging 2.13, 2.15, 2.17, 2.20 and 2.22 into 2.12.
- Emission intensity (2.23): $f_8(x(j), u(j)) = x_8 e^{-g_\sigma (1-\delta_\sigma)^\Delta (x_1 - 1)\Delta}$
- Population (2.25): $f_9(x(j), u(j)) = x_9 \left(\frac{L_a}{x_9}\right)^{l_g}$
- Total factor productivity (2.26): $f_{10}(x(j), u(j)) = \frac{x_{10}}{1 - g_a e^{-\delta_A \Delta (x_1 - 1)}}$
- Land emissions (2.27): $f_{11}(x(j), u(j)) = E_{L0}(1 - \delta_{EL})^{x_1}$
- External forcings (2.29): $f_{12}(x(j), u(j)) = f_0 + \min\left(f_1 - f_0, \frac{f1 - f0}{t_{force}} x_1\right)$
- Emissions: the expression would be too long to write down but it can be obtained by plugging 2.24, 2.15 and 2.28 into 2.30
- Consumption (2.31): $f_{14}(x(j), u(j)) = \Delta(1 - x_{16}(i+1))Q(i+1)$
- Abatement rate: $f_{15}(x(j), u(j)) = u_1(i)$
- Savings rate: $f_{16}(x(j), u(j)) = u_2(i)$
- Negative social welfare (2.33):
  $f_{17}(x(j), u(j)) = x_{17}(i) - x_9(i) \frac{(\frac{1000}{x_9(i)} x_{14}(i))^{1-\alpha} - 1}{(1-\alpha)(1+\rho)^{\Delta(i-1)}} + s_w(x_2(i) - T_{AT,max})$
- Rate of μ (2.34): $f_{18}(x(j), u(j)) = |x_{15}(i+1) - x_{15}(i)|$

- Growth of μ (2.35): $f_{19}(x(j), u(j)) = \frac{x_{15}(i+1)-x_{15}(i)}{x_{15}(i)}$
- Equilibrium climate sensitivity: $f_{20}(x(j), u(j)) = x_{20}(i)$
- Rate of ECS (2.36): $f_{21} = |x_{20}(i+1) - x_{20}(i)|$

The equality constraint for the ECS is worth noting because what it tells us is that, during the minimization phase, the ECS is considered to be constant. This is not the case during the entire algorithm, as the ECS gets chosen during the maximization phase (and is, therefore, not constant) but then, once the ECS at a certain step is chosen, it is kept constant during the minimization.
As stated in the problem definition, these equality constraints are applied for j that goes from 1 to N, where N is the prediction horizon length. Considering this and considering the fact that the objective function is evaluated at the time instant $N+1$, what we are doing is basically simulating the system from time 1 to time N with the goal of having an optimal objective function at the time instant right after the end of the prediction horizon. In other words, during the prediction horizon the problem computes optimal control inputs with the goal of optimizing the objective function only at time $N+1$ but, to do that, it has of course to simulate the system in the time interval from time instant 1 to time instant N.

3.1.2 Equality constraints for the initial conditions

The second equality constraint serves the purpose of setting the initial conditions for the problem (i.e. setting the value for $x(1)$). The quantity $x^*(2|i-1)$ can be seen as the first step of the solution computed at the previous time step. The sentence "first step of the solution" refers to the fact that the solution at each time step is a vector and we are taking only the first value of this vector. Since every value of the vector refers to a specific time instant, we are taking only the value related to the first time instant, i.e. "the first step". To give an intuitive idea of what this means, we can consider the following scenario:

1. we solve the problem at time $i-1$, obtaining the solution $x^*(i-1)$;
2. since we apply a receding horizon approach (described more in details below in section 3.4), we only take the first step of this solution, i.e. $x^*(2|i-1)$ (why the index 2 if it is the first step? Because $x^*(1|i-1)$ represents the initial condition for the problem at time $i-1$ and so it is not the first step of the computed solution, but rather the initial value which the optimization in the previous step started from);
3. we assign this value as an initial condition to the problem to be solved at time i.

This whole procedure is described more in details in the section related to the receding horizon approach (3.4).

3.1.3 Inequality constraints for the bounds

The inequality constraints represent bounds for the state variables. Here, we will list all the inequality constraints implemented for the minimization. However, it must be noted that not all of these constraints are always active (as opposed to the equality constraints that are always all active). This means that, for a certain execution of the algorithm, the actual set of the active inequality constraints is a subset of what follows. In the Results chapter (4), it will be said, for every result, which inequality constraints were active. Regardless, in the list that follows it will be mentioned if some constraints can be activated or deactivated.

- Time index: $x_1 \geq 0$ (i.e. $h_1(x,i) = x_1(i)$ and $l_{b,1} = 0$)
- Atmospheric temperature: $x_2 \geq 0$ (i.e. $h_2(x,i) = x_2(i)$ and $l_{b,2} = 0$: we don't enforce a specific upper bound on T_{AT}, instead, we embed a soft constraint on it in the objective function, as described in the Model section (2.14))
- Lower ocean temperature: $x_3 \geq 0$ (i.e. $h_3(x,i) = x_3(i)$ and $l_{b,3} = 0$)
- Mass of carbon in the atmosphere: $x_4 \geq 0$ (i.e. $h_4(x,i) = x_4(i)$ and $l_{b,4} = 0$)
- Mass of carbon in the upper ocean: $x_5 \geq 0$ (i.e. $h_5(x,i) = x_5(i)$ and $l_{b,5} = 0$)
- Mass of carbon in the lower ocean: $x_6 \geq 0$ (i.e. $h_6(x,i) = x_6(i)$ and $l_{b,6} = 0$)
- Capital: $x_7 \geq 0$ (i.e. $h_7(x,i) = x_7(i)$ and $l_{b,7} = 0$)
- Consumption: $x_{14} \geq 0$ (i.e. $h_8(x,i) = x_{14}(i)$ and $l_{b,8} = 0$)
- Abatement rate: $x_{15} \geq 0$ (i.e. $h_9(x,i) = x_{15}(i)$ and $l_{b,9} = 0$) and $x_{15} \leq 1$ (i.e. $g_1(x,i) = x_{15}(i)$ and $u_{b,1} = 1$)
- Savings rate: $x_{16} \geq 0$ (i.e. $h_{10}(x,i) = x_{16}(i)$ and $l_{b,10} = 0$) and $x_{16} \leq 1$ (i.e. $g_2(x,i) = x_{16}(i)$ and $u_{b,2} = 1$)
- μ_{rate}: $x_{18} \geq 0$ (i.e. $h_{11}(x,i) = x_{18}(i)$ and $l_{b,11} = 0$) and, if we activate the upper bound on μ_{rate}, $x_{18} \leq \Delta_\mu$ (i.e. $g_3(x,i) = x_{18}(i)$ and $u_{b,3} = \Delta_\mu$)
- μ_{growth}: if we don't constrain μ_{growth}, $x_{19} \geq 0$ (i.e. $h_{12}(x,i) = x_{19}(i)$ and $l_{b,12} = 0$), otherwise, $x_{19} \geq -\Gamma_\mu$ (i.e. $h_{12}(x,i) = x_{19}(i)$ and $l_{b,12} = -\Gamma_\mu$) and $x_{19} \leq \Gamma_\mu$ (i.e. $g_4(x,i) = x_19(i)$ and $u_{b,4} = \Gamma_\mu$)

The constants indicated here are available in the Appendix (6.8).

3.1.4 Issues with the initial conditions

The initial conditions for each state can be obtained by evaluating each state equation for $i = 0$. This leads to no problem for most of the states. However, when we do this for the 13^{th} and for the 14^{th} states, i.e. the emissions and the consumption respectively (so, what we are doing is evaluating 2.31 and 2.30 for $i = 0$), we get the following for the emissions:

$$E(1) = \Delta \left(x_8(1)(1 - x_{15}(1)) x_{10}(1) x_8(1)^\gamma \left(\frac{x_9(1)}{1000} \right)^{1-\gamma} + E_{L0} \right) \tag{3.2}$$

And the following for the consumption:

$$C(1) = \Delta \left(\frac{1 - \theta_1(1) x_{15}(1)^{\theta_2}}{1 + a_2 x_2(1)^{a_1}} \right) x_{10}(1) x_8(1)^\gamma \left(\frac{x_9(1)}{1000} \right)^{1-\gamma} (1 - x_{16}(1)) \tag{3.3}$$

The details of the two expressions are not relevant at the moment, what is important is that $E(1)$ depends on $x_{15}(1) = \mu(1)$ and that $C(1)$ depends on both $x_{15}(1) = \mu(1)$ and $x_{16}(1) = s(1)$, which are the two control inputs. This means that the optimization has the constraint of initialising E and C using $\mu(1)$ and $s(1)$ but these last two values are decision variables and so they will be obtained after the problem is solved. This issue was brought up in [1] and the solution suggested there is that of solving first an initial problem that serves the purpose of properly initialising all the states variables so that then we are able to proceed with the resolution of the direct and adversary problems. This initialization problem is described more in details in the section that follows.

3.2 Initialization problem

As mentioned in the previous section, the goal of this problem is to compute feasible initial conditions that will serve as a starting point for the MinMax problem previously described.
This problem is formulated as follows:

$$\begin{aligned} &\min_{\mu,s,\nu} && x_{17}(N+1) \\ &\text{subject to} \\ & & x(j+1) = f(x(j),u(j)), & \qquad j = 1,...N \\ & & x(1) = \nu & \\ & & \nu_k = x_k(1) & \qquad k\epsilon\{1,...,21\} \setminus \{15,16\} \\ & & \nu_k \epsilon [0,1] & \qquad k = 15,16 \\ & & \mu \ \epsilon \ [0,1] & \\ & & s \ \epsilon \ [0,1] & \end{aligned} \tag{3.4}$$

3.2.1 New decision variable: ν

The key differences of this problem, with respect to the minimization problem described above, are the additional decision variable $\nu \ \epsilon \ \mathbb{R}^{21}$ and the constraints related to it.
From the constraint $x(1) = \nu$, we can see that this ν is used to initialise the state vector (which was the issue mentioned in the previous section).
The constraint $\nu_k = x_k(1)$, $k \ \epsilon \ \{1,...,21\} \setminus \{15,16\}$ means that all the values of the vector ν, except for ν_{15} and ν_{16}, can be obtained by evaluating the state equations for $i = 0$. Why are ν_{15} and ν_{16} excluded? Because they refer to the 15^{th} and 16^{th} states respectively, i.e. the two control inputs μ and s, which are decision variables of the problem.
The way ν_{15} and ν_{16} are dealt with is described in the constraint $\nu_k \ \epsilon \ [0,1]$, $k = 15,16$, which tells us that we are not setting a specific value for ν_{15} and ν_{16} but we are actually just bounding them in a set so that the problem computes feasible values for them.

3.2.2 Equality constraints for the state dynamics

The first set of the equality constraints, i.e. the ones related to the states dynamics, are exactly the same as the ones described in the section related to the minimization problem (3.1.1).

3.2.3 Inequality constraints for the bounds

As far as the inequality constraints are concerned, they are, again, the same as the ones for the minimization problem. However, there is an additional constraint which is needed to force the initial condition of the control input μ to be the one that we need. So, what we have is that $x_{15}(1) \epsilon [0, \mu_0]$. The reason this is written as an inequality constraint is that if there is no constraint at all, μ starts immediately from a value which is too high. This happens because our goal is to reduce emissions and so the problem will tend to pick higher values for μ instead of lower ones. So, we never have the problem of having an initial value of μ that is too low but rather the opposite. So, in conclusion, by writing it this way, we always get that $\mu(1) = \mu_0$, hence we achieve forcing an initial value for the control input μ without using a strict equality constraint.

3.3 Maximization: the adversary problem

The so called adversary problem (i.e. the "bad guy") has the previously mentioned goal of putting the direct problem (i.e. the "good guy"), at every iteration, in the worst possible situation by increasing the equilibrium climate sensitivity ECS. For this adversary agent, the control inputs μ and s are constant and decided in the previous iteration.
This problem is defined as follows:

$$\begin{aligned} \max_{ECS} \qquad & x_{17}(N+1) \\ x(j+1) = f(x(j), u(j)),& \quad j = 1, ...N \\ x(1) = x^*(2|i-1)& \\ g_{min}(x, i) \leq u_b& \\ h_{min}(x, i) \geq l_b& \\ ECS \,\epsilon\, [ECS_{min},\, ECS_{max}]& \end{aligned} \tag{3.5}$$

So, the adversary agent aims to maximize the negative social welfare (the opposite of what we want) by acting on the ECS (i.e. by increasing it). We can see that the decision variable for this problem (i.e. the ECS) is bounded between $ECS_{min} = 2°C$ and $ECS_{max} = 4°C$. This is according to what has been previously stated in the section related to the estimation of the ECS (2.17.2).

3.3.1 Equality constraints

The equality constraints are almost exactly the same as the ones for the direct problem because they define the way states are allowed to evolve over time. The only differences are related to the fact that the adversary problem has a different decision variable and considers μ and s to be constant, so:

- The adversary problem does not have the constraint $ECS(i + 1) = ECS(i)$ because the ECS is the decision variable for the problem and is, therefore, not constant.

- The adversary problem has two additional constraints that force the two control inputs μ and s to remain constant: $\mu(i+1) = \mu(i)$ and $s(i+1) = s(i)$.

3.3.2 Inequality constraints

The inequality constraints for the adversary problem are almost the same as the ones for the direct problem, with just a few differences:

- $ECS \in [ECS_{min}, ECS_{max}]$, since now the ECS is varying and bounded.
- Allowing the ECS to vary at an arbitrary rate brought up feasibility problems and so it was necessary to constrain ECS_{rate}. This is, obviously, only necessary in the adversary problem, since in the direct problem the ECS is constant. So, in the adversary problem we have that $ECS_{rate} \in [0, ECS_{rate,max}]$.
- The adversary problem does not need bounds on μ and s since they are considered as constants.

3.4 Receding horizon: implementation of the Min-Max approach

The three problems (initialization, direct and adversary) are non-convex infinite-horizon optimal control problems [1]. This means that they are computationally and analitically difficult to solve. For this reason, in [1] a model-predictive control (or receding-horizon) strategy was employed, based on the conjecture that

> in the case of undiscounted optimal control problems, receding horizon control likely provides an approximate solution to the infinite horizon optimal control problem [1].

This means that we will not reach the optimal solution with such a strategy but we will get close enough to it (the larger the prediction horizon, the closer we get to the infinite-horizon solution) and, at least, we have a computationally feasible algorithm. So, with such problems, the options are either to have a sub-optimal solution or to have no solution at all. Following this, we employed the same strategy for the MinMax problem solved in this thesis.
From a high level perspective, the algorithm through which we implemented the MinMax approach goes as follows:

```
adversary = false;
for k = 1,...,t_f
    if k == 1
        solve initialization problem (with prediction horizon N)
        save the 1st step of the solution
    else
        if adversary
            use the 1st step of the previous solution as a starting point
            solve adversary problem (with prediction horizon N)
            save the 1st step of the solution
            adversary = false
```

```
        else
            use the 1st step of the previous solution as a starting point
            solve direct problem (with prediction horizon N)
            save the 1st step of the solution
            adversary = true
        end
    end
end
```

Here, t_f determines how far in the future does the prediction go. For instance, as shown in the Appendix 6.1, we are using a value of $t_f = 40$. Since the DICE model has a time step of 5 years, this means that our simulation will predict $40 \cdot 5 = 200$ years into the future.

The prediction horizon is given by N. The more we increase N, the closer we get to the optimal solution given by the infinite-horizon problem. However, this comes at a price, as the more we increase N, the more computationally demanding the problem becomes.

The algorithm shows that, at every iteration, we compute a solution of length N but then we only feed the first step of this solution to the next iteration. Such a strategy is known as 1-step receding horizon. This means that we sample the system at a time k and we compute the optimal control path in the horizon $[k, k+N]$. Then, we only apply the first step of this optimal control path and we re-iterate the process (sampling, optimization, application of the first step) for the next time instant. Only applying the first step of the solution is what makes this algorithm computationally feasible.

4

Results

This chapter lists various scenarios considered as relevant results.

4.1 Monte Carlo analysis of the MinMax approach with normally distributed ECS

In order to test the robustness of the MinMax approach, we ran the following Monte Carlo simulation:

1. Solve the MinMax problem once and collect the resulting control inputs (since these control inputs are obtained by solving the MinMax problem, they will assume an always increasing ECS, due to the action of the adversary agent).
2. Run a simulation of the system using the previously obtained control inputs but with a different ECS, sampled randomly from a normal distribution.
3. Repeat step 2 a significant number of times (10000 in our case).

The goal of this Monte Carlo simulation is to test how well the controller performs in scenarios in which the ECS is different than the one that was assumed. This process is summed up by the following diagram:

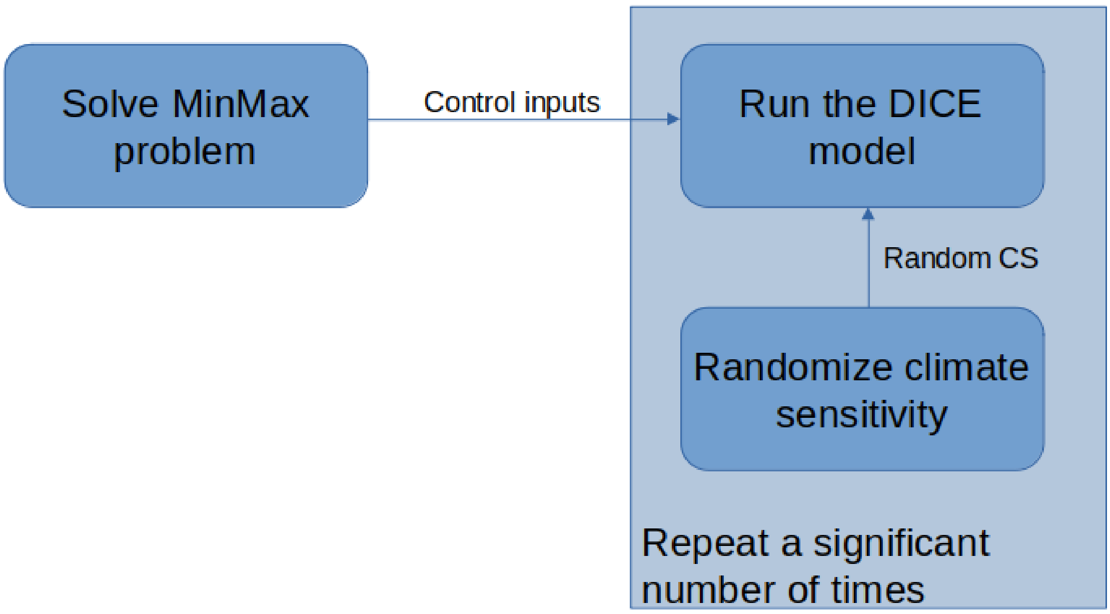

Figure 4.1: Diagram that sums up the procedure used to perform the Monte Carlo simulation in a MinMax approach

As previously mentioned, the samples of the ECS are normally distributed with a mean value of $3°C$ and a standard deviation of $0.3°C$ (i.e. $ECS \sim N(3, 0.3)$). This makes it possible to have around 99% of the times values of the ECS in the range [$2°C$, 4°C], which is the range of all plausible values of the ECS (as previously mentioned in 2.17.2). More precisely, using the properties of the gaussian distribution, we have that the interval $[\mu - 3\sigma,\ \mu + 3\sigma] = [2.1°C,\ 3.9°C]$ has a probability of around 99.7%.
After solving the MinMax problem, we get the following control inputs:

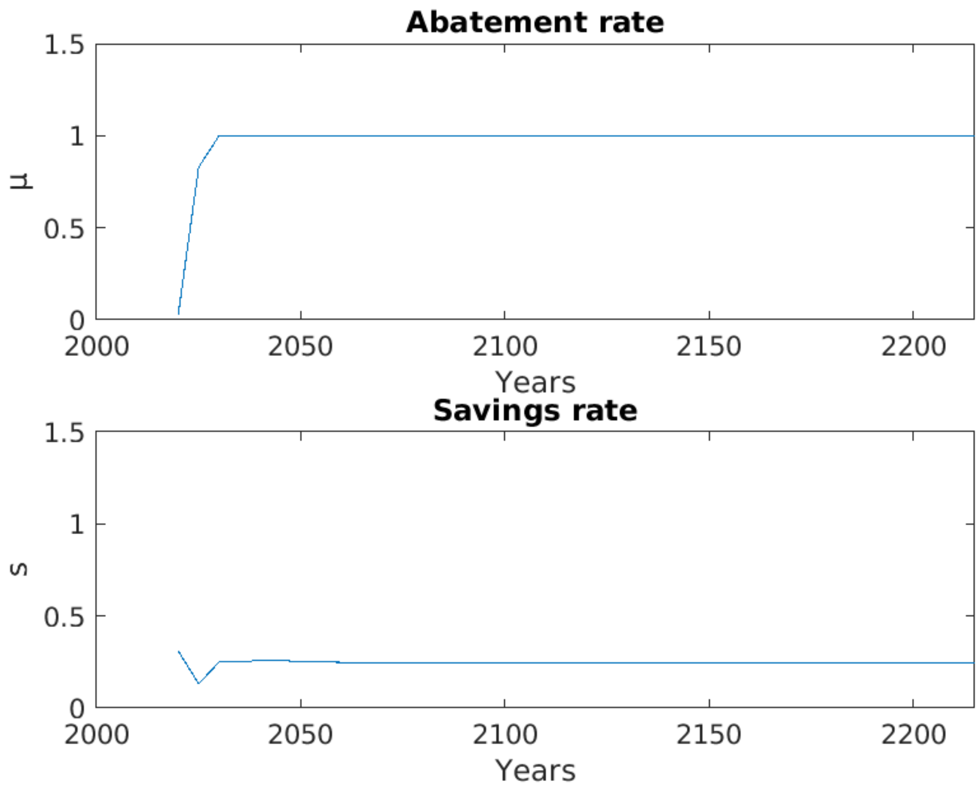

Figure 4.2: Control inputs obtained by solving the MinMax problem, where the controller assumes an always increasing ECS

These control inputs will be used for the entire Monte Carlo simulation (at every iteration, the only thing that changes is the ECS, which is sampled from a normal distribution).
By running the Monte Carlo simulation, we get the following trajectories for the atmospheric temperature:

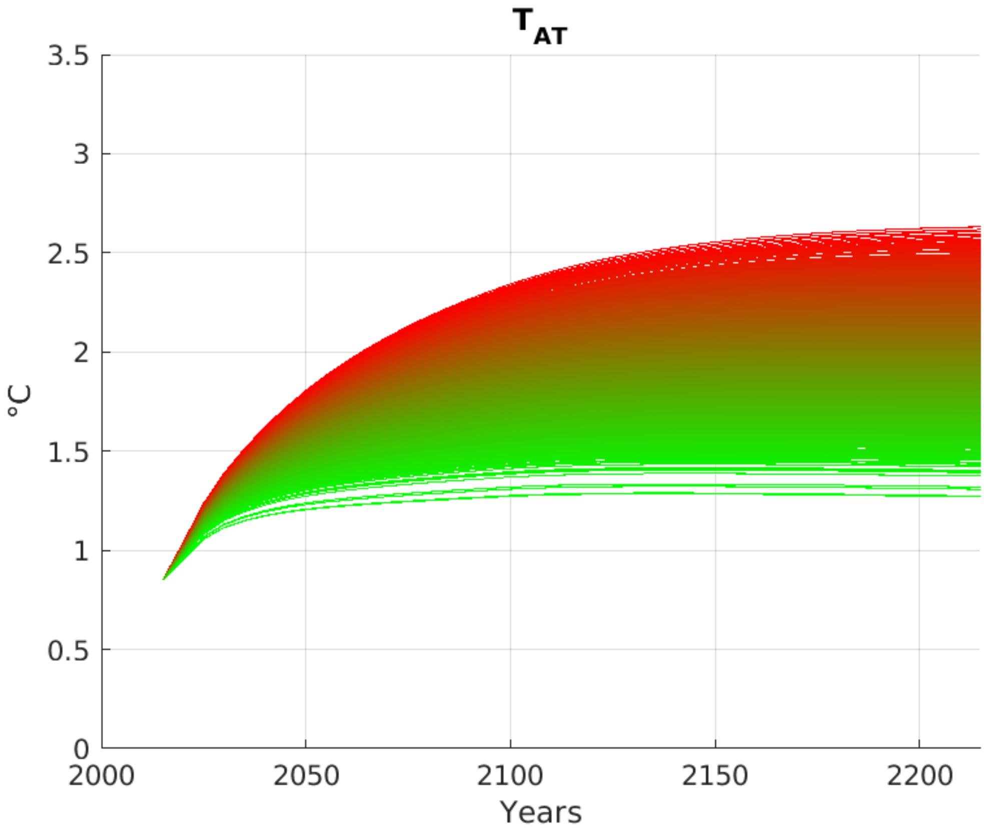

Figure 4.3: Atmospheric temperature trajectories for different values of the ECS ϵ [2°C, 4°C]

Every color in Fig. 4.3 is related to a specific sample of the ECS. A color close to green refers to a lower ECS, while a color close to red refers a higher ECS. Knowing this, the results are not surprising: we get that having a higher ECS leads to higher atmospheric temperatures and this is in line with what was said in 2.17. From these simulations, we get that the final atmospheric temperatures range between around 1.3°C (with the lowest ECS) and 2.63° (with the highest ECS).
For the Monte Carlo simulation to be meaningful, enough iterations need to be executed, such that the ECS is distributed in a way that resembles an ideal normal distribution (because this is the kind of distribution that was used to sample the various values of the ECS). The following plot shows the probability density function of the samples of the ECS against an ideal one computed with Matlab with a mean value of 3°C and a standard deviation of 0.3°C:

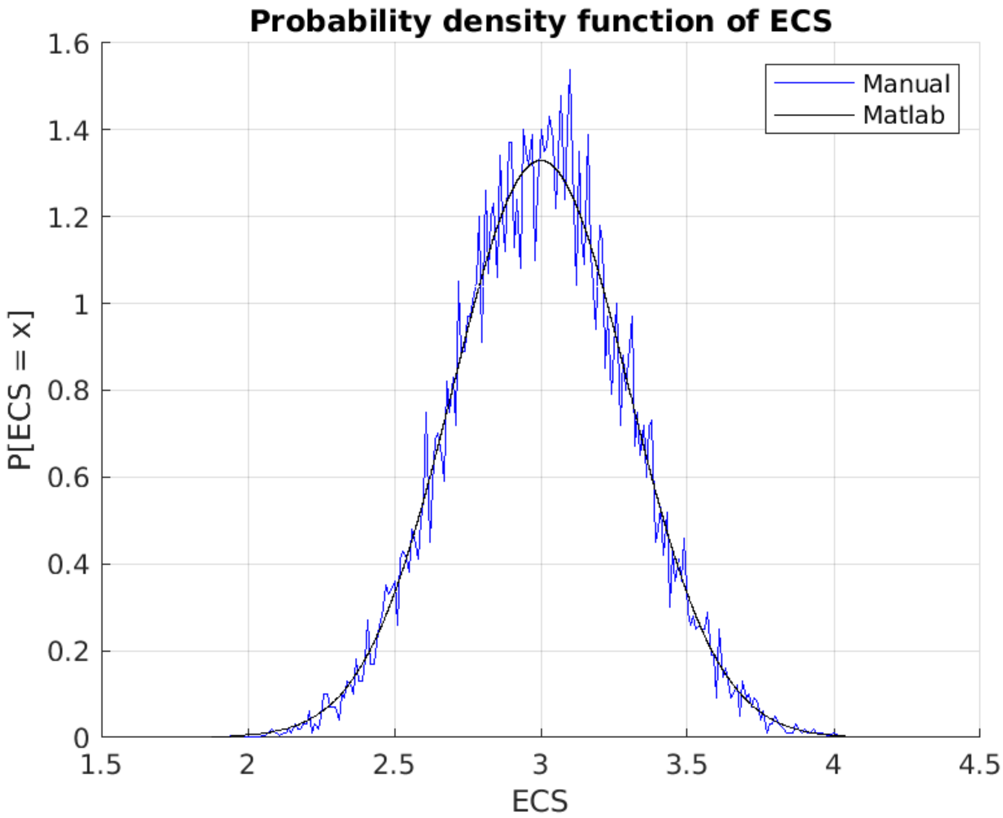

Figure 4.4: Comparison between the probability density function computed manually from the samples of the ECS and the ideal one obtained with Matlab as $N \sim (3, 0.3)$

We can see that the probability density function computed manually does resemble the ideal one obtained with Matlab. Another thing to notice is that, by using such a normal distribution, we do get more than 99% of the times values contained in the interval [2°C, 4°C].
The probability density function shown is Fig. 4.4 was obtained from the cumulative density function shown in the following figure (plotted against an ideal one computed with Matlab):

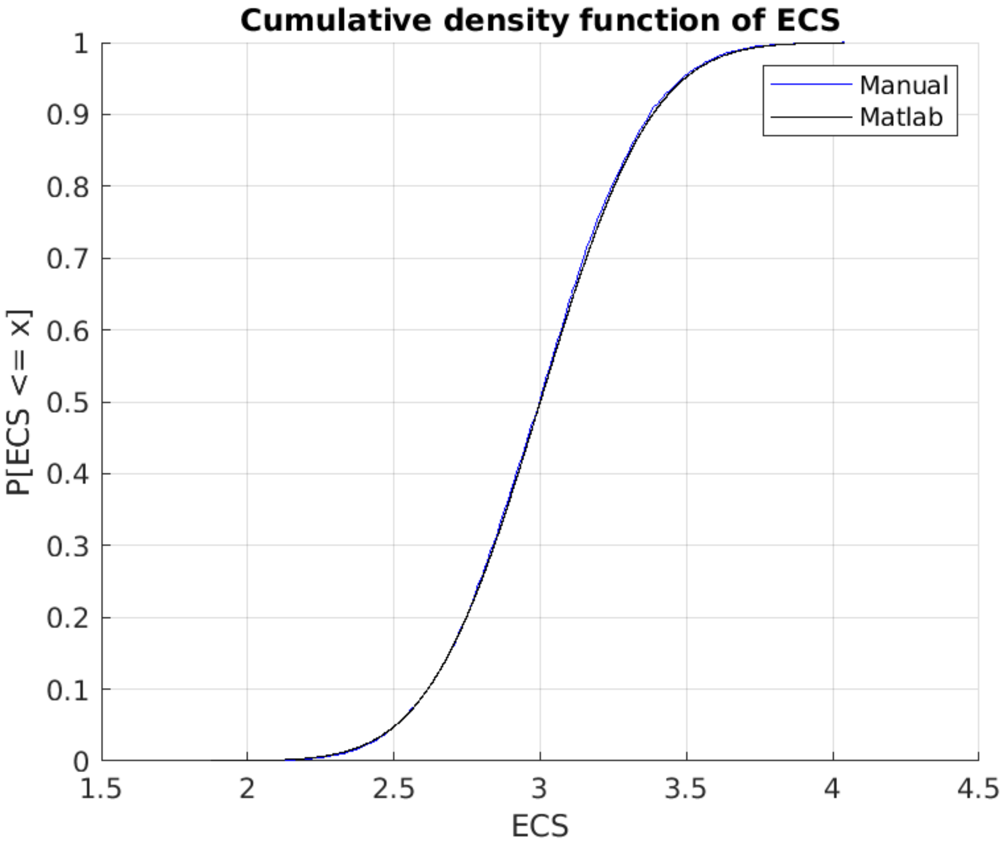

Figure 4.5: Comparison between the cumulative density function computed manually from the samples of the ECS and the ideal one obtained with Matlab as $N \sim (3, 0.3)$

As a reference, the probability density function and the cumulative density function have been computed with the following algorithm:

1. sort the values of the ECS
2. count the number of times every value occurs
3. normalize that count such that it sums up to 1
4. take a cumulative sum of these counts to obtain the cdf
5. take the discrete derivative of the cdf to obtain the pdf

Having built such a stochastic setting, it is interesting to see what is the probability of the atmospheric temperature to rise above $2°C$ (as a reminder, this specific value comes from the UN Paris Agreement, as mentioned in the abstract). We can do that by collecting the final values of the atmospheric temperature shown in Fig. 4.3 and apply the previously described algorithm to obtain a probability density function and a cumulative density function (we actually only need this second one) for them. The following plots show the results of such a procedure:

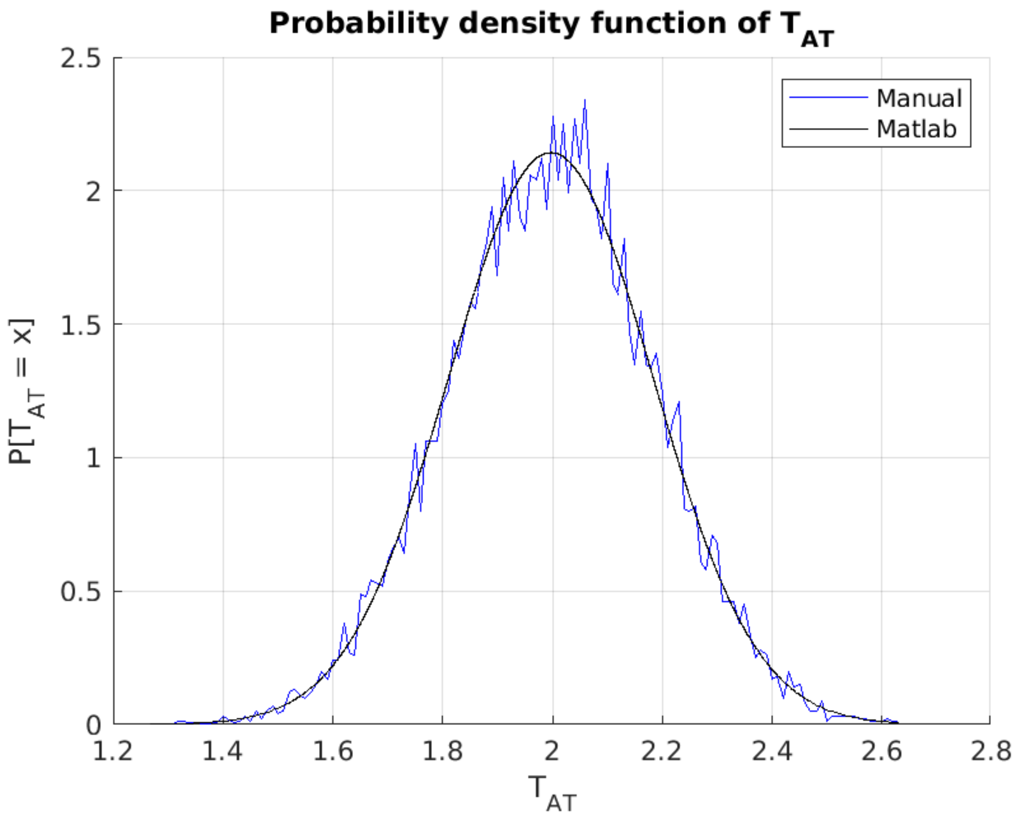

Figure 4.6: Comparison between the probability density function computed manually from the final values of T_{AT} and the ideal one obtained with Matlab

From Fig. 4.6, we can see that the probability density function of the final values of the atmospheric temperature is very similar to an ideal gaussian distribution. However, to compute $P[T_{AT} > 2°C]$, we need the cumulative density function, which is shown in the following figure:

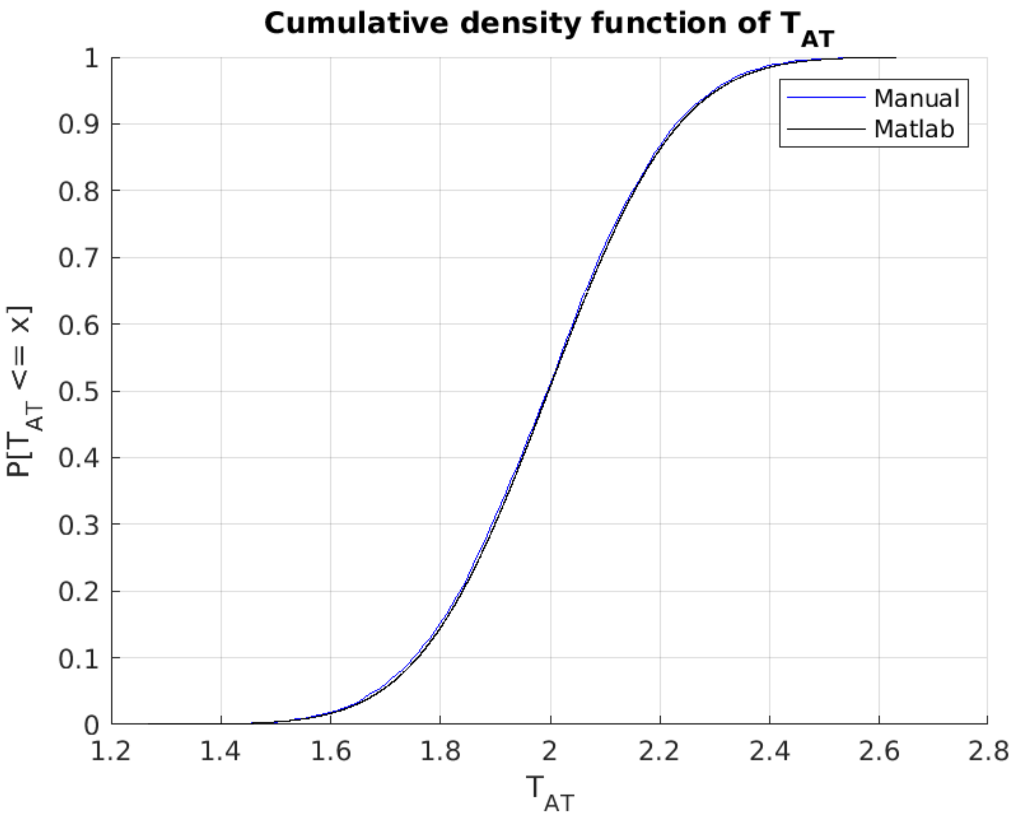

Figure 4.7: Comparison between the cumulative density function computed manually from the final values of T_{AT} and the ideal one obtained with Matlab

From Fig. 4.7 we, first of all, have the confirmation of the final values of the atmospheric temperature being very closely distributed like a normal random variable. Secondly, we can see that:

$$P[T_{AT} > 2°C] = 1 - P[T_{AT} \leq 2°C] = 1 - 0.512 = 0.488 \tag{4.1}$$

So, basically, the UN Paris Agreement is violated a little less than half of the times. Now, this may seem like a good result. After all, we manage to comply with the agreement for slightly more than half of the times and, even when we do not, the maximum atmospheric temperature that we reach is around 2.63°. However, we cannot ignore the control input trajectories shown in Fig. 4.2. Due to the fact that the MinMax approach is very conservative (we are using it precisely because we want to achieve robustness) and due to the soft constraint on T_{AT} that encourages the solver to keep the atmospheric temperature as low as possible, the trajectory for the abatement rate μ is very steep: according to these results, we should reduce emissions by almost 83% in 2025 and eliminate them completely in 2030. This means that, by applying these control inputs, we obtain very good results in term of the atmospheric temperature but, unfortunately, such control inputs are simply not feasible in reality.
A way to try to overcome such a problem is by constraining μ_{rate}, as described in the Optimization chapter (3.1.3). This way, we expect to obtain a less steep trajectory for μ which, in turn, implies worse results for the atmospheric temperature (because

we are not allowing μ to change arbitrarily fast). The upper bound for μ_{rate} is the same one used in [1] and can be found in the Parameters chapter (6.8). The results obtained this way are shown in the following figure:

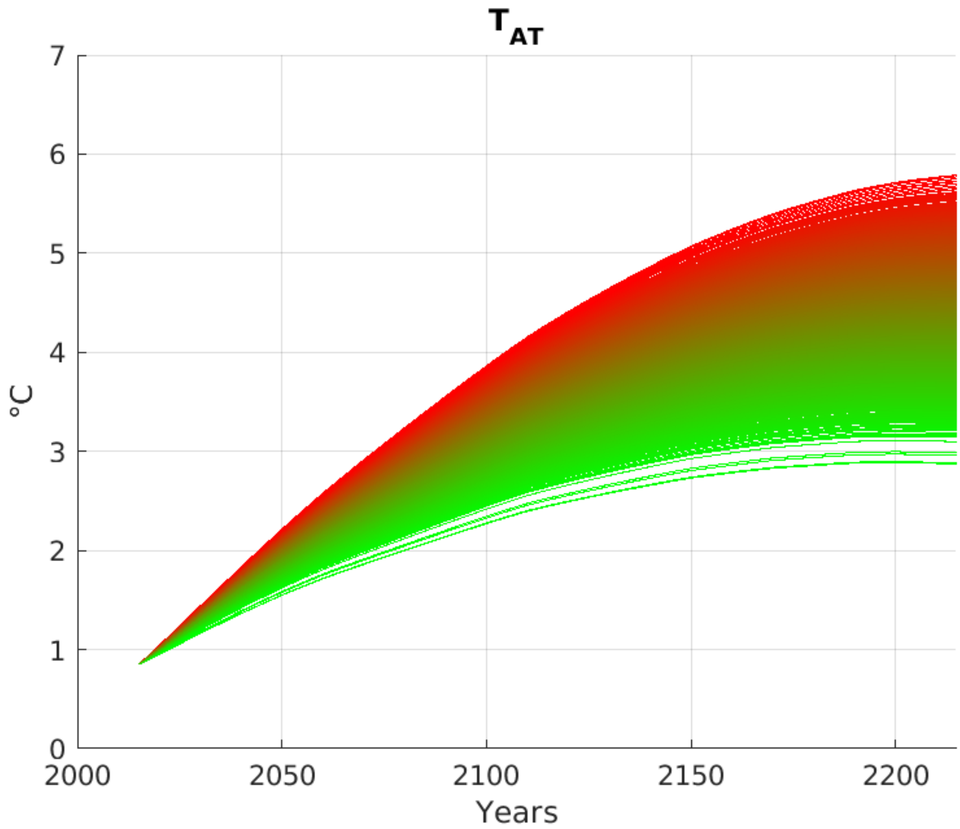

Figure 4.8: Atmospheric temperature trajectories for different values of the ECS ϵ $[2°C,\ 4°C]$, with μ_{rate} constrained

We can see that, as expected, the results are way worse than before. Now, the final value for the atmospheric temperature ranges between around $2.87°C$ and $5.79°C$, which means that the UN Paris Agreement is violated 100% of the times. The corresponding abatement rate μ is shown in the following figure (plotted against the previous one in order to show the difference):

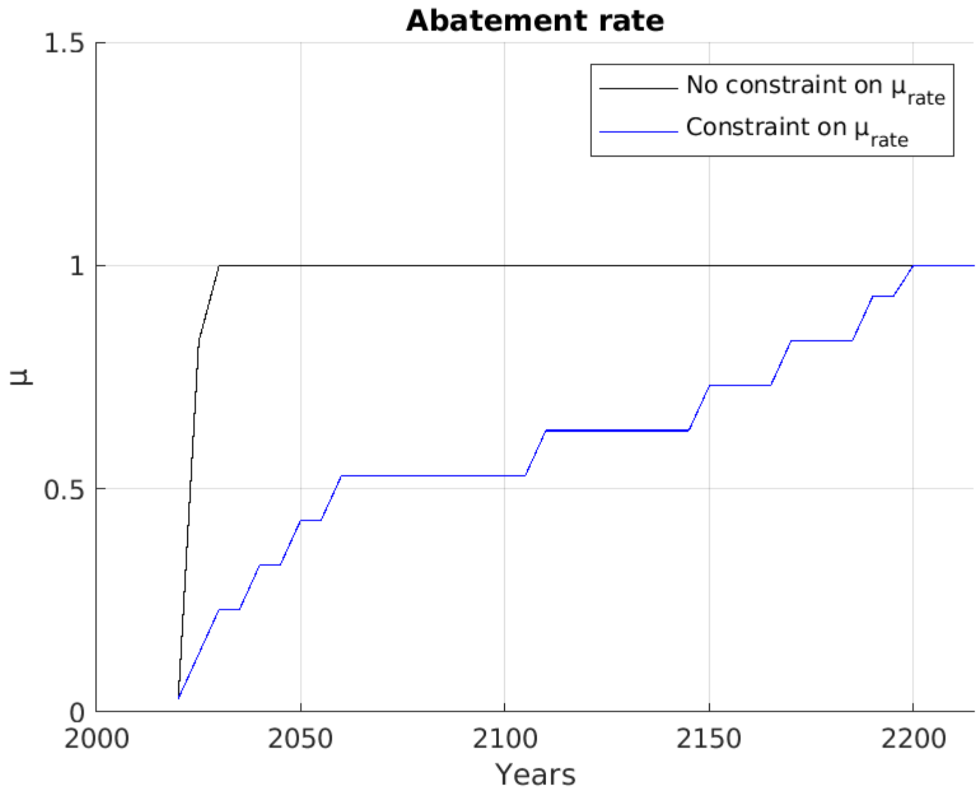

Figure 4.9: Abatement rate μ obtained by solving the MinMax problem with and without a constraint on μ_{rate}

It is clear that now the control action is much more feasible in terms of the abatement rate μ. Another interesting thing to consider is the other control input, i.e. the savings rate s:

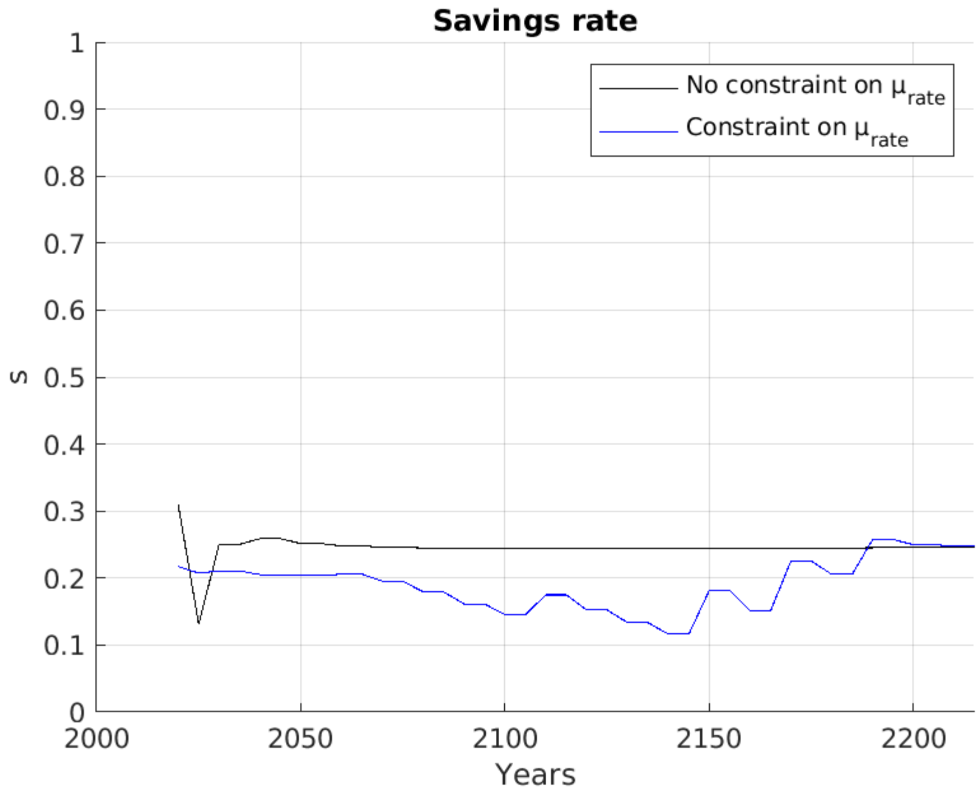

Figure 4.10: Savings rate s obtained by solving the MinMax problem with and without a constraint on μ_{rate}

We can see that when we enforce a constraint on μ_{rate} the savings rate s tends to be lower. This is not surprising because, by constraining μ_{rate}, we are basically reducing the capabilities of the abatement rate μ to fulfill our goals. However, our goals remain the same, and so if one of the two control inputs is less able to fulfill them, then the other one needs to step in. This is precisely what happens: lower values of the savings rate mean that we are trying to keep the atmospheric temperature down by directly reducing the economic activity (as a reminder, the savings rate indicates the fraction of the net economic output that gets reinvested into the economy (2.13)).
One could argue that imposing a constraint on μ_{rate} such that $\mu_{rate} \in [0, \Delta_{\mu}]$ poses a tuning problem: how can we determine the value of Δ_{μ}? The value used in this thesis is $\Delta_{mu} = 0.1$ and it comes from [1]. Just for testing purposes, we can try to increase this Δ_{mu} (i.e. to soften the constraint on μ_{rate}) to see if we get both feasible control inputs trajectories and also good performances regarding the atmospheric temperature. By running the same Monte Carlo simulation as before but with a $\Delta_{mu} = 0.3$, we get the following trajectories for the atmospheric temperatures:

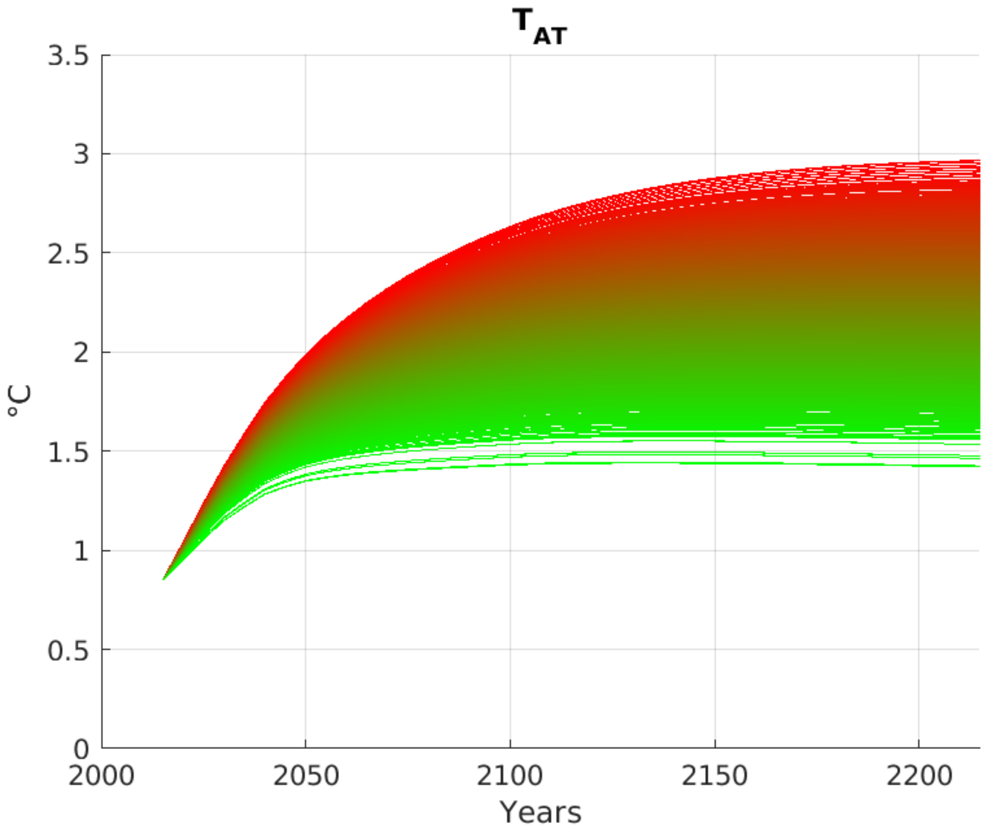

Figure 4.11: Atmospheric temperature trajectories for different values of the ECS ϵ [$2°C$, $4°C$], with μ_{rate} constrained more softly

The final value of the atmospheric temperature has the following features:

- ranges in the interval [$1.42°C$, $2.97°C$]
- has a mean value of $2.24°C$
- has a standard deviation of $0.21°C$

By looking at the cumulative density function of these values

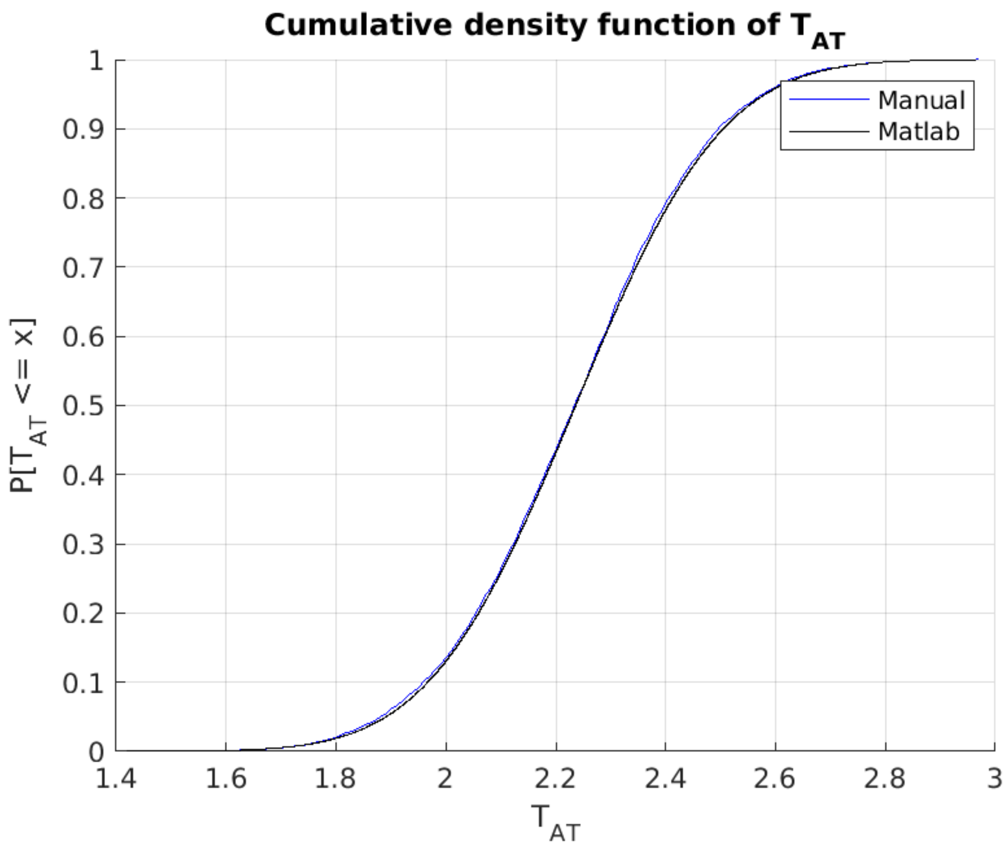

Figure 4.12: Comparison between the cumulative density function computed manually from the final values of T_{AT} and the ideal one obtained with Matlab, with μ_{rate} constrained more softly

We can see that

$$P[T_{AT} > 2°C] = 1 - P[T_{AT} \leq 2°C] = 1 - 0.123827 = 0.8762 \tag{4.2}$$

Which means that we do not comply with the UN Paris Agreement in more than 87% of the cases. Before drawing our conclusions, we should also compare the control inputs of the two previous cases with the control inputs in this one:

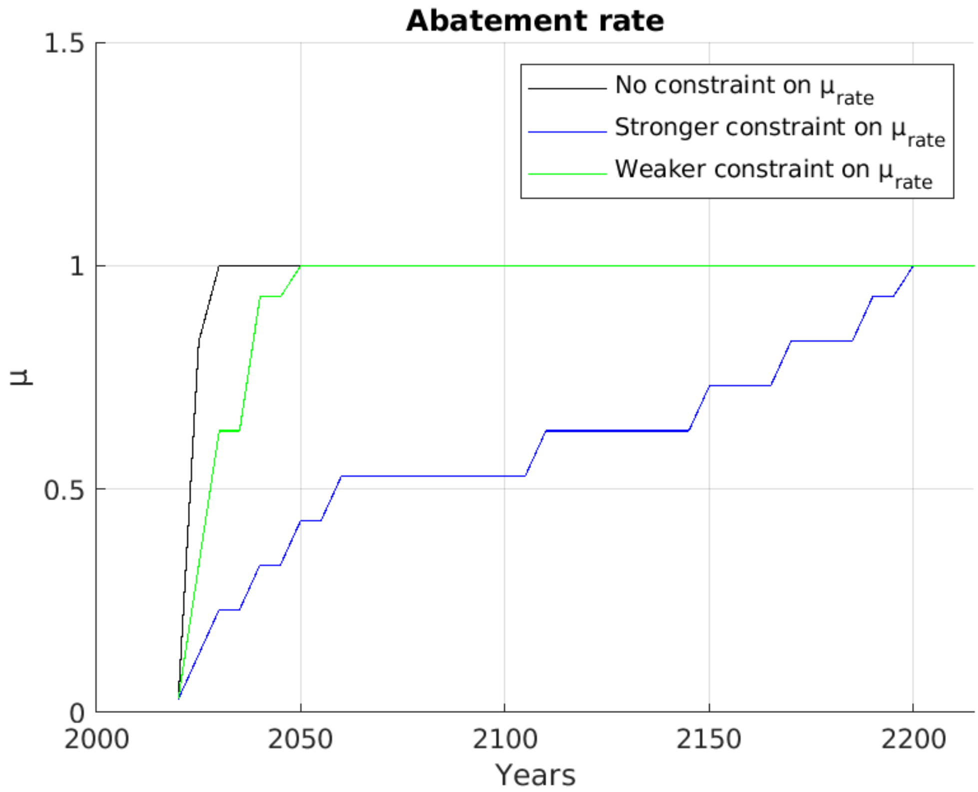

Figure 4.13: Abatement rate μ obtained by solving the MinMax problem with two different constraints on μ_{rate} and without any constraint

We can see that in this case (the green line) we are in between the first two, which is not a surprise, since we are considering a more constrained situation with respect to the first one (black line, where there was no constraint at all on μ_{rate}) but not as constrained as the second one (blue line, where the constraint on μ_{rate} was stronger because Δ_μ was lower). This means that the control inputs obtained in this third case are more feasible that the one obtained in the first and also more effective that the one obtained in the second. However, it should be kept in mind that this increase from $\Delta_\mu = 0.1$ to $\Delta_\mu = 0.3$ was just for testing purposes and we do not have the data to say that such a constraint is actually applicable in reality or not. In conclusion, if such a control input trajectory is actually feasible, we would be able to get an atmospheric temperature that most likely will not comply with the UN Paris Agreement but that will not be too much higher than the limit of $2°C$ imposed by said agreement.

4.2 Monte Carlo analysis of the pure minimization approach with normally distributed ECS

It is interesting to compare the results obtained with the MinMax approach (the novelty of this master's thesis) and the ones obtained with a pure minimization approach (i.e. without an adversary agent that tries to make the situation worse).

To this end, the following Monte Carlo analysis (analogous to the one in the previous section (4.1)) is run:

1. Solve the minimization problem, assuming $ECS = 3.1°C$ (since this is the most common value found in the literature) and collect the resulting control inputs.
2. Run a simulation of the system using the previously obtained control inputs but with a different ECS, sampled randomly from a normal distribution.
3. Repeat step 2 a significant number of times.

With this Monte Carlo simulation, we want to test how robust a pure minimization approach is, when the controller assumes an ECS different than the real one, and how different the results are compared to when we use a MinMax approach. This process is summed up by the following diagram:

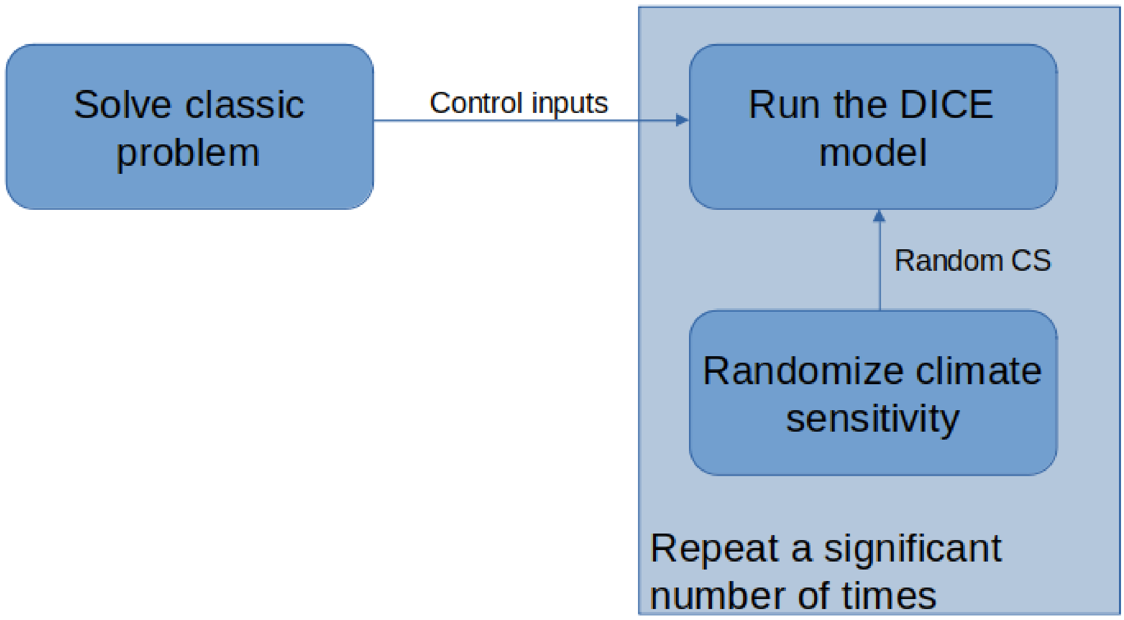

Figure 4.14: Diagram that sums up the procedure used to perform the Monte Carlo simulation in a classic approach (i.e. without adversary agent)

Just like the Monte Carlo simulation that was run to test the MinMax approach, this Monte Carlo simulation uses, for the same reasons, normally distributed samples of the ECS with a mean value of $3°C$ and a standard deviation of $0.3°C$ (i.e. $ECS \sim N(3, 0.3)$).
After solving the minimization problem, we get the following control inputs:

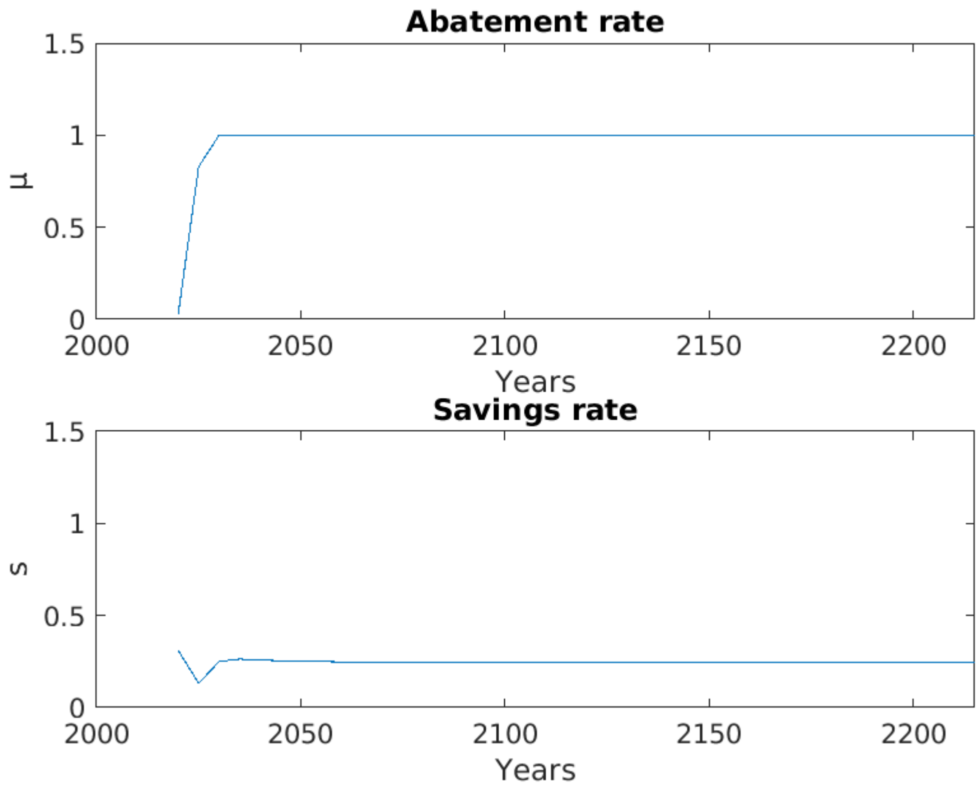

Figure 4.15: Control inputs obtained by solving the Min problem

These control inputs are exactly the same as the ones obtained by solving the MinMax problem (Fig. 4.2). Why does that happen? In both cases, there was a soft constraint in the objective function that pushed the solver into keeping the atmospheric temperature below $2°C$ (section 2.14) and the fact that the control inputs trajectories are the same in the two cases tells us that this soft constraint has more impact than the value of the ECS. In other words, the UN Paris Agreement is so demanding that it does not matter if we consider a constant ECS equal to $3.1°C$ or an increasing one: complying with it is equally as hard in the two cases. These control inputs will be used for the entire Monte Carlo simulation, since the only thing that changes in every iteration is the value of the ECS (just like the previous Monte Carlo simulation (4.1)). Obviously, given that the control inputs are the same as the ones obtained by solving the MinMax problem, we expect to get more or less the same behaviour for the atmospheric temperature in both cases (not precisely the same because there's a random sampling involved). In fact, these are the atmospheric temperature trajectories that we get by running the Monte Carlo simulation:

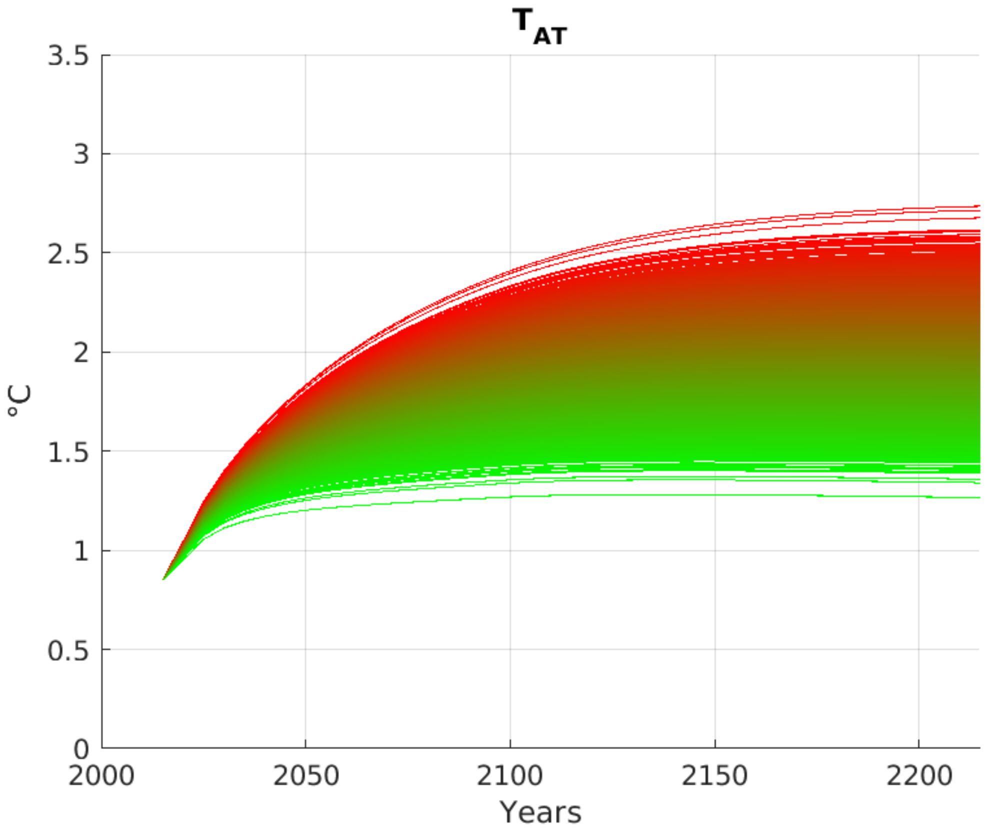

Figure 4.16: Atmospheric temperature trajectories for different values of the ECS ϵ $[2°C,\ 4°C]$

Again, every color in Fig. 4.16 is related to a specific sample of the ECS, exactly the same way as Fig. 4.3. The final values of the atmospheric temperatures that we get are inside the interval $[1.27°C,\ 2.74°C]$, which is not that different from the interval $[1.3°C,\ 2.63°C]$ obtained by performing the Monte Carlo simulation on the MinMax approach. We can then conclude that the slight differences are due to the fact that the ECS is sampled randomly and so, even though the control inputs used are the same, it is very unlikely to get the exact same results.
To verify that the soft constraint has a bigger impact with respect to the value of the ECS, the following section will analyse the same two Monte Carlo simulations of sections 4.1 and 4.2 but without said soft constraint.

4.3 Repetition of the two previous Monte Carlo analyses without a soft constraint on T_{AT}

In this section, we will analyse how the results in sections 4.1 and 4.2 change when we remove the soft constraint on T_{AT} (i.e. we use 2.32 as an objective function instead of 2.33). This is done to show that the reason the results in those two section are basically the same is that the effect of the soft constraint is much stronger than the one of the ECS. Removing this soft constaint means thatwe do not care anymore

about complying with the UN Paris Agreement. So, here we want to show that it does not matter whether we have a constant $ECS = 3.1°C$ or an increasing one: complying with the UN Paris Agreement is so demanding that the results are the same in both cases.
The following two plots show the control inputs obtained by running the same Monte Carlo simulation of sections 4.1 and 4.2 but without enforcing a soft constraint on T_{AT}:

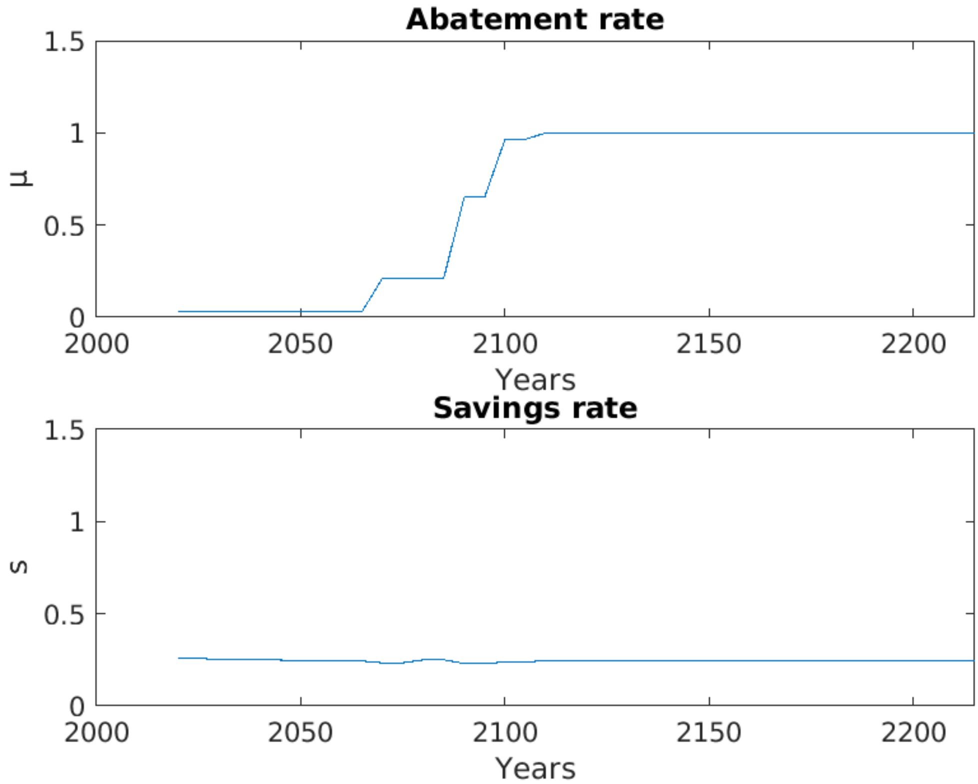

Figure 4.17: Control inputs obtained by solving the MinMax problem without soft constraint on T_{AT} (Monte Carlo simulation of section 4.1 without soft constraint on T_{AT})

Fig. 4.17 is what we obtain when we solve the MinMax problem without enforcing a soft constraint on T_{AT}. To no surprise, removing the soft constraint makes the control action much less aggressive with respect to what was shown in Fig. 4.2. We can see that there is no abatement at all until the year 2070 (where emissions get cut by just 20%) and that a complete elimination of the emissions will not be reached until the year 2110.

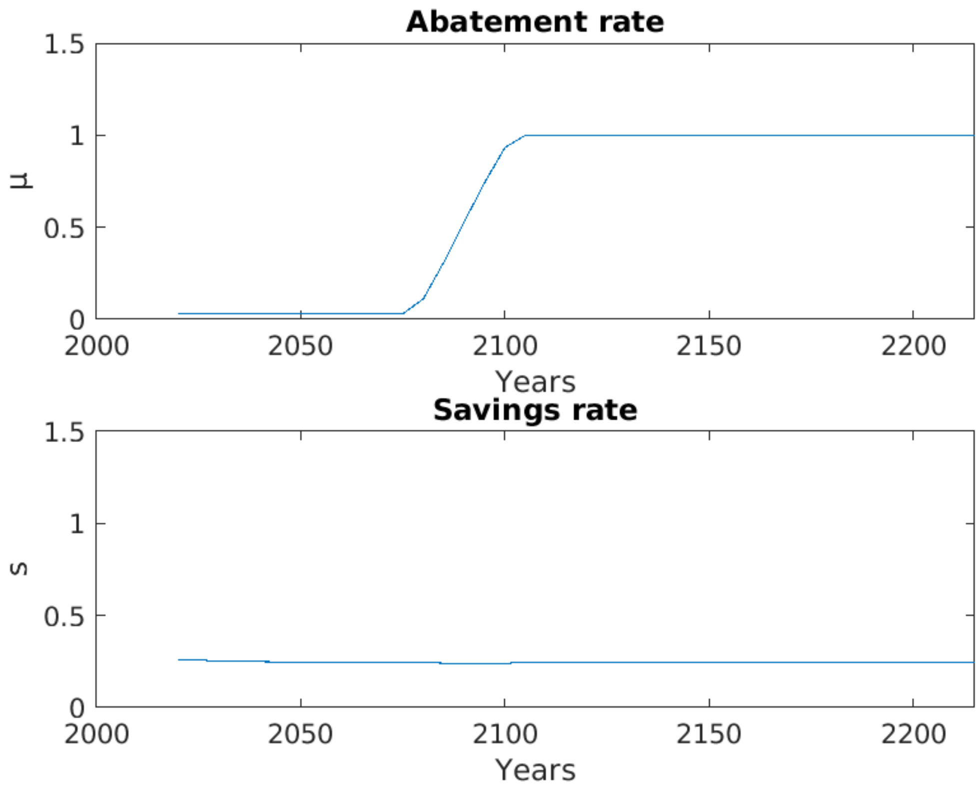

Figure 4.18: Control inputs obtained by solving the minimization problem without soft constraint on T_{AT} (Monte Carlo simulation of section 4.2 without soft constraint on T_{AT})

Fig. 4.18 is what we obtain when we solve the minimization problem without enforcing a soft constraint on T_{AT}. Obviously, here we also have a much less aggressive control action with respect to what was shown in Fig. 4.15. In fact, abatement action does not start until the year 2075 and we reach a full elimination of the emissions only in the year 2105. Another thing worth noting is that Fig. 4.18 depicts a much smoother abatement rate evolution with respect to what is shown in Fig. 4.17. This happens because Fig. 4.18 portrays a scenario where the ECS is considered constant (and equal to $3.1°C$) while 4.17 shows a scenario in which the ECS changes due to the adversary agent that always increases it. So the "jumps" that happen in Fig. 4.17 are due to increased values of the ECS.
The following picture shows the result of said Monte Carlo simulation for the Min-Max scenario:

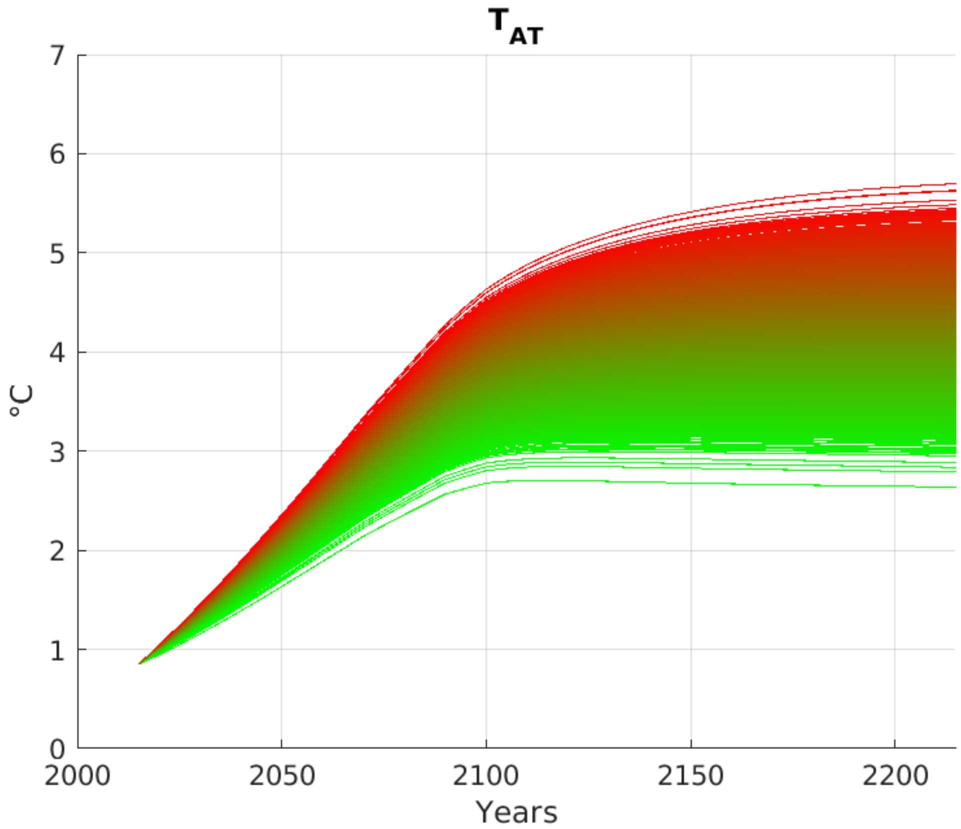

Figure 4.19: Atmospheric temperature trajectories for different values of the ECS ϵ [2°C, 4°C] for the MinMax scenario

Exactly as in sections 4.1 and 4.2, each color represent a value of the ECS, where colors closer to green refer to lower values of the ECS while colors closer to red refer to higher values of the ECS. The final value of the atmospheric temperature depicted here have the following properties:

- ranges in the interval [2.75°C, 5.69°]
- has a mean value of around 4.25°C
- has a standard deviation of around 0.4°C

The results of the Monte Carlo simulation for the minimization scenario are quite similar:

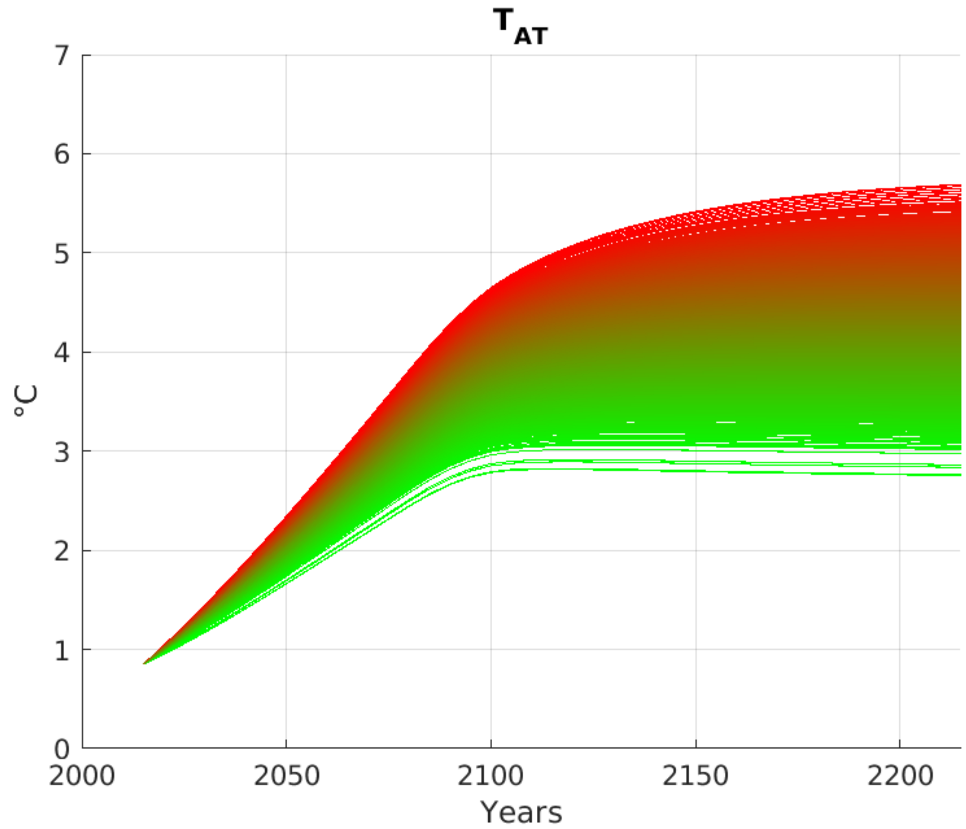

Figure 4.20: Atmospheric temperature trajectories for different values of the ECS ϵ $[2°C, 4°C]$ for the minimization scenario

The final values of the atmospheric temperatures shown here have the following properties:

- ranges in the interval $[2.75°C, 5.69°]$
- has a mean value of around $4.31°C$
- has a standard deviation of around $0.4°C$

These results are mostly the same as the ones obtained for the MinMax approach, except for the mean value, which appears to be slightly higher in the minimization scenario, which makes sense because the minimization scenario leads to less aggressive control inputs trajectories and so the atmospheric temperature is able to grow a little bit more. But also, since there is a random sampling involved, we cannot expect the exact same results in the two simulations.
So, in conclusion, the only real difference is related to the control inputs, where it is clear that the MinMax approach leads to a more aggressive control action due to the fact that it analyses a worse scenario than the one analysed with the minimization approach.

4.4 Comparison between different fractions of forcings that can be abated

As mentioned in the section related to the radiative forcings (2.2.2), this thesis uses a model where the external forcings can be directly abated by acting on the control input μ. The magnitude of this abatement is determined by the parameter $a_{frac} \in [0, 1]$ (the higher it is, the more we can abate forcings by acting on μ), which in this thesis is assumed to be equal to 0.6 (according to [4]).
Since this parameter poses a tuning problem, it is interesting to see the effects of using different values for it.
The figure that follows depicts a scenario in which we ran the previously described MinMax approach (chapter 3). For each plot, the only thing that changes is the value of a_{frac}:

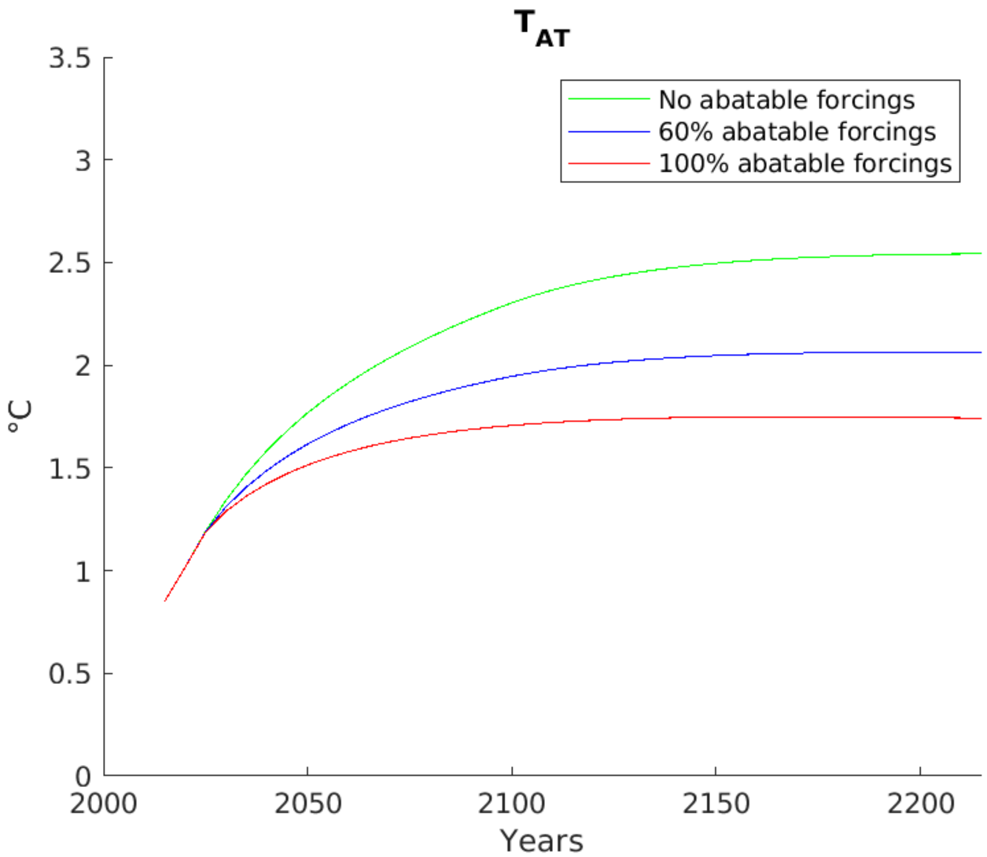

Figure 4.21: Atmospheric temperature trajectories for different fractions of abatable forcings

The results are all but surprising: being able to directly abate forcings through the control input μ is a good thing (since forcings are negative for the environment). Knowing this, it is not a surprise that lower values of a_{frac} lead to higher atmospheric temperatures.

5
Conclusion

In this thesis, we analysed how feasible the UN Paris Agreement's goals are by employing a robust model predictive control approach and how important it is to consider the equilibrium climate sensitivity in a robust way in order to compensate for the uncertainty behind its estimation. We also expanded the model both to add robustness and to make it more realistic, in order to have more reliable conclusions. The results seem to indicate that, while a higher value of the climate sensitivity does lead to worse results in term of the atmospheric temperature, the goals of the UN Paris Agreement are so ambitious and our current situation is so critical that it does not really matter which value of the climate sensitivity we assume: if we want to comply with said goals, we need to act quickly and effectively, regardless of the actual value of the climate sensitivity. As stated in [8]:

> We're emitting carbon dioxide so fast that the difference between a low and a high value of climate sensitivity is largely irrelevant in climate policy terms.

Which means that whatever the actual value of the climate sensitivity is, an urgent action in terms of emissions abatement needs to be taken. So, having a more precise estimate of the climate sensitivity may only be needed to fine-tune the actions to be taken ([6]) but not to question whether or not this actions **need** to be taken. In fact, according to [9], an equilibrium climate sensitivity closer to $2°C$ would extend the deadline of reaching net-zero emissions only by about a decade and, moreover, there is no real reason to consider low values of the equilibrium climate sensitivity instead of high ones, since there are just as many studies claiming that it is around the lower end as many claiming that it is around the higher end [6]). So, the question of "if the equilibrium climate sensitivity turns out to be on the lower end of the estimation interval, do we have more time to take actions to reduce emissions?" has an obvious answer of "No, we do not", the situation is too critical to pretend to have time. Reaching said goals is theoretically feasible and optimal in the setting analysed by this thesis, but the control inputs trajectories that we get are too aggressive. This leads to the need of constraining them, but as soon as we do so, we lower significantly our chances of complying with the UN Paris Agreement.

In conclusion, it does not really matter whether or not we can precisely reach the UN Paris Agreements. What matters is that we cannot anymore lie to ourselves about having time to fix the state of things. We do not. Which does not mean that we are doomed, but rather that no more time needs to be wasted when it comes to taking actions against climate change. Each one of us has the responsibility of conducting a more sustainable lifestyle but who can really make the critical difference are policy makers and big economic actors, who both need to act realising that

short-term economic profit should not be put in front of the long-term well-being of the planet and, consequently, of the entire human race.
Whether or not the die is cast is up to us.

> "The best time to act on climate change was 50 years ago, the second best time is now." (Sommer Ackerman, 2022)

5.1 Further work

An effort that has begun with the conception of the DICE model and that still continues is that of updating the model, so that it remains relevant and such that its prediction are reliable and accurate. This direction of work is always interesting and useful to pursue, so any new updates to the model can only benefit the predictions obtained in this thesis.
Other research can also be devoted to treating the ECS in a different way than how it was done in this thesis. For instance, keeping the scope within the control field, the controller could assume the ECS as a random variable. This way, stochastic control can be employed and its results may be compared with the ones of this thesis.
Lastly, what has been done in this thesis for the ECS can also be done for other parameters whose exact value is not known. For instance, such parameters could be the fraction of abatable external forcings a_{frac} or the pliability of the abatement costs p (both parameters that have been added from [4] and that introduce tuning problems).

6

Appendix: parameters and constants

The tables that follow list the values of all the parameters and the constants used in the model and in the implementation, as well as a short description of what the value represents and in which equations does the value appear (if the unit is not mentioned, it means that the parameter has no unit):

6.1 General parameters

Parameter	Value	Description	Equations
N	60 time steps	Horizon length	
t_0	2015	Initial year	
Δ	5 years	Time step	2.12, 2.23, 2.24, 2.26, 2.30, 2.31, 2.32
t_f	40	Final simulation time	
s_w	40	Weight for the soft constraint on T_{AT} in the objective function	

6.2 Initial conditions for the states

Parameter	Value	Description	Equations
i_0	1	Initial iteration index	
$T_{AT,0}$	0.85 °C	Initial atmospheric temperature	
$T_{LO,0}$	0.0068 °C	Initial lower ocean temperature	
$M_{AT,0}$	851 GtC	Initial mass of carbon in the atmosphere	
$M_{UP,0}$	460 GtC	Initial mass of carbon in the upper ocean	
$M_{LO,0}$	1740 GtC	Initial mass of carbon in the lower ocean	
K_0	223 trillions 2010USD	Initial capital	
σ_0	0.3503 GtC/trillions 2010USD	Initial emissions intensity	
L_0	7403 millions people	Initial population	
A_0	5.11	Initial total factor productivity	
$E_{L,0}$	2.6 $GtCO_2$/yr	Initial land emissions	
f_0	0.5 W/m^2	Initial forcings of non-CO2 GHGs	2.29
E_0	191.7019	Initial emissions	2.27, 2.28
C_0	3.89	Initial consumption	
μ_0	0.03	Initial abatement rate	
s_0	0.259	Initial savings rate	
J_0	0	Initial social welfare	

$\mu_{rate,0}$	0	Initial rate of the abatement rate	
$\mu_{growth,0}$	0	Initial growth of the abatement rate	
ECS_0	3.0 °C	Initial equilibrium climate sensitivity	
$ECS_{rate,0}$	0	Initial rate of the equilibrium climate sensitivity	

6.3 Climate diffusion parameters

Parameter	Value	Description	Equations
c_3	0.088	Model diffusion parameter	2.5, 2.6
c_4	0.025	Model diffusion parameter	2.5

6.4 Carbon cycle diffusion parameters

Parameter	Value	Description	Equations
ζ_{11}	0.88	Carbon cycle diffusion parameter	2.7, 2.8, 2.9, 2.11
ζ_{12}	0.196	Carbon cycle diffusion parameter	2.7, 2.8, 2.9, 2.11
ζ_{21}	0.12	Carbon cycle diffusion parameter	2.11
ζ_{22}	0.797	Carbon cycle diffusion parameter	2.11
ζ_{23}	0.001465	Carbon cycle diffusion parameter	2.7, 2.8, 2.9, 2.11
ζ_{32}	0.007	Carbon cycle diffusion parameter	2.11
ζ_{33}	0.99853488	Carbon cycle diffusion parameter	2.11

6.5 Geophysical parameters

Parameter	Value	Description	Equations
η	3.6813 W/m^2	Forcings of equilibrium CO2 doubling	2.5, 2.6, 2.7, 2.8, 2.9
ξ_1	0.1005	Climate equation coefficient for upper level	2.5, 2.6, 2.7, 2.8, 2.9
ξ_2	0.2727 GtC/$GtCO_2$	Conversion factor from GtC to $CtCO_2$	2.10, 2.7, 2.8, 2.9
$M_{AT,1750}$	588 GtC	Carbon concentration in the atmosphere in the year 1750	2.7, 2.8, 2.9
f_1	1.0 W/m^2	Forcings of non-CO2 GHGs in 2100	2.29
t_{force}	17 time steps	Slope of non-CO2 GHG forcings	2.29
δ_{EL}	0.115	Land use emissions decrease rate	2.27, 2.28
$T_{AT,max}$	$2°C$	Maximum atmospheric temperature according to the UN Paris Agreement	2.33

6.6 Socioeconomic parameters

Parameter	Value	Description	Equations
δ_K	0.1	Capital depreciation (5 year)	2.12
γ	0.3	Capital elasticity in production function	2.14, 2.15
θ_2	2.6	Exponent of control cost function	2.18, 2.19, 2.20, 2.21, 2.22
a_2	0.00236	Damage multiplier	2.16, 2.17
a_3	2	Damage exponent	
α	1.45	Elasticity of marginal utility of consumption	2.32
ρ	0.015	Initial rate of social time preference per year	2.32
L_a	11500 millions people	Asymptotic population	2.25
l_g	0.134	Population growth rate	2.25
g_A	0.076	Initial TFP rate	2.26
δ_A	0.005	TFP increase rate	2.26
p_b	550 2010 USD/tCO_2	Initial backstop price	2.21, 2.22
δ_{PB}	0.025	Decline rate of backstop price	2.21, 2.22
g_σ	0.0152	Emissions intensity base rate	2.23, 2.24
δ_σ	0.001	Decline rate of emissions intensity	2.23, 2.24

6.7 Model updates from [4]

Parameter	Value	Description	Equations
a_{frac}	0.6	Fraction of abatable external forcings	2.8, 2.9
$\hat{t}$	30 years	Time scale for technological transitions	2.19, 2.20
p	0.75	Pliability of the abatement costs	2.19, 2.20

6.8 Bounds

Parameter	Value	Description	Equations
Δ_μ	0.1	Rate bound on the input μ	
Γ_μ	0.53	Growth bound on the input μ	
ECS_{min}	2 °C	Lower bound for the equilibrium climate sensitivity	
ECS_{max}	4 °C	Upper bound for the equilibrium climate sensitivity	3.5
$ECS_{rate,max}$	0.1	Upper bound on the rate of the equilibrium climate sensitivity	3.5

Bibliography

[1] Kellett, Christopher M and Weller, Steven R and Faulwasser, Timm and Grüne, Lars and Semmler, Willi (2019) Feedback, dynamics, and optimal control in climate economics. Annual Reviews in Control, volume 47, pages 7–20, Elsevier.

[2] Hänsel, Martin C and Drupp, Moritz A and Johansson, Daniel JA and Nesje, Frikk and Azar, Christian and Freeman, Mark C and Groom, Ben and Sterner, Thomas (2020) Climate economics support for the UN climate targets. Nature Climate Change, volume 10, pages 781–789, Nature Publishing Group.

[3] Azar, Christian and Johansson, Daniel JA (2021) DICE and the carbon budget for ambitious climate targets. Earth's Future, volume 9, number 7, pages e2021EF002041, Wiley Online Library.

[4] Grubb, Michael and Wieners, Claudia and Yang, Pu (2021) Modeling myths: On DICE and dynamic realism in integrated assessment models of climate change mitigation. Wiley Interdisciplinary Reviews: Climate Change, volume 12, number 3, pages e698, Wiley Online Library.

[5] Johansson, Daniel JA and O'Neill, Brian C and Tebaldi, Claudia and Häggström, Olle (2015) Equilibrium climate sensitivity in light of observations over the warming hiatus. Nature Climate Change, volume 5, number 5, pages 449–453, Nature Publishing Group.

[6] Zeke Hausfather (2018) Explainer: How scientists estimate 'climate sensitivity'. https://www.carbonbrief.org/explainer-how-scientists-estimate-climate-sensitivity.

[7] Kerry Emanuel, David Chandler (2020) Radiative Forcing. https://climate.mit.edu/explainers/radiative-forcing.

[8] Roz Pidcock (2014) Your questions on climate sensitivity answered. https://www.carbonbrief.org/your-questions-on-climate-sensitivity-answered.

[9] Rogelj, Joeri and Meinshausen, Malte and Sedláček, Jan and Knutti, Reto (2014) Implications of potentially lower climate sensitivity on climate projections and policy. Environmental Research Letters, volume 9, number 3, pages 031003, IOP Publishing.

[10] National Oceanic and Atmospheric Administration (2022) What is the carbon cycle? https://oceanservice.noaa.gov/facts/carbon-cycle.html

[11] Nordhaus W. D. (2017) Revisiting the social cost of carbon. Proceedings of the National Academy of Sciences, volume 114, number 7, pages 1518–1523, National Acad Sciences.

[12] United Nations (2015) What is the Paris Agreement? https://unfccc.int/process-and-meetings/the-paris-agreement/the-paris-agreement