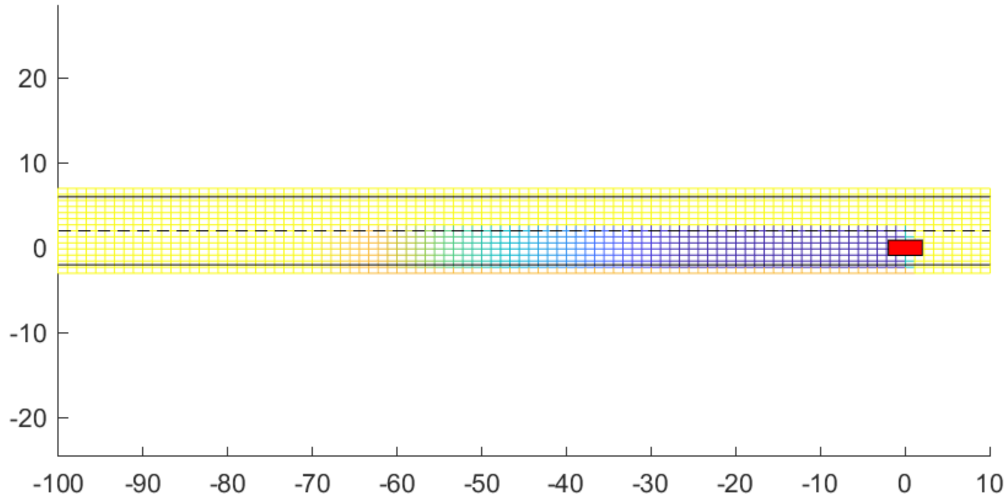
CHALMERS
UNIVERSITY OF TECHNOLOGY



The Oskillator
Artificial Force Field Highway Chauffeur

An Algorithm for Autonomous Driving on Highways

Master's thesis in Computer Science, Algorithms, Language and Logic

OSKAR LARSSON

Mathematical Sciences
CHALMERS UNIVERSITY OF TECHNOLOGY
Gothenburg, Sweden 2019

Master's thesis 2019:NN

The Oskillator
Artificial Force Field Highway Chauffeur

An Algorithm for Autonomous Driving on Highways

OSKAR LARSSON

Mathematical Sciences
Division of Division name
Name of research group (if applicable)
Chalmers University of Technology
Gothenburg, Sweden 2017

The Oskillator, Artificial Force Field Highway Chauffeur
An Algorithm for Autonomous Driving on Highways
OSKAR LARSSON

Supervisor: Viktor Svensson, Feature Function Engineer, Aptiv
Examiner: Alexei, Heintz

Master's Thesis 2019:NN
Mathematical Sciences
Division of Division name
Name of research group (if applicable)
Chalmers University of Technology
SE-412 96 Gothenburg
Telephone +46 31 772 1000

Cover: Force field visualization constructed in Matlab showing the artificial longitudinal force field around a vehicle.

Typeset in LaTeX
Printed by [Name of printing company]
Gothenburg, Sweden 2017

The Oskillator, Artificial Force Field Highway Chauffeur
An Algorithm for Autonomous Driving on Highways
OSKAR LARSSON
Mathematical Sciences
Chalmers University of Technology

Abstract

The fully autonomous vehicle is not here yet, but a first step would be a completely autonomous feature for a simpler subset of scenarios. In this research work, the Oskillator is presented, a combined behavior and trajectory algorithm for a Highway chauffeur based on artificial force fields. At each point in time an acceleration vector is determined by generating an artificial force based on the road and the other vehicles. The force is generated from a set of simpler force components, such as the Lane Component for centering in lane, and the Pass Component for switching lanes behind a slower vehicle, etc. The components are not additive as the logic for traffic behavior is not. Instead, a composition method was developed based on min and max operations. This way the components from different vehicles on the road prevents the influence of the others, a single car is as much an obstruction as three. The force is continuous w.r.t. all inputs and a damping is analytically determined as to avoid oscillations. The host is proven to never engage in a lane switch to a lane of another vehicle with an unsafe longitudinal distance and to stop in time to avoid crashing upon entering the unsafe longitudinal distance behind another vehicle. Experiments have been performed in a simulation environment to assess the behavior of the model. It shows that the host is able to follow the road in lane and pass when appropriate, or approach the desired headway smoothly if unable to pass. The emergent lane switching manoeuvres have a very low lateral velocity, thus the switch takes a long time. Several components could be extracted for standalone use, such as Adaptive Cruise Control or Lane Keep Assistance.

Keywords: Autonomous, Self-driving cars, Highway Chauffeur, Highway Pilot, behaviour planning, trajectory planning.

Acknowledgements

A special thanks to all my colleagues at Aptiv for the motivation during the long months.

Many thanks to my supervisor at Chalmers, Alexey Heintz, for help with structuring the thesis for presenting the work.

Oskar Larsson, Gothenburg, Jan 2020

Term Definitions

A description of domain specific terms used in the paper.

- **Host** - The vehicle controlled by the proposed algorithm.
- **Object, Vehicle** - Other road-user.
- **Longitudinal** - Forward direction in curvilinear road coordinate system, *Along the road.*
- **Lateral** - Sideways direction in curvilinear road coordinate system, *Across the road.*
- **Bias** - Lateral offset from lane center.

Letter convention for variables and parameters.

- x - longitudinal positions,
- y - lateral position,
- Δ - distances,
- d - distances,
- v - velocity component,
- a - acceleration component,
- b - deceleration parameter,
- t - time parameter or variable,
- A - force amplitude,
- k - scaling coefficient,
- o - subscript denoting other road-user property.

Contents

List of Figures

1
Introduction

This master thesis is a collaboration between Aptiv and Chalmers University of Technology on development of an algorithm for autonomous driving on highways by use of artificial force fields. Certain artificial force field components are designed based on the road and other road-users, and a way of composing them to deduce an acceleration that will yield a trajectory on the road that is both safe and comfortable.

In this chapter an introduction to the subject and the proposed approach is presented. The selected approach is motivated and the goals of the developed algorithm is outlined.

1.1 Background

The *Highway Chauffeur* or *Highway Pilot* is a feature that is becoming more common in modern vehicles. Companies such as Tesla and Bosch have developed popular versions of the feature that are available on the market today. The feature allows the driver to provide the car with a desired velocity v_{des} and headway time t_{des}, and hand over the control of both acceleration and steering to the autonomous system. The system will then try to achieve the desired velocity through acceleration and follow the lane with steering. When a slower car appears ahead, the system will decide on whether to pass or to slow down to keep the desired headway distance to the vehicle ahead. The feature may also switch lanes to follow a desired route. More advanced versions of the feature may even predict the behavior of other road-users to find the appropriate actions and reactions.

The feature is sometimes pipelined into four steps, Sensing, Behavior Planning, Trajectory Planning and Control. The sensing step uses equipment such as radars, cameras, inertial measurement units etc., to collect data about the host vehicle and its environment. It also involves a process of *fusing* and *tracking* the data into a high-level environment model describing the road and the traffic on it. The behavior module will determine what manoeuvre to perform and the trajectory module how to execute it. The control model then calculates the appropriate control signals, acceleration and steering angle, for the vehicle to realize the decided trajectory.

The subject of this thesis has been to develop a module for combined behavior and trajectory planning based on artificial force fields. Given a desired velocity and headway time as driver input, a range of acceptable lanes as route input, and enough information about the host, the road and other road-users, it determines an appropriate acceleration vector in the world coordinate system for each point in time, to drive the car safely and comfortably on highways.

The algorithm consists of calculating a longitudinal acceleration, *along the road*, by considering the desired velocity, properties of the vehicles ahead, such as distance, velocity and acceleration, and the curvature of the road ahead. The lateral acceleration, *across the road*, is then calculated by considering the road curvature, the acceptable lanes and the other cars on the road.

The thesis has been focused on finding a set of *force components* to achieve the different behaviors, e.g. the Lane Component to center in lane, the Trail Component to slow down to a vehicle in front, etc, and a way of composing the components, with an analytically determined damping, to achieve an acceleration that is feasible, safe and comfortable. The composition is based on min and max operations rather than just addition. The different vehicles around the host can then prevent each other's influence, a single vehicle in the way is as much an obstruction as three. The output acceleration is continuous with respect to all inputs.

In this paper the different components and the composition method is defined. The resulting differential equations defining the trajectories are analysed to prove certain assertions of safe behavior. The algorithm has been implemented and tested in a simulation environment for subjective assessment. The proposed highway chauffeur algorithm can perform basic highway control, including Adaptive Cruise Control and lane keeping, along with several of the necessary high-level manoeuvres required to travel on a highway, such as passing and merging.

1.2 Motivation for the Force Field approach

There are several reasons why the force field approach for determining behavior and trajectory on highways is promising. First of all, Safety.

By law, the highway chauffeur features still require human supervision and potential intervention. Before private cars are allowed to drive completely autonomously both the sensing equipment and the algorithms used need to be proven to significantly increase safety.

While the safety's dependency on sensing equipment only can be proven with respect to likelihood, the algorithms used can be proven safe w.r.t. formal models. Responsibility-Sensitive Safety(RSS) is a formal model describing allowed behavior to guarantee safety. The model is sufficient to prove safety with respect to emergency manoeuvres, however it lacks respect to lanes. In the setting of both autonomous agents *and* human drivers, it is important that the behavior of the autonomous agents match the behavior expected by human drivers. The proposed algorithm is shown to never switch into the lane of another vehicle with an unsafe longitudinal distance. It is also is shown to stop in time to avoid crashing as long as other road-users respect a safe longitudinal distance when cutting in.

The two main approaches to autonomous driving algorithms are rule-based and statistical models including those based on machine learning. The advantage of rule-based models is the transparency. It is possible to predict what will happen in any given situation, and if something happens the choices made can be motivated. In contrast a crash caused by decision made by a machine learning model is harder to defend.

The advantage of a force field- compared to a rule-based model lies in the transitions between states. Any function from continuous inputs to one of several discrete outputs suffers the problem of discontinuity. As an example, consider the boundary between the conditions that allow and do not allow lane changing. When on the boundary small disturbances may cause flickering between the two states. This has to be avoided and is usually done through a method called hysteresis[reference]. The design of these transitions is not always an easy task. As you will see in the approach proposed in this paper, the force field can be made completely continuous thus avoiding this problem.

A common approach today is to divide the autonomous driving pipeline into 4 steps: Sensing, behavior planning, trajectory planning[references] and control. The behavior is from a set of predefined high-level manoeuvres, such as cut-in-front, etc. However, as more complex situations arise more complex manoeuvres are needed. Another issue is the tight connection between behaviour and trajectory planning which makes this structure even harder to maintain. Whether the manoeuvre is even feasible as a choice for the behavior module, depends on the implementation of the trajectory module. The approach proposed in this paper combines the two steps in a way that is both natural and efficient.

In other applications of force field trajectory planning, such as the shortest path problem, a common issue is the existence of local optima. However, in the current application it is considered perfectly acceptable to stay in a local optima as it will follow the flow of traffic on the road.

1.3 Related Works

The proposed approach was inspired by the method proposed in the paper *Artificial Potential Functions for Highway Driving with Collision Avoidance*[1] from 2008, by Wolf and Burdick. Their solution involves an artificial force field determined by the road and the state of the host and other road-users. The parameters and functions describing the forces were chosen to suit the application and lacks formal justification, but in some simpler scenarios the behavior by the method provides a smooth solution to the behavior-trajectory planning dilemma.

The main challenge to this thesis was to achieve a more general solution without increasing the complexity too much. Throughout the thesis, many of the problems encountered could be solved by adding more forces taking care of special cases. However, the trivial way of doing this would increase the complexity in behavior analysis and prediction. To avoid this trap, the algorithm has specifically been developed with a small set of force components that maintain invariant statements about safe behaviors.

In the paper *Steering Behaviors of autonomous characters*[2] it is described how the trajectory of a particle in a force field tend to orbit the desired state instead of directly converging[2]. Many of the algorithms used in pursuit, escape and approach algorithms therefore use forcing based on velocity vectors instead of positions. This turns the second degree differential equation, with oscillatory solutions, into first order, giving the problem exponential solutions which then directly reaches the target state without oscillations for all such equations. However, by introducing a damping term to the linear second order equation the solutions to the characteristic polynomial can be real, and thus exponential without oscillations, whenever the damping coefficient is large enough w.r.t the lateral derivative, and the system is said to be *critically damped*, or *overdamped*. By design, the derivatives of the proposed method are bounded and the damping coefficient is analytically determined to avoid any oscillations.

1.4 Scope and Limitations

The purpose of the thesis is to find a dynamic force field on the road as a function of the properties of the road and the objects on it. The force field should be continuous w.r.t. all inputs. It should then be possible to deduce a trajectory from the force field that displays human-like behavior. The behavior should comply with the following specifications:

- Longitudinal, ACC
  - Keep cruise control speed
  - Stay at safe distance
  - Allow merging with other lanes
- Lateral movement within lane
  - Follow road
  - Avoid objects
  - Do not make illegal turn endangering other drivers even if someone enters your lane dangerously.
- lane-changing. to fast lane(+), to slow lane(−), stay(.)
  −. No other objects
  \+ Slow leading vehicle with free fast lane
  . Slow leading vehicle with obstructed fast lane
  . Slow leading vehicle but switching would cause obstruction in other lane
  . Slow leading vehicle but lane priority
  +− Safely Merge into another lane if obstruction on current lane

- Additional Details
    1. No unexpected behaviour: oscillations, accelerating or turning weirdly
    2. Do not decelerate if passing
    3. Comfortable trajectories: Limited Jerk
    4. Never cause a danger to a third part even if a second car brakes safety assumptions.

The algorithm assumes a noise free representation of the required inputs. The inputs include the bounding box, position, velocity and acceleration in the Road Coordinate System of all road-users, and the road curvature at the hosts position.

It also assumes there are actuators capable of yielding the desired acceleration to the vehicle. The simulator used for testing is based on a kinematic model, thus the exact control signals to follow the desired trajectory is calculated and used.

The trajectories generated have low lateral accelerations and are thus feasible for most velocities. However, in traffic jams and similar cases when the velocity of the host reaches 0 the force field may produce infeasible trajectories. The behavior in these situations is not investigated.

The algorithm is designed for highways. The following assumptions are made about the road and other road-users.

- The road consists of one or more forward lanes with the same width,
- All traffic has a longitudinal velocity in forward direction, $v_x \geq v_{min}$ for some unspecified $v_{min} > 0$.

1.5 Thesis Overview

In chapter 2 we describe the Road Coordinate System, mathematical theory regarding solutions to damped piecewise linear force fields, the model of Responsibility-Sensitive Safety(RSS), and define a set of *helper* functions. The mathematical theory is referenced in the results when the algorithm is proven to never allow the host vehicle to switch into the lane of another vehicle *dangerously close*. The RSS defines what *dangerously close* means.

In chapter 3 the artificial force field generation is defined. Each component is defined and the composition method is stated.

In chapter 4 the results are presented. We prove that the host will never switch into the lane of another vehicle dangerously close, and that the host will stop in time to avoid crashing with the vehicle in front from all distances that are not dangerously close. The general behavior of the host is evaluated by subjective analysis in randomly generated traffic scenarios in a simulation environment. The host is able to perform many of the required manoeuvres of a highway chauffeur. The lane switches are slow due to the limited lateral velocity.

In chapter 5 I discuss the results about safety and comfort. The slow lane switches and their implications are discussed along with potential ways to overcome the issue. A section is on how to modify the RSS to define lateral safety w.r.t. lanes. The modified version could potentially be a better framework for what is considered safe behavior on highways.

In chapter 6 I conclude the main points brought up in the paper in a brief summary.

2

Theory

In the beginning of this chapter the road coordinate system in which the forces are defined is presented. By expressing the forces and trajectory in road coordinates the forces can be designed with respect to longitudinal and lateral behavior.

Furthermore, the mathematical theory needed to reason about the trajectory of solutions to the dynamic force fields, is presented.

Lastly, The parts of the formal RSS model[3] that apply to highway driving is presented. The parameters for RSS chosen for this paper are presented and motivated.

2.1 Road Coordinate System, RCS

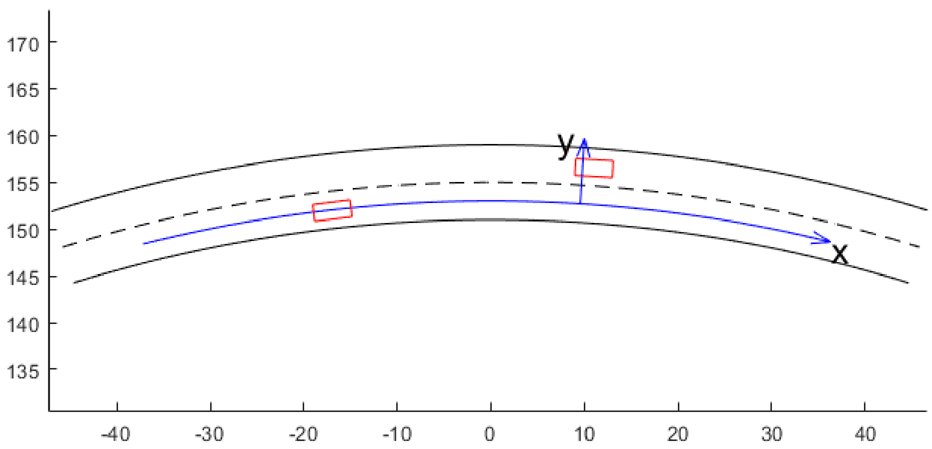


Figure 2.1: The curvilinear road coordinate system reduces the problem to a straight road.

The force field is defined on the curvilinear coordinate system based on the road, with the two components x, the longitudinal direction or forward *along* the road, and y, the lateral direction or *across* the road to the left(the fast lane).

The positions are defined with respect to the center of the bounding box of the

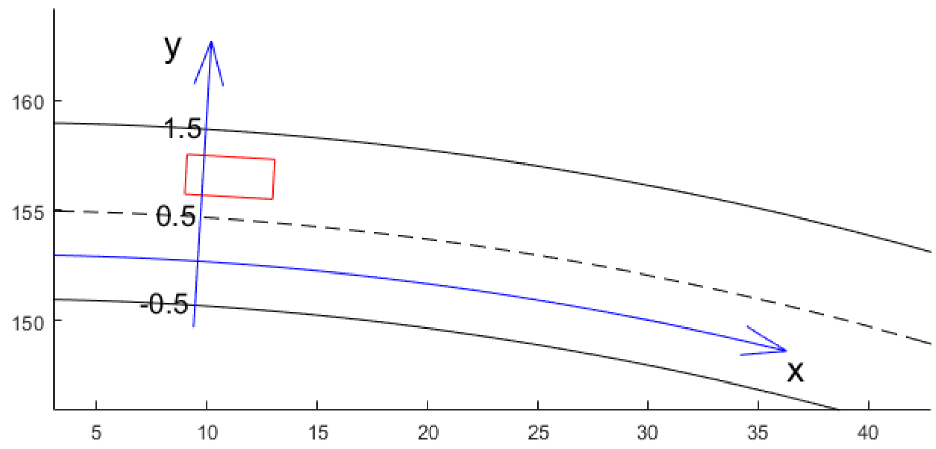


Figure 2.2: By rescaling the lateral position with respect to the lane width, it is trivial to adapt the lateral extent of the force components to position the host correctly in its lane.

vehicle. The length and width of vehicles are also defined w.r.t. this bounding box.

The x-axis is always parallel to the lanes of the road and $x = 0$ denotes the position of the host. An object at the same lateral position as the host with longitudinal position x_o is thus x_o meters ahead of the host on the road.

The contraction effect of the x-axis at lateral offsets due to curvature are ignored due to the low curvature on highways, but could be considered in a more sophisticated model.

The y-axis is at every x orthogonal to the x-axis. The y coordinates are scaled to the lane width so that $y = 1$ is one whole lane to the left. $y = 0$ is the center of some lane. Which one is not important as all functions dependent on lateral position is either periodic with period 1 or determined from positions relative to the host. When clarification is needed, a lateral position can be specified w.r.t. a lane l and then $y = 0$ at the center of lane l.

The velocity and acceleration vectors of all road-users are defined from the velocity in world coordinate system projected onto the road axes. The components are expressed in lateral and longitudinal direction, however the lateral components are still expressed in $[m/s]$ and $[m/s^2]$ and not in $[lanes/s]$ or $[lanes/s^2]$ respectively.

Thus, for an object with position $x(t)$, we have $\frac{\partial x}{\partial t} = v - v^h$, where v and v^h are the longitudinal components of velocity of the object and host respectively. And laterally, $\frac{\partial y}{\partial t} = v/w_{lane}$, where v is the lateral component of the objects velocity and w_{lane} is the width of its lane.

Some of the functions are only dependent on the relative position to the current lane. The notation $\tilde{}$ is used to describe this value and can be calculated by using the modulo operator.

Lane Coordinate - Let y be the lateral position on the road. The corresponding position in the lane is then given by

$$\tilde{y} = (y + 0.5)\%1 - 0.5. \tag{2.1}$$

2.1.1 Road Curvature

To let the lateral force components describe the lateral behavior relative to the road, the centripetal force to follow road curvature is added.

The signed curvature, κ relates to the radius of the circle that approximates the curve to the second degree in a point, that is $\kappa = 1/r$, where r is the signed distance to the epicenter of the circle. When the road is straight the road thus has $\kappa = 0$.

Given a function to transform the coordinates from the road coordinate system to world coordinates, $rcs2wcs \in \mathbb{R}^2 \to \mathbb{R}^2$, and a distance h, The Menger curvature κ of the curve with lateral offset y, in longitudinal position x can be approximated with three points as

$$\begin{aligned}
p_1 &= rcs2wcs(x-h, y), \\
p_2 &= rcs2wcs(x, y), \\
p_3 &= rcs2wcs(x+h, y), \\
v_{1,2} &= p_2 - p_1, \\
v_{2,3} &= p_3 - p_2, \\
v_{1,3} &= p_3 - p_1, \\
\kappa &= \frac{2\|v_{1,2}, \times v_{1,3}\|}{\|v_{1,2}\|\|v_{2,3}\|\|v_{1,3}\|},
\end{aligned} \tag{2.2}$$

where $\times$ is the signed area of the parallelogram spanned by the vectors defined as $x \times y = x_1 y_2 - x_2 y_1$.

2.2 Harmonic Oscillators

The forces of the proposed algorithm are piecewise linear with respect to position on the road at each time point. In each interval it is described by a damped harmonic oscillator, see figure 2.3. Even though the field is changing dynamically, an analysis of the behavior in a stationary environment is useful as the *autonomous*(time independent) system is easier to analyse and can describe the behavior in certain scenarios well enough, such as trailing a leading vehicle. The dynamic system can also be bounded by autonomous systems during certain time intervals, so statements about the trajectory can be made that would have been hard otherwise.

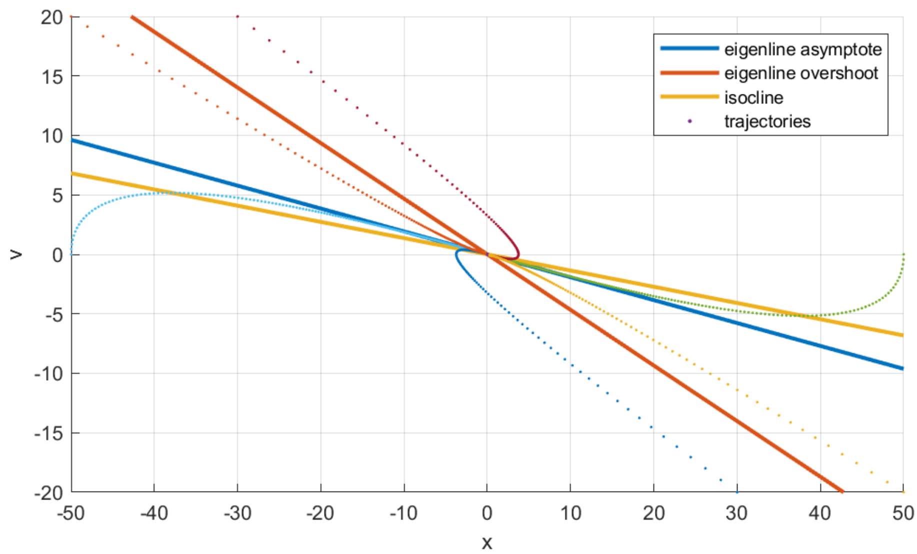


Figure 2.3: The phase space of an overdamped harmonic oscillator, with $\omega = 0.3$ and $\eta = 1.1$ as in the trail component. The solutions pass the equilibrium at most once.

A linear autonomous differential equation describing a force field with damping describes what is called a damped harmonic oscillator. Consider the 1-dimensional second order differential equation, where $\eta > 1$, $\omega > 0$, describing an overdamped harmonic oscillator

$$x''(t) = -2\eta\omega x'(t) - \omega^2 x(t). \tag{2.3}$$

The characteristic polynomial to the initial equation (2.3) has 2 real roots, which allows the solutions to decrease exponentially instead of oscillating that comes with complex roots. The non-oscillatory property of the overdamped oscillator allows to make several assertions about the behavior of the vehicle. As an example, an ACC based on an overdamped oscillator around the desired distance behind a leading vehicle with constant velocity will approach smoothly without oscillating.

The system can be expressed as a system of first order differential equation with two variables $x_1(t) = x(t)$, $x_2(t) = x'(t)$. Calling $x = [x_1, x_2]^T$ it can be written in matrix form as

$$x' = Ax \tag{2.4}$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\eta\omega \end{bmatrix}. \tag{2.5}$$

The equation with initial conditions $x(\tau) = \xi$ has the solution $x = \xi exp(A(t-\tau))$, where exp denotes matrix exponentiation. If A has $n = 2$ linearly independent eigenvectors, v_1, v_2, with corresponding eigenvalues λ_1, λ_2, the expression can be written as

$$x = a_1 v_1 e^{\lambda_1(t-\tau)} + a_2 v_2 e^{\lambda_2(t-\tau)}, \tag{2.6}$$

where $\xi = a_1 v_1 + a_2 v_2$, where a exists and is uniquely determined since v_1, v_2 are linearly independent.

When $\eta > 1, \omega > 0$, the geometric multiplicity of A is 2. The eigenvalues λ_1, λ_2 with corresponding eigenvectors v_1, v_2 are given as

$$\lambda_1 = -w(\eta - \sqrt{\eta^2 - 1}), \tag{2.7}$$

$$\lambda_2 = -w(\eta + \sqrt{\eta^2 - 1}), \tag{2.8}$$

$$v_1 = [\frac{\lambda_2}{\omega^2}, 1]^T, \tag{2.9}$$

$$v_2 = [\frac{\lambda_1}{\omega^2}, 1]^T. \tag{2.10}$$

The position over time x_1 with the initial position and velocity x_0, v_0 at time $\tau = 0$ is then given as

$$x_1(t) = ae^{\lambda_1 t} + be^{\lambda_2 t}, \tag{2.11}$$

where

$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{\lambda_2 - \lambda_1} \begin{bmatrix} \lambda_2 & -1 \\ -\lambda_1 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ v_0 \end{bmatrix}. \tag{2.12}$$

Since the $\lambda_2 << \lambda 1$ the b-term goes to zero faster than the other. The solution can overshoot the equilibrium at most once, and it only happens if $\|b\| > \|a\|$ and a and b have different signs. Specifically, if $v_0 = 0$ then $\|a\| \geq \|b\|$ and so it will not overshoot the equilibrium, which is proven in the following lemma.

Lemma 1. *Let x_0 be the initial position. If the initial velocity $v_0 = 0$, the entire future orbit of the trajectory will be inside the interval between x_0 and 0.*

The proof is done by showing that the solution has a lower(upper) bound of 0 and monotonically decreasing(increasing), when x_0 is positive(negative).

Proof. If $v_0 = 0$, the values for a and b are given as

$$a = \frac{\lambda_2}{\lambda_2 - \lambda_1} x_0, \tag{2.13}$$

$$b = \frac{-\lambda_1}{\lambda_2 - \lambda_1} x_0. \tag{2.14}$$

Note first that $\|a\| \geq \|b\|$. Secondly, since $\lambda_2 < \lambda_1$, then $e^{\lambda_2 t} < e^{\lambda_1 t}$ for all $t > 0$. Also note that $a\lambda_1 + b\lambda_2 = 0$.

Consider first $x_0 \geq 0$, then $a \geq 0, b \leq 0$ and $a + b \geq 0$. Then the position is non-negative for all future times

$$x_1(t) = ae^{\lambda_1 t} + be^{\lambda_2 t} \geq (a + b)e^{\lambda_1 t} \geq 0. \tag{2.15}$$

The velocity is non-positive for all future times, since

$$x_2(t) = x_1'(t) = a\lambda_1 e^{\lambda_1 t} + b\lambda_2 e^{\lambda_2 t} \tag{2.16}$$

$$\leq (a\lambda_1 + b\lambda_2)e^{\lambda_1 t} \tag{2.17}$$

$$= 0. \tag{2.18}$$

□

The trajectory for $x_0 \leq 0$ follows symmetrically and is hence omitted.

Notice how the results above are valid for other equilibriums than 0

$$x' = A(x - a), \tag{2.19}$$

since we can make a substitution $\xi = x - a$, then $\xi' = A\xi$ and $x = \xi + a$.

2.2.1 Bounded Trajectories of Dynamic Systems

To be able to form statements about the dynamically changing systems we can find autonomous systems that bound the dynamic system during certain time intervals. The lateral force equations will describe a continuous piecewise linear force at each time point. The derivative of each line segment is bounded by δ_{max} and the damping coefficient is set to make the system overdamped in all points, $k_{damping} = 2\eta\sqrt{\delta_{max}}$ for some $\eta > 1$. The force is also saturated at maximal accelerations of A_{max}, which bounds the lateral velocity.

Consider such a system dependent on time t

$$f(t, y) = interp([y_1..y_n](t), [f_1..f_n](t), y) \tag{2.20}$$

$$y'' = -2\eta\sqrt{\delta_{max}} y' + f(t, y). \tag{2.21}$$

where *interp* is the 1-dimensional linear interpolation function defined in (2.36), the spacial derivative of each line segment is bounded $\left\| \frac{f_{k+1} - f_k}{y_{k+1} - y_k}(t) \right\| < \delta_{max}$, and the force is saturated $\|f_k(t)\| < A_{max}$ for all k and t.

If for all t in some interval $I = (0, a)$, the force is above a certain value f_+ in some point y_+, $f(t, y_+) \geq f_+ > 0\, \forall t \in I$, the function $f(t, y)$ can be bounded from below for all $t \in I$ by a time independent function $g(y)$ which decreases from f_+ with rate δ_{max} on both sides of y_+ until reaching the lower bound $-A_{max}$. A visualization of $g(y)$ can be seen in figure 2.4.

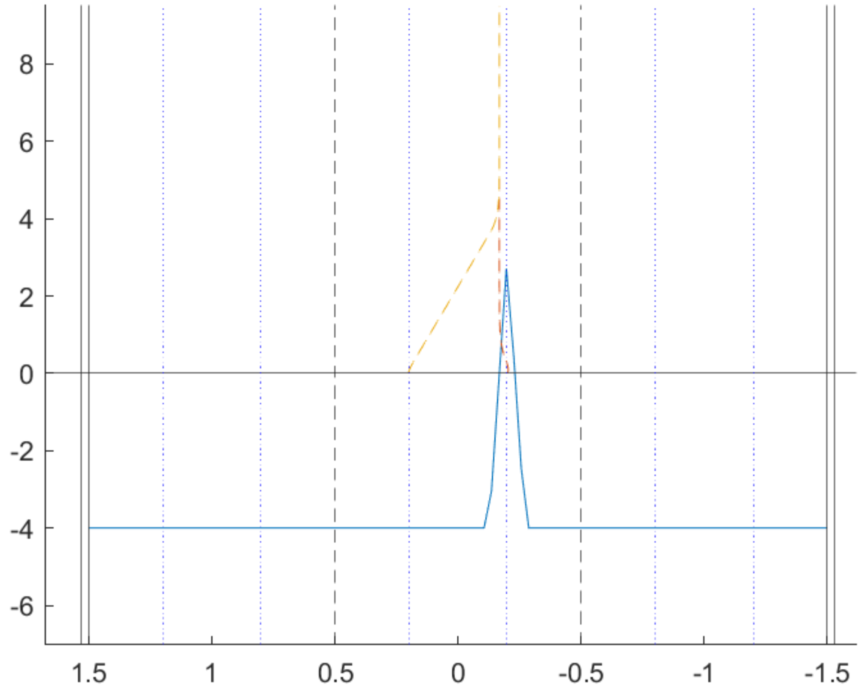


Figure 2.4: The solution to the lower bound system stays to the left of the positive region and so the actual solution must as well. The horizontal axis marks lateral position on the road. The vertical axis marks lateral acceleration for the force and time for the trajectories. The peak of of the lower bound force happens at $y_+ = -0.2$

$$g(y) = \max(-A_{max}, f_+ - \delta_{max}(\|y_+ - y_{auto}\|)) \tag{2.22}$$
$$f(t, y) \geq g(y) \qquad \forall t \in I. \tag{2.23}$$

Consider the autonomous system

$$y'' = -2\eta\sqrt{\delta_{max}}y' + g(y). \tag{2.24}$$

The solution to the actual system (2.21) $y(t)$, will be larger than the solution to the system (2.24) $y_{auto}(t)$, for the same initial conditions $y(0) = y_{auto}(0), y'(0) = y'_{auto}(0)$, and times $t \in I$.

Lemma 2 (Bounded Solutions)**.** *The solution to the actual system* (2.21)*, will be larger than the solution to the system* (2.24)*, for the same initial conditions* $y(0) = y_{auto}(0), y'(0) = y'_{auto}(0)$*, and times* $t \in I$*.*

Proof. Let $y(t)$, $y_{auto}(t)$ denote the solutions to (2.21) and (2.24), with intial conditions $y(0) = y_{auto}(0), y'(0) = y'_{auto}(0)$. Consider the function $d(t) = y(t) - y_{auto}(t)$. By linearity of differentiation $d(t)$ must fulfil the differential equation

$$\begin{aligned} d'' &= y'' - y''_{auto} && (2.25) \\ &= f(t, y) - g(y) && (2.26) \\ &\geq 0. && (2.27) \end{aligned}$$

And so, by integrating twice, $d(t)$ can be expressed as

$$d'(t) = d'(0) + \int_0^t d''(\tau)\mathrm{d}\tau \geq d'(0) \tag{2.28}$$

$$d(t) = d(0) + \int_0^t d'(\tau)\mathrm{d}\tau \geq d(0) + d'(0)t. \tag{2.29}$$

Since the initial conditions are given by

$$d(0) = y(0) - y_{auto}(0) = 0 \tag{2.30}$$

$$d'(0) = y'(0) - y'_{auto}(0) = 0, \tag{2.31}$$

we get that $d(t) \geq 0$ and so $y(t) = d(t) + y_{auto}(t) \geq y_{auto}(t)$. □

Correspondingly if there is a time-independent function that bounds the force from above $g(y) \geq f(t, y) \forall t \in I$, we can see that the solution to the actual system will be less than the autonomous system for the same initial conditions.

For the autonomous system of the lower bound, if $y(0) > y_+$, then $y(t) > y_+ \, \forall t \in I$, if at time $t = 0$ the velocity is at least $y'(0) \geq 0$, as can be seen in figure 2.4. These properties are used to prove that the host will never engage in a lane switch to an occupied lane dangerously close to other vehicles, see lemma (5).

2.3 Responsibility-Sensitive Safety

Responsibility-Sensitive Safety[3] defines a formally safe model for a multi-agent environment. Since it is possible for other road-users to suddenly start to ram the host vehicle, achieving absolute safety is futile. Instead the model makes assumptions on traffic behavior and driver capabilities and defines a protocol for safe behaviors, so that the fault of all accidents can be put on the vehicles braking the protocol. Given all road-users respect the protocol, no accidents can occur.

The proposed algorithm does not fulfil the exact protocol of RSS. Nevertheless, the longitudinal safety is maintained, and the algorithm will respect lateral safety if other vehicles do not enter the hosts lane with an unsafe longitudinal distance.

2.3.1 Responsibility-Sensitive Safety

As discussed in the paper *On a Formal Model of Safe and Scalable Self-driving Cars*[3], any practical model describing safety must satisfy the following 2 aspects:

1. **Soundness:** when the model says that the self-driving car is not responsible for an accident it should clearly match the "common sense" of human judgement. Note that we allow the model to assign responsibility on the self-driving car in fuzzy scenarios, possibly resulting in extra cautiousness, as long as the model is still useful.
2. **Usefulness:** it is possible to efficiently create a policy that guarantees to never cause accidents while still maintaining normal flow of traffic.

The formal model described in the paper is called a Responsibility-Sensitive Safety(RSS), and is described in terms of the algorithm(called Policy in the paper) that abides to it. The model includes protocol for safe behavior in various settings including crossings, occluded pedestrians and unstructured environments such as parking lots. In the highway setting however only the protocol for safe longitudinal and lateral distance with same direction traffic is needed.

The formal model of Responsibility-Sensitive Safety on highways defines safe longtudinal and lateral distances and a protocol for behavior with respect to the distances that can guarantee safety. The model sets up a minimal required response to dangerous situations and calculates the required distances to safely actuate the response, assuming a *worst case scenario*. By placing the blame of an accident on the vehicle not conforming to the minimal required response it has thus defined a responsibility-sensitive safety.

It should be noted that there are possible *worse* cases than the assumed *worst case*. For example the worst case scenario for longitudinal safe distance is based on the front vehicle actuating an upper bound on the car's braking capabilities in current weather conditions. The acceleration of the front vehicle could be lower if it for example would hit a truck standing still. It is thus the responsibility of the front vehicle, in this case, to never cause a *worse case*. This way of defining *worst case* is necessary to consider normal traffic behavior safe, which is necessary for the model to be useful if the road includes both human and autonomous drivers.

All distances are defined w.r.t. the nearest edges of the bounding box of the vehicle in RCS.

2.3.2 RSS: Longitudinal Safety

In the Responsibility-Sensitive Safety model the longitudinal safe distance is defined with respect to a longitudinal emergency manoeuvre. The front vehicle can at any time brake with its maximal braking deceleration, b_{max}. The rear vehicle must then react within time ρ and respond by braking with at least b_{min}.

The value for b_{max} should be a justified *upper bound* on the braking capabilities of the car. It can be estimated dependent on weather conditions, car model, etc. It is then the responsibility of the rear vehicle to stay at a distance that allows it to react and brake to stop in time. The distance required to brake is estimated with a *lower bound* on the braking capabilities, b_{min}, of the car in current road conditions, and is also not dependent on the driver.

The behavior during the reaction time is harder to estimate as it *is* dependent on the driver. In the paper it is based on two parameters, a duration called *reaction time* ρ, as well as a maximal longitudinal acceleration a_{max}. As the subject of driver intention modelling is outside the scope of this thesis, the acceleration is based on the Cruise Control Component and the reaction time for other road-users are set as conservative as possible to still allow merging under normal headway distances.

The parameters can be different for different agents. The autonomous vehicle can have a low ρ, and heavy vehicles could be given a less demanding brake response b_{min}, etc. All parameters used during testing and simulation are listed below.

- $\rho = 0.2[s]$ - The reaction time of the host,
- $\rho_o = 0.5[s]$ - the reaction time of other road-users,
- $a_{max} = 2[m/s^2]$ - The maximal acceleration during reaction time,
- $b_{min} = 6.9[m/s^2]$ - The minimal braking response of the host,
- $b_{max} = 7[m/s^2]$ - The maximal braking response of the host,
- $b_{o,min} = 6.5[m/s^2]$ - The minimal braking response of other road-users,
- $b_{o,max} = 7.5[m/s^2]$ - The maximal braking response of other road-users.

RSS Definition 1 (Safe Longitudinal Distance - same direction)**.** *A longitudinal distance between a car c_r that drives behind another car c_f, where both cars are driving in the same direction, is considered safe w.r.t. a response time ρ if for any braking of at most b_{max} performed by c_f, if c_r will accelerate by at most a_{max} during the response time, and from there on will brake by at least b_{min} until a full stop then* *it won't collide with c_f. The minimal safe longitudinal distance is thus given as*

$$d_{min}(v_r, v_f) = \left[v_r\rho + \frac{a_{max}\rho^2}{2} + \frac{(v_r + a_{max}\rho)^2}{2b_{min}} - \frac{v_f^2}{2b_{max}} \right]_+ . \tag{2.32}$$

The unsafe area is the bounding box in the road coordinate system of the minimal safe longitudinal and lateral distances. The protocol for behaviour depends on whether the unsafe area is entered from the side or from behind. Not only must a vehicle brake in time to avoid collision, but it must never cut in closer behind or in front of another car than the safe longitudinal distance. Put more formally, a vehicle must comply with the protocol for safe lateral behavior upon coming to an unsafe lateral distance to a vehicle with an unsafe longitudinal distance.

RSS Definition 4 (Proper Longitudinal Response)**.** *If a vehicle c_r enters the unsafe area of another car c_f from behind at time $t = 0$, c_r can have any acceleration of at most a_{max} during the time interval $[0, \rho)$ and then apply a braking deceleration of at least b_{min} until at a safe longitudinal distance or standing still to comply, the front vehicle is to not to exceed a braking deceleration of b_{max}.*

It is clear that, for any vehicle, $b_{max} > b_{min}$ since it would not be able to comply with the braking requirements with a trailing vehicle behind it otherwise.

2.3.3 RSS: Lateral Safety

The protocol for safe lateral distance is defined by the distance required to perform a lateral emergency manoeuvre. The lateral emergency manoeuvre is based on reducing lateral velocity until driving straight. The parameters are the same as for longitudinal safety, but the equation describing the distance is different as the velocity can be either positive or negative. The distance to perform the emergency manoeuvre assumes a maximum lateral acceleration a_{max}, a reaction time ρ and a minimum lateral braking response b_{min}.

The safe lateral distance is more easily defined with an additional definition of stop deviation not present in the paper.

RSS Definition (Lateral Stop Deviation)**.** *Let v be the lateral velocity of a car, the outmost possible lateral deviation to stop, $\Delta y_{max}, \Delta y_{min}$ w.r.t. parameters ρ, a_{max}, b_{min} is given as*

$$\Delta y_{max}(v) = (v + \frac{a_{max}\rho}{2})\rho + \frac{max(0, v + a_{max}\rho)^2}{2b_{min}} - \frac{min(0, v + a_{max}\rho)^2}{2b_{min}}, \tag{2.33}$$

$$\Delta y_{min}(v) = (v - \frac{a_{max}\rho}{2})\rho + \frac{max(0, v - a_{max}\rho)^2}{2b_{min}} - \frac{min(0, v - a_{max}\rho)^2}{2b_{min}}. \tag{2.34}$$

The lateral manoeuvre requires an additional parameter μ which represents an upper bound for the lateral fluctuations when driving *straight*.

RSS Definition 7 (Safe lateral distance)**.** *The lateral distance between cars* c_1, c_2*, driving with lateral velocities* v_1, v_2 *is considered safe w.r.t. parameters* ρ, a_{max}, b_{min} *if during the time interval* $[0, \rho)$ *the two cars accelerate laterally with* a_{max} *toward each other, and after that the two cars will brake laterally with* b_{min} *until they reach a lateral velocity of* 0*, then the final lateral distance between them is at least* μ*. The minimal safe lateral distance,* d_{min}*, between a car* c_l *with lateral velocity* v_l *to the left of a car* c_r *with lateral velocity* v_r*, is given by*

$$d_{min}(v_r, v_l) = 2\mu + [\Delta y_{max}(v_r)]_+ + [\Delta y_{min}(v_l)]_- \,. \tag{2.35}$$

Due to symmetry both vehicles always enter the unsafe lateral distance of each other simultaneously and are thus both constrained by the proper lateral response.

RSS Definition 9.2 (Proper Lateral Response)**.** *If two cars,* c_l, c_r*, enters the unsafe area of one another from the sides at time* $t = 0$*, where* c_l *is to the left of* c_r*, during time interval* $[0, \rho)$*, each car can have a lateral acceleration,* a *s.t.* $\|a\| \leq a_{max}$*, after which* c_l *has to apply a lateral acceleration of at least* b_{min} *until its velocity* $v_l \geq 0$*, and* c_r *has to apply a lateral acceleration of at most* $-b_{min}$ *until its velocity* $v_r \leq 0$*. After reaching a velocity of* 0 *any acceleration away from the other vehicle is allowed,* $a_l \geq 0$*,* $a_r \leq 0$*, until they are at a safe lateral distance.*

It is proven in the paper that, if all road-users comply to the behavior protocols in definitions (4) and (9.2), no accidents can occur[3].

2.3.4 Discussion on Parameters

Even though the parameters could be set with a conservative mindset, a long reaction time, low required response, etc., it is important that the parameters are chosen so that normal traffic behavior is considered safe.

Consider for example a human driver cutting in front of the host vehicle, if the parameters are too conservative it might force an emergency braking response by the host, which may cause an accident if the vehicle behind does not respond in time. The formal model will assign the blame to the vehicle behind that does not brake, however, human judgement would assign the blame to the host for unnecessary braking. To avoid any emergency response to normal traffic behavior, the host has to have parameters set with low reaction time and high minimum braking response.

Many necessary manoeuvres, such as merging from an on-ramp, will also not be possible if the parameters for other road-users are too conservative. A normal headway on a highway can be as low as $1.5s$, thus the safe distance to the vehicle behind, in front and the length of the host vehicle has to be less than that to perform a merge. As an example, with host and object vehicles travelling at $110[km/h]$ this would yield a safe distance of $12.4m$ behind other vehicles and $24m$ ahead of them. This would leave a gap of $9.4 - l$, where l is the length of the host vehicle, to perform and merge if the two objects in the lane have a distance corresponding to a $1.5s$ headway.

If the rear vehicle has a higher velocity than the front vehicle, the safe longitudinal distance does in no way represent a *comfortable* cut-in distance, as the rear vehicle will enter the unsafe region directly after the cut-in and would be required to perform emergency braking. The RSS is a sound model for what *can* be done, not what is optimal. The behavior of a highway chauffeur should therefore make sure that all vehicles can comfortably avoid entering the safe distance if no strong accelerations are occuring.

Since some of the parameters describe guessed *worst case* scenarios of what *could* happen, it makes sense to

2.4 Linear Ramping Functions

To ensure the forces are continuous, each component is continuous and defined for all positions. To limit the range of the components, their effect is linearly decreased at desired longitudinal and lateral distances. A few special cases of continuous piecewise linear functions are given names as they have a clear intuitive meaning and are used to describe many of the components.

Some functions are defined with linear interpolation. The notation used is $interp \in [D] \to [R] \to D \to R$, where the list of domain values are strictly increasing. Defined as

$$interp([x_1, x_2...x_n], [y_1, y_2...y_n], x) = \begin{cases} y_1, & x \leq x_1 \\ y_k + a(y_{k+1} - y_k), & x_k \leq x \leq x_{k+1}, \\ a = (x - x_k)/(x_{k+1} - x_k) & \\ y_n, & x \geq x_n \end{cases}. \tag{2.36}$$

Many values are saturated to appropriate ranges. The saturation function has many different names, but in this thesis it uses the name *clip* as the function is called in numpy.

Clip Function - Let a and b denote the lower and upper saturation bounds, the clip function is then defined as

$$clip(x, a, b) = min(max(a, x), b). \tag{2.37}$$

The lateral force components can be decreased from full effect to 0 and using the trapezoid function.

Trapezoid Function - Let a and b be parameters, the trapezoid function is then defined as

$$trapezoid(x, a, b) = clip(1 - (x - a)/(b - a), 0, 1). \tag{2.38}$$

Some functions only depend on the position in the lane and must thus be periodic with period 1. They are described using the continuous triangle function.

Triangle Function - let $a \in [0, 0.5]$ be a parameter. The triangle function is then given by

$$triangle(x, a) = max(0, min(\tilde{x}/a, 1 - (\tilde{x} - a)/(0.5 - a))) \tag{2.39}$$

$$- max(0, min(-\tilde{x}/a, 1 - (-\tilde{x} - a)/(0.5 - a))). \tag{2.40}$$

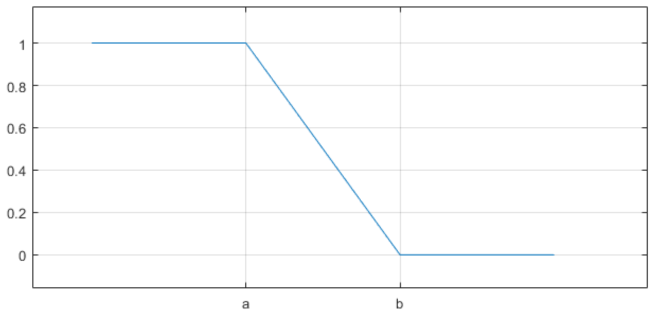


Figure 2.5: The trapezoid function is used to limit the range of influence for lateral force components in a continuous manner.

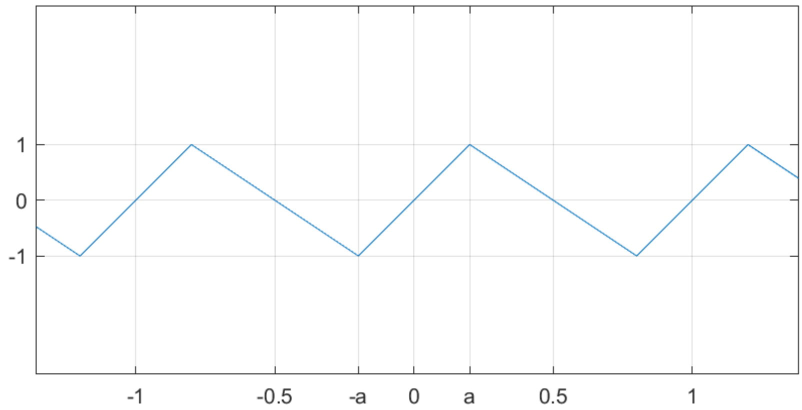


Figure 2.6: The triangle function is used to describe functions that only depend on the position in the lane. The function is anti-symmetric in lane, reaching its maximum value at $\pm a$.

Even the longitudinal force components for braking need to allow accelerations outside of their range of influence and thus the value must change sign. The drop function with parameters enables this and is defined as

$$drop(x, a, b) = min(1, 1 - (x - a)/(b - a)). \tag{2.41}$$

When the value is ramped down both longitudinally and laterally, it is scaled by the minimum of the ramping values. For example, say a component f_{ex} is in full effect with an amplitude of a in the area of full effect $(x, y) \in [-5, 0] \times [-1, 1]$. Let the lateral ramping distance be 0.2 and the longitudinal ramping distance be 1. The component is then given by $f_{ex} = a \min(k_x, k_y)$ where

$$k_x = min(trapezoid(x_o, 5, 6), trapezoid(-x_o, 0, 1)), \tag{2.42}$$

$$k_y = min(trapezoid(y_o, 1, 1.2), trapezoid(-y_o, 1, 1.2). \tag{2.43}$$

The the result of f_{ex} for different positions around the object is seen in figure 2.8.

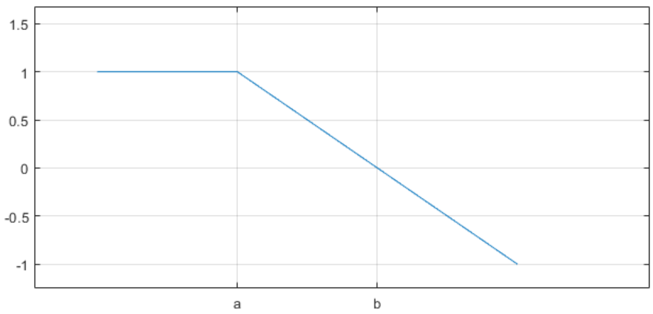


Figure 2.7: The drop function is used to limit the range of influence for longitudinal force components in a continuous manner.

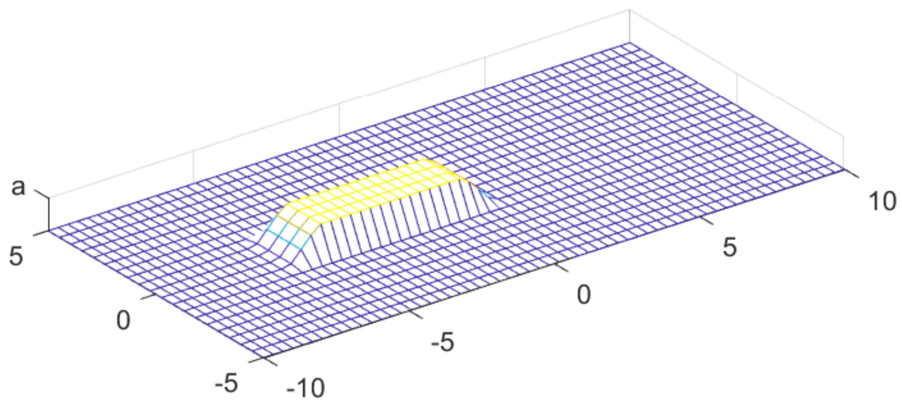


Figure 2.8: The linear ramping is applied in two dimensions by taking the minimum of the ramping coefficients. The region where both coefficients are 1 is called the area of full effect.

2.5 Analytical solutions to differential equations

There are two distinct analytical types of solutions to the differential equation

$$x'' = clip(k(v_{des} - x'), a_{min}, a_{max}). \tag{2.44}$$

If the initial velocity $x'(0) = v_0$ is outside of the convergence range

$$V_{conv} = [v_{des} - a_{max}/k, v_{des} - a_{min}/k], \tag{2.45}$$

there will be a constant acceleration for some time, as the trajectory approaches the convergence range, after which the velocity will exponentially approach v_{des}.

the time the acceleration stays at a_{max} is then given by

$$t_{a,max} = max(0, (v_{des} - a_{max}/k - v_0)/a_{max}). \tag{2.46}$$

The corresponding time for a_{min} is

$$t_{a,min} = max(0, (v_0 - (v_{des} - a_{min}/k))/a_{max}). \tag{2.47}$$

A closed form for the time spent outside of the convergence region is given by

$$t_{full} = max(t_{a,max}, t_{a,min}). \tag{2.48}$$

The predicted velocity and distance travelled during this time is used to determine the appropriate distance to begin a lane switch to pass in time to avoid reducing acceleration.

Host Prediction - Let v and v_{des} denote the hosts current and desired velocity, respectively. The hosts predicted velocity t seconds in the future $v_{cc}(t)$, and distance travelled during time t, $s_{cc}(t)$ are then given by

$$\begin{aligned} v_1(t) &= v + a_{max} \min(t, t_{a,max}) + a_{min} \min(t, t_{a,min}), \\ v_{cc}(t) &= v_{des} + (v_1(t) - v_{des}) e^{-k \max(0, t - t_{full})}, \end{aligned} \tag{2.49}$$

$$\begin{aligned} s_1(t) &= \frac{v + v_1(t)}{2} \min(t, t_{full}), \\ s_{cc}(t) &= s_1(t) + v_{des} \max(0, t - t_{full}) + \frac{v_1(t) - v_{des}}{k}(1 - e^{-k \max(0, t - t_{full})}). \end{aligned} \tag{2.50}$$

3

Proposed Highway Chauffeur Algorithm

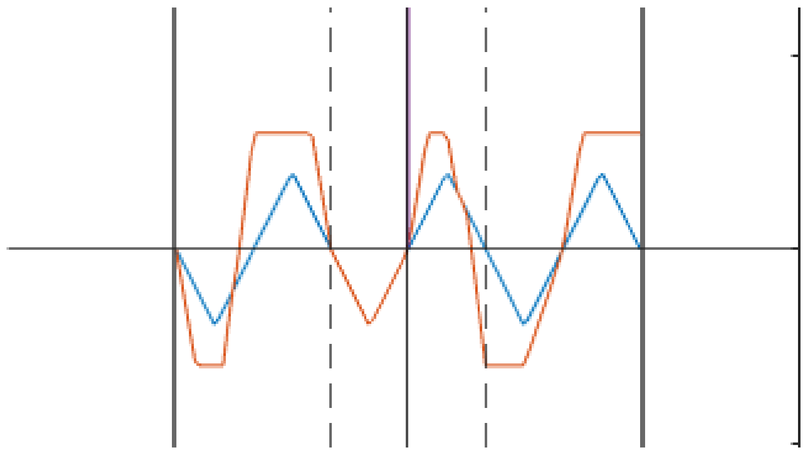


Figure 3.1: The lateral force on the different positions on the road. In blue, the Lane Component. In orange, the final lateral force by composition of the lane, preference, pass and No-Cut Components in a certain scenario.

The proposed method for a safe and comfortable highway chauffeur consists of determining a longitudinal and lateral acceleration as a function of the surrounding environment. The acceleration is determined from different components and way of composing them into a *force* before deducing controls. Inspired by Artificial Potential Fields for autonomous driving[1], the idea is to solve the complex problem of behaviour and trajectory planning by designing solutions to simpler subproblems such as LKA and ACC and composing them. The components and the composition method are designed to enable formal assertions of the final behavior.

The road coordinate system described in section 2.1 facilitates the algorithm design and the ability to prove assertions of safety of the behavior. The trajectories are developed indepedently along the road, longitudinally, and across the road, laterally. This division makes sense since the velocities and accelerations in the different directions are of different magnitudes. The force in each of these dimensions are determined by a composition of simpler components, and different damping.

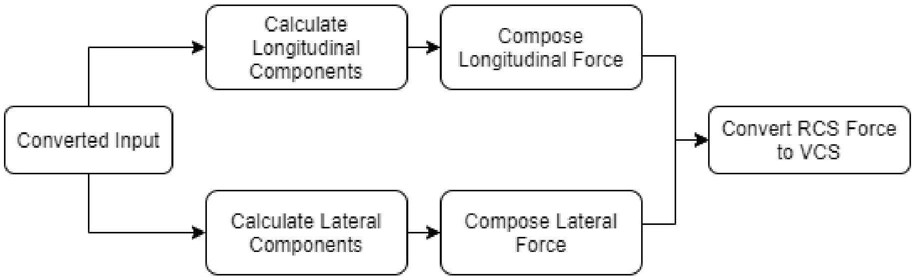


Figure 3.2: A high-level description of the algorithm.

The components themselves are not forces, as the composition method will show. Longitudinally they are rather an upper bound on acceleration. Laterally the composition is more sophisticated. After the final composition however, the force gives an acceleration vector used to describe the trajectory.

The components can be divided into two groups, road components and object components. The road components are dependent on the properties of the road and the hosts position on it. The object components are dependent on the position, velocity and acceleration vectors of other road-users. They are intended to allow safe distance keeping, lane switching and general obstacle avoidance. All designed components are listed below.

Longitudinal

1. Cruise Control Component - A component which allows the vehicle to stay at the desired velocity.
2. Trail Component - A component that allows the vehicle to keep a safe distance to vehicles ahead.
3. High Curvature - A component to slow down so that the maximum lateral force to follow road curvature never exceeds $a_{y,max}$.

Lateral

1. Curvature component - A centripetal road component which allows the trajectory to follow the curvature of the road.
2. Lane Component - A weak road component which pulls the trajectory to the center of a lane and away from the lines.
3. Preference Components - Weak or strong road components used to return to the slow lane after passing or to only enter lanes acceptable by route input.
4. Pass Component - An object component that enables passing slower vehicles or obstacles.
5. No-Cut Component - A strong object component that prohibits lane changing too close in front or behind the vehicle.
6. Damping Component - A damping component that limits lateral movement and prohibits oscillations in lane.

The input to the algorithm comes from the state of the environment, as well as driver input and route input. The state of the environment should be expressed using the road coordinate system(RCS) described in section 2.1. The driver and route inputs could be set as parameters instead.

Inputs

- Road
  - host position, x,
  - curvature, κ,
  - lane width, $w = 3.8$.
- Objects
  - position in RCS, x, y,
  - bounding box in RCS, (l, w),
  - velocity in RCS, v,
  - longitudinal component of acceleration in RCS, a.
- Driver input
  - Set desired speed, v_{des},
  - Set desired headway, t_{des}.
- Route input
  - Preferred lane, l_{pref},
  - Leftmost acceptable lane, l_{left},
  - Rightmost acceptable lane, l_{right}.

There is a set of parameters for the algorithm. Some values are free to vary while others must respect certain inequalities.

Parameters

- Cruise Control Component
  - $k = 0.7$ - scale parameter,
  - $a_{min} = -2 < 0$ - $[m/s^2]$ maximum brake deceleration,
  - $a_{max} = 2 > 0$ - $[m/s^2]$ maximum acceleration.
- Trail Component
  - $\omega = 0.3$ - scale parameter oscillator,
  - $\eta = 1.1 > 1$ - damping parameter oscillator,
  - $margin = 5$ - $[m]$ minimum distance to vehicle ahead,
  - $b_{max} = 7$ - $[m/s^2]$ maximum braking deceleration.
- Sharp Turn Component
  - $a_{y,max} = 3$ - $[m/s^2]$ highest comfortable centripetal acceleration to follow

road curvature.

- Lateral Extent
  - $\Delta y_{bias} = 0.2 \in (0, 0.25)$ - acceptable deviation from lane center called *bias leeway*,
  - $v_\mu = 0.2$ - $[m/s]$ highest lateral velocity without intent of lane switching,
  - $v_{min,switch} = 0.3$ - $[m/s]$ velocity signifying intent of lane switching.
- Longitudinal Extent
  - $\Delta x = 2$ - minimal longitudinal ramping distance.
- Auxiliary Components
  - $A_{max} = 4 > A_{lane}$ - $[m/s^2]$ maximum lateral acceleration.
- Lane Component
  - $A_{lane} = 3$ - $[m/s^2]$ maximum acceleration from lane component.
- Pass Component
  - Δv_{pass} - $[m/s]$ velocity difference to allow passing,
  - $t_{switch} = 5$ - $[s]$ approximate time to switch lanes,
  - $t_a = 4$ - $[s]$ object behavior parameter,
  - $v_{min,switch} = 0.5$ - $[m/s]$ object behavior parameter.
- Lateral Damping
  - $\eta_{lat} = 1.1 > 1$ - redundancy in lateral damping.

3.1 Limiting Extent of Components

To avoid discontinuities, all components are defined for all positions. To limit the area of effect or *extent* of the components to the appropriate ranges, we utilise the linear ramping functions described in section 2.4. For example, the braking effect of the Trail Component should only affect the host if the vehicle is in the same lane and in front of the host. This is assured by ramping up the component as the lateral distance is far enough or the vehicle's longitudinal position gets negative.

The lateral extents of the components are based on the lanes of the road. The host should stay near the lane center when not switching lanes. The invariant used to assure safety is tightly coupled with the bias leeway parameter, Δy_{bias}. The parameter represents the maximal lateral deviation from the lane center where the host may stay when not switching lanes.

Bias Region - Let $y_{lane,center} \in \mathcal{Z}$ be the lateral position of a certain lane l. The bias region of the lane l is then given by

$$Y_{bias} = [y_{lane,center} - \Delta y_{bias}, y_{lane,center} + \Delta y_{bias}]. \tag{3.1}$$

Since all vehicles are allowed to move freely within the bias region it is important that the value selected for the bias leeway, Δy_{bias}, ensures enough distance to the line so the entire bounding box of the vehicle is in its lane. In section 5.2 the Lateral Stop Deviation from stand still, $\Delta y_{max}(0)$, is defined. Given a lane width of w_{lane} and a car width of w_{cars}, an upper bound on Δy_{bias} is set as $(1 - 2\Delta y_{bias} - w_{cars}/w_{lane} - 2\Delta y_{max}(0)/w_{lane} > 0) \implies \Delta y_{bias} < 0.5 - w_{cars}/w_{lane}$. Throughout this thesis the value for the bias leeway is $\Delta y_{bias} = 0.2$.

The lateral force component design is one of the key achievements of the algorithm. The lateral extent of the lateral components are designed to ensure two key properties of the final composed lateral force field:

- The size of the lateral derivative is bounded

$$\left\| \frac{\partial f}{\partial y} \right\| \leq \delta_{max}, \tag{3.2}$$

- There exists no stable equilibrium outside of the bias region if all other trafficants remain within their bias region with low lateral velocity

$$\begin{aligned} \forall y \notin Y_{bias} \qquad & \text{either} \\ f(y) \neq 0 \qquad & \text{or} \qquad (3.3) \\ \frac{\partial f}{\partial y}(y) > 0. \qquad & \qquad (3.4) \end{aligned}$$

The bounded derivative allows to set a lateral damping component which ensures the trajectory never goes past a stationary stable equilibrium point. The bounded derivative is ensured by ramping up and down all functions over the distance Δy_{bias}, and by a special method of composition.

By ensuring the lateral derivative is positive outside of the bias region, there can exist no stable equilibrium there. The road components are thus designed to have a positive, or non-negative, lateral derivative in this region. The object components should not break this invariant. The object components pushing into the next lane must thus reach into the bias region before declining. The lateral range function, $\Delta y_{range}(y, v)$, is a continuous function dependent on the objects lateral position in lane and velocity that enables this. All object components calculate their lateral range using the lateral range function, the left range is given by $\Delta y_{range}(y, v)$, while the right range follows by anti-symmetry as $\Delta y_{range}(-y, -v)$.

The two lateral object components, the Pass Component and the No-Cut Component, both extend into the adjacent lane in some direction and their extent thus make use of the lateral range function. The left range, shown in figure 3.3, is 1 at a lateral position of $-\Delta y_{bias}$ to reach to the beginning of the adjacent bias region, while it is $1 - \Delta y_{bias}$ at the lateral position Δy_{bias} to be able to ramp down with a limited derivative before reaching the end of the bias region, as displayed in figure 3.4. At position 0.5, when an object is on a line, the range is the longest reaching $1.5 - \Delta y_{bias}$ which prohibits the host from initiating a switch to any of the two lanes the vehicle is currently occupying.

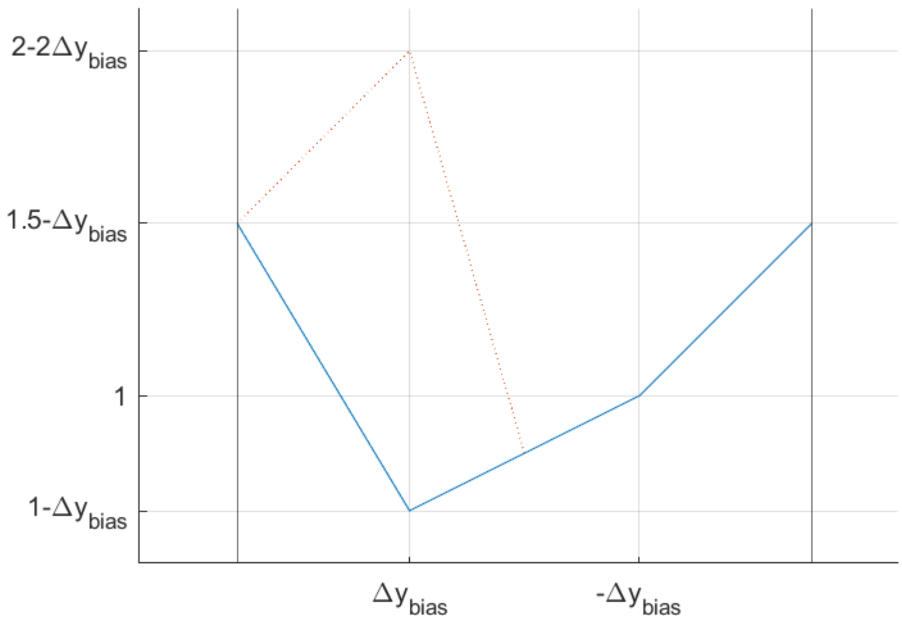


Figure 3.3: The left range of full effect for object components depending on positions in lane as to ensure the component is constant outside of the bias region whenever the vehicle is in the bias region. In blue, the range with lateral velocity $v \leq v_{\mu}$, in orange the range with lateral velocity $v \geq v_{min,switch}$. The range to the right is defined symmetrically.

The final function is the piece-wise linear continuous periodic interpolation between the points, with an additional distance with respect to lateral velocity $\Delta y_{v,range}$. The host will avoid switching lanes as soon as another vehicle initiates a lane switch from the other side as the lateral extent of the vehicle is increased with lateral velocity in a certain region.

The increase of lateral range should not cause uncomfortable behavior by small oscillations of an object in the adjacent lane. The range increase thus starts at a lateral velocity of v_μ until reaching a velocity that signifies lane switching $v_{min,switch}$. The range is only increased on the side of the lane in which the velocity is pointing.

Lateral Velocity Range - Let y and v be the lateral position and velocity of an object respectively. Then the lateral velocity range is given by

$$\Delta y_{v,range}(y,v) = interp([0, \Delta y_{bias}, 0.5], [0, 1 - \Delta y_{bias}, 0], \tilde{y}) clip(\frac{v - v_\mu}{v_{min,switch}}, 0, 1). \tag{3.5}$$

Lateral Range Functions - let y, v be the lateral position and lateral velocity respectively of an object on the road. The left lateral range is then given by

$$\begin{aligned} ys &= [-0.5, -\Delta y_{bias}, \Delta y_{bias}, 0.5] \\ rs &= [1.5 - \Delta y_{bias}, 1, 1 - \Delta y_{bias}, 1.5 - \Delta y_{bias}] \\ \Delta y_{base}(y) &= interp(ys, rs, \tilde{y}) && (3.6) \\ \Delta y_{range}(y,v) &= \Delta y_{base}(y) + \Delta y_{v,range}(y,v). && (3.7) \end{aligned}$$

The right lateral range is calculated by symmetry as $\Delta y_{range}(-y, -v)$.

This definition of the range function allows us to state the following lemma.

Lemma 3. *The left and right ranges reach into the bias region of the adjacent lane.*

The proof is omitted but you can convince yourself that it is true by looking at figure 3.4, or reading the paragraph before the definition.

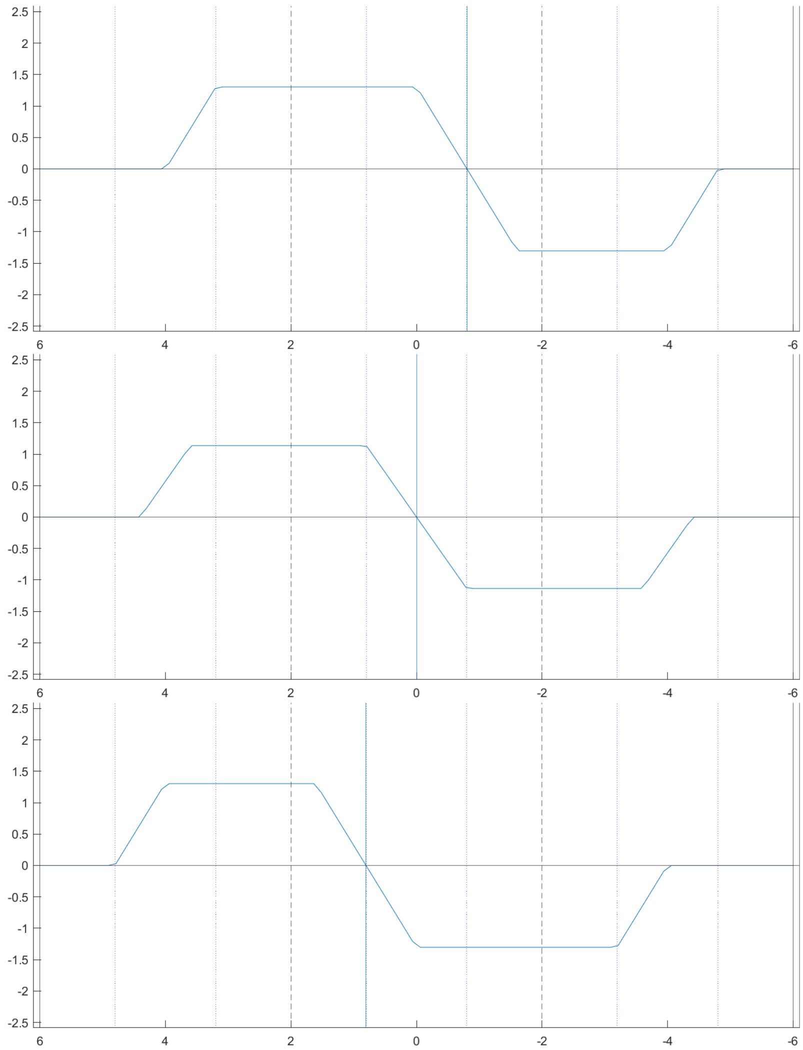


Figure 3.4: The lateral extent of the no-cut component displays how the lateral components that reach into the next lane are defined with the Δy_{range} function to have a non-negative derivative outside of the bias leeway region.

3.2 Longitudinal Force

The longitudinal force consists of three components. The Cruise Control Component f_{cc} that keeps the car at the desired speed, the Trail Component f_{trail} that keeps the vehicle with a desired headway to the leading vehicle and the Sharp Turn Component slows down the host to limit lateral acceleration due to road curvature. The longitudinal acceleration is given by the minimum of the longitudinal components. Together they compose a module for adaptive cruise control(ACC).

3.2.1 Cruise Control Component

The component for reaching the desired velocity is given by a saturated p-regulator. The parameter k determines the jerk with which the regulator stops accelerating. The parameters a_{min} and a_{max} determines the smallest and largest comfortable acceleration to reach a certain velocity.

Cruise Control Component - Let v_{des} be the desired velocity as provided by the driver. Let v be the longitudinal component of the hosts velocity. The Cruise Control Component is then given by

$$f_{cc}(v) = clip(k(v_{des} - v), a_{min}, a_{max}). \tag{3.8}$$

3.2.2 Trail Component

The Trail Component should let the host smoothly approach the desired distance to the vehicle ahead, while reacting to an emergency with the required response. Since the host should only slow down if the object is in front of the host, the final description of the component is described in section 3.1. The behavior caused by the component in the area of full effect is described here.

The component is based on an overdamped harmonic oscillator which allows the velocity and distance difference to decay exponentially:

$$x'' = -2\eta\omega x' - \omega^2 x. \tag{3.9}$$

If the host approaches from far enough behind, the solution never overshoots the desired headway for reasonable velocities ($\leq 346km/h$). However, since the initial distance to the vehicle ahead may be less than the desired headway as a result of a cut-in, two important modifications to the oscillator are made.

The first is to limit the reaction to uncomfortable yet safe cut-ins. Since the distance at which a vehicle is allowed to cut in can be a lot less than the desired headway, the response according to the oscillator would prompt a strong braking, which is both uncomfortable and potentially dangerous, thus the braking force when the host is at the same velocity as the host and not decelerating is saturated at a_{min}, the minimum comfortable acceleration from the Cruise Control Component.

The trail component depends on the desired headway given as driver input t_{des}, as well as the algorithm parameters η, ω and $margin$.

Trail Component Behind Object - Let x, v_o, a_o be the longitudinal position, velocity and acceleration of the leading vehicle relative to the host, respectively. Let l_o and l be the longitudinal component of the bounding boxes of the object and host, respectively. The strength of the Trail Component is then given by A_{trail} as

$$d_{des}(v_o, l_o) = l/2 + l_o/2 + margin + v_o t_{des}, \tag{3.10}$$

$$A_{trail} = a_o + 2\eta\omega(v_o - v) + max(a_{min}, \omega^2(x - d_{des})). \tag{3.11}$$

The second is due to the smooth decrease of braking before stopping. Since the function is agnostic to absolute distance it may reduce braking before impact as the relative velocity gets lower than $(a_{min} - a_o)/2\eta\omega$. To ensure full braking until the velocities are equal in dangerous situations another component is appended. This force component will only affect the host in dangerous situations as a result of a cut-in.

Full Brake Distance - Let x_o, v_o be the longitudinal position, velocity and acceleration of the object respectively. Let v be the longitudinal velocity of the host. Let l_o, l be the longitudinal extent of the host and object respectively. The full brake distance is then given by

$$d_{emr} = \frac{l_o}{2} + \frac{l}{2} + margin + \frac{\max(0, v - v_o)^2}{2b_{max}}. \tag{3.12}$$

Consider first the 1-dimensional case of a straight road with a single lane. Let the acceleration of the host be the minimum of the two components, $a = min(f_{cc}, A_{trail})$. As the two functions are continuous for all x the composition is as well, giving a smooth trajectory with low jerk. With far distance to the leading vehicle, the car will exponentially approach the desired velocity, after a constant acceleration period if the actual velocity is far from the desired one. As it gets closer the leading vehicle it will instead be controlled by the Trail Component. It will then approach the desired distance exponentially. An example trajectory in phase space can be seen in figure, 3.5, and the corresponding acceleration, velocity and distance can be seen in figure 3.6.

The Trail Component should affect the host when an object is in front of the host. To limit the extent of the component, the component ramps up to allow acceleration as the lateral distance increases, or the longitudinal position of the object becomes negative. The parameters for lateral extent will make sure the vehicle will always

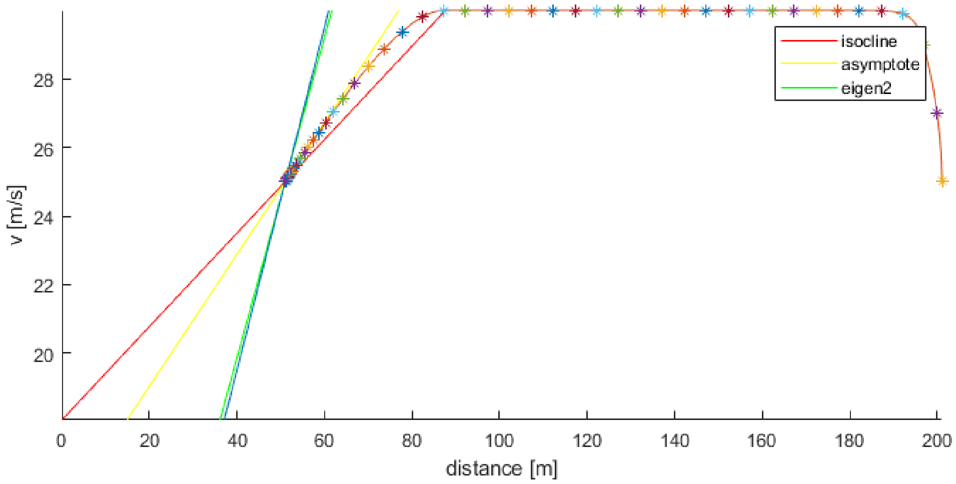


Figure 3.5: A phase diagram displaying the exponential convergence of the ACC functionality emergent from the composition of the two components f_{cc} and A_{trail}.

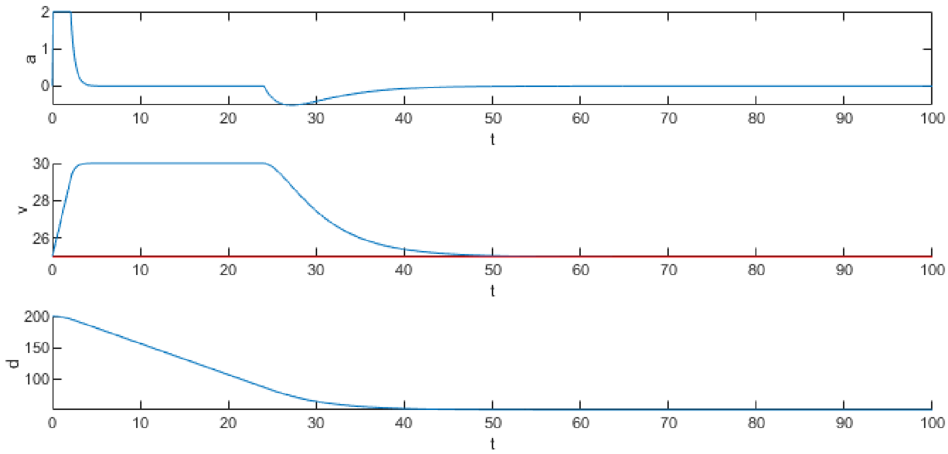


Figure 3.6: A display of the acceleration(top), velocity(middle) and distance(bottom) over time as the host approaches a slower vehicle from behind, controlled by the ACC functionality emergent from the composition of the two components f_{cc} and A_{trail}.

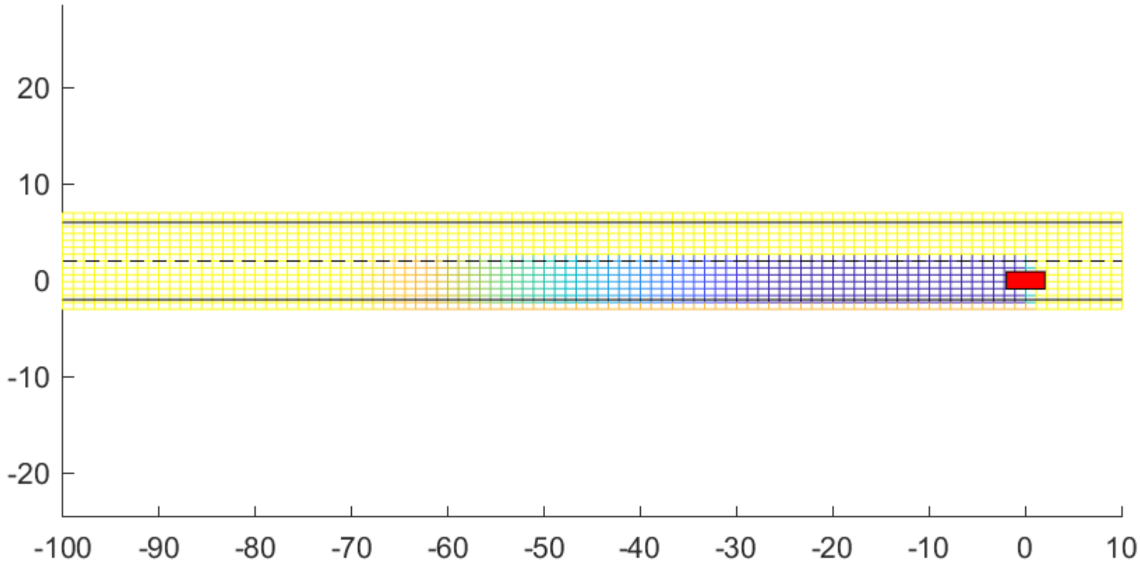


Figure 3.7: The composed effect of the basic longitudinal components, Cruise Control Component and Trail Component. Together they compose a module for adaptive cruise control(ACC).

brake enough when the object is in the same lane as the host, and make a smooth transition if the host cuts in behind the object or the object cuts in in front of the host. The values used for ramping the Trail Component are calculated as

$$l_0 = \Delta y_{range}(y, v), \tag{3.13}$$
$$l_1 = \Delta y_{range}(y, v) + \Delta y_{bias} - 0.5, \tag{3.14}$$
$$r_1 = \Delta y_{range}(-y, -v) + \Delta y_{bias} - 0.5, \tag{3.15}$$
$$r_0 = \Delta y_{range}(-y, -v). \tag{3.16}$$

Trail Component - Let x, y be the longitudinal and lateral position relative to the host respectively of the object. The Trail Component is then given as

$$\begin{aligned} k_x &= drop(-x, -1, 0), \\ k_y &= min(drop(y, r_1, r_0), drop(-y, l_1, l_0)), \\ f_{trail,i} &= max(min(A_{trail}, b_{max} drop(x, d_{emr}, d_{emr} + margin)), b_{max} min(k_x, k_y)). \end{aligned} \tag{3.17}$$

3.2.3 Sharp Turn Component

If the curvature of the road is high, it will be uncomfortable to drive with a high velocity due to the centripetal force. By utilizing the same oscillator used for A_{trail} we can ensure the lateral acceleration due to road curvature is low. The parameter $a_{max,lat}$ denotes the highest comfortable lateral acceleration. The parameters ω, η are the same as for A_{trail}.

Sharp Component - Let κ_i be the absolute road curvature, at every longitudinal position $x_i \in 0, 10, 20...100$. Let v be the longitudinal velocity of the host. The longitudinal sharp component is then given as

$$f_{sharp} = min(2\eta\omega(\sqrt{\frac{a_{y,max}}{\kappa_i}} - v)) + \omega^2 x_i). \tag{3.18}$$

3.2.4 Longitudinal Composition

The longitudinal components are composed by a *min*-composition, that is, taking the least accelerating(or most braking) component as longitudinal acceleration. Since all force components are continuous in the entire domain, their *min*-composition is as well, limiting the jerk, giving a smooth and comfortable trajectory.

Composed Longitudinal Force - Let N be the number of vehicles around the host. Let $f_{trail,i}$, $i \in 1..N$ be the Trail Component from i:th vehicle. The composed longitudinal force is then given as

$$f_{long} = min(f_{cc}, f_{trail,i}, f_{sharp}). \tag{3.19}$$

3.3 Lateral Force

The lateral force determines the trajectory's acceleration in the direction orthogonal to the lane. The force is calculated from a set of components that are composed with the lateral composition method described in section 3.4. The components can be divided into 2 categories: road components and object components. The road components allow the host to center in lane, return to the desired lane after passing and never switch out of the outermost lanes. The object components allow passing if the distance to the vehicles in the target lane is sufficient.

All components except the lane component are *auxiliary*. The auxiliary components give rise to high-level manoeuvres such as lane changing and biasing. By the lateral force composition two opposing auxiliary components will cancel out at certain levels, leaving the vehicle to stay in lane and complying to safety specifications using only longitudinal force. This way, the host will never cause a danger to a third part even if another road-user would cause a danger to the host.

The composed auxiliary force component is saturated at $\pm A_{max}$ to allow a lane switch to occur within reasonable time without causing uncomfortably large lateral accelerations. The weak preference force has an amplitude of $\pm A_{max}$. To allow the pass component and strong preference component to switch lanes against the weak preference component, the amplitude of the *strong* components are $\pm 2A_{max}$. Since the No-Cut component should cancel out the other auxiliary forces it also has an amplitude of $\pm 2A_{max}$. By design of the lateral composition method, if any force component is at $\pm 2A_{max}$ it will cancel out all other force components in the opposite direction.

The Curvature Force is a centripetal force that makes the trajectory follow the curvilinear system, so that all other lateral components can be designed as if the road was straight.

Curvature Force - Let κ be the signed Menger curvature of the lane at the hosts current lateral position y. Let v be the longitudinal component of the velocity of the host. The curvature force is given by

$$f_{curve} = \kappa v^2. \tag{3.20}$$

3.3.1 Road Components

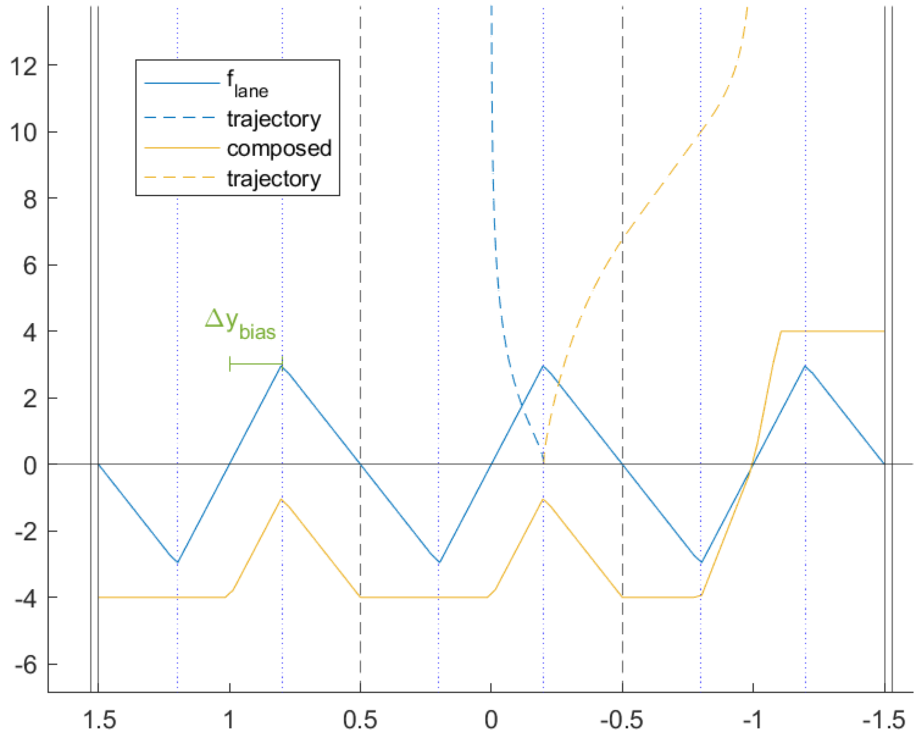


Figure 3.8: In blue, the Lane Component over the lateral positions of the road with example trajectory. In yellow, the composed lateral force on an empty road consisting of Lane and Preference components with an example trajectory. The vertical axis for the solid lines are an acceleration $[m/s^2]$, and for the dashed lines is time $[s]$.

The two road components are meant to keep the vehicle near the center of its lane, to favor the desired lane and to never drive off the road. The force that pulls the vehicle towards the center of the lane is called the Lane Component and is lower in amplitude than the auxiliary components allowing the host switch lanes when appropriate. The Lane Component is a piece-wise linear force reaching its maximum values at $\pm\Delta y_{bias}$ from the lane centers as can be seen in figure 3.8.

Lane Component - Let y be the lateral position of the host on the road. Then the Lane Component is given by

$$f_{lane} = -A_{lane} triangle(y, \Delta y_{bias}). \tag{3.21}$$

The preference components are meant to guide the host with strong and weak rules. Weak rules allows the host to return to the preferred lane after passing, while strong rules could be used to stop the host from turning into an on-ramp, exit-ramp or similar. The weak preference components need an amplitude of A_{max} to enable proper lane switching. The strong preference components limits lane switching to a subset of the lanes, to prohibit entering ramps, lanes that are about to end, or simply off the road. It has an amplitude of $2A_{max}$ to prohibit any composed auxiliary

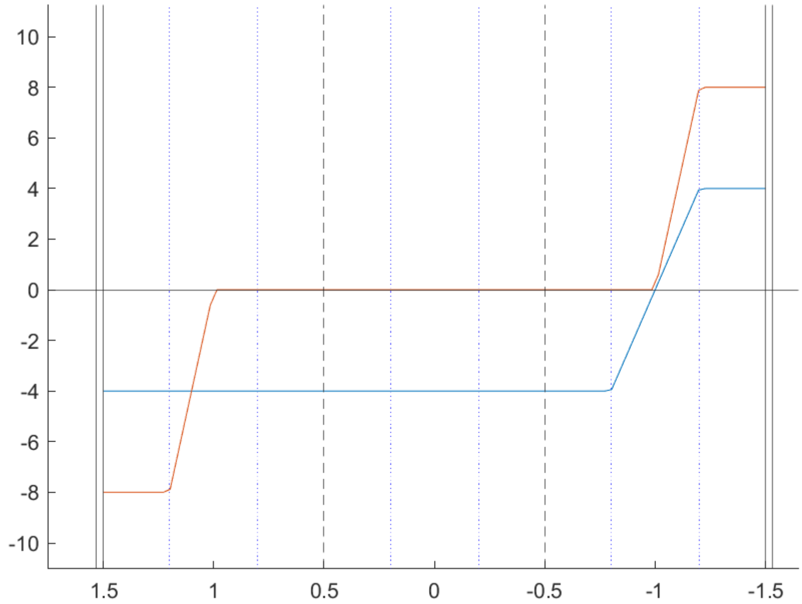


Figure 3.9: In blue, the weak preference component. In orange, the strong preference component with $y_{left} = 1, y_{right} = -1$.

component against that direction. The components ramp up from zero in the center of the lane to their maximum values at $\pm\Delta y_{bias}$. The components are visualized in figure 3.9.

Preference Component - Let $y_{right} \leq y_{pref} \leq y_{left} \in \mathcal{Z}$ be the lateral position of the rightmost acceptable, preferred and leftmost acceptable lane center respectively. Let y be the lateral position of the host on the road. Then the Preference Components are given by

$$\begin{aligned} f_{pref,weak} &= A_{max}(trapezoid(y - y_{pref}, -\Delta y_{bias}, 0) \\ &\quad - trapezoid(y_{pref} - y, -\Delta y_{bias}, 0)), \quad (3.22) \\ f_{pref,strong} &= 2A_{max}(trapezoid(y - y_{right}, -\Delta y_{bias}, 0) \quad (3.23) \\ &\quad - trapezoid(y_{left} - y, -\Delta y_{bias}, 0)). \quad (3.24) \end{aligned}$$

3.3.2 Object Components

There are two lateral object components, the Pass Component and the No-Cut Component. The Pass Component allows the host to switch to the fast lane behind a slow vehicle and the No-Cut Component ensures the host will not switch into the lane if there is another vehicle there with an uncomfortable longitudinal distance.

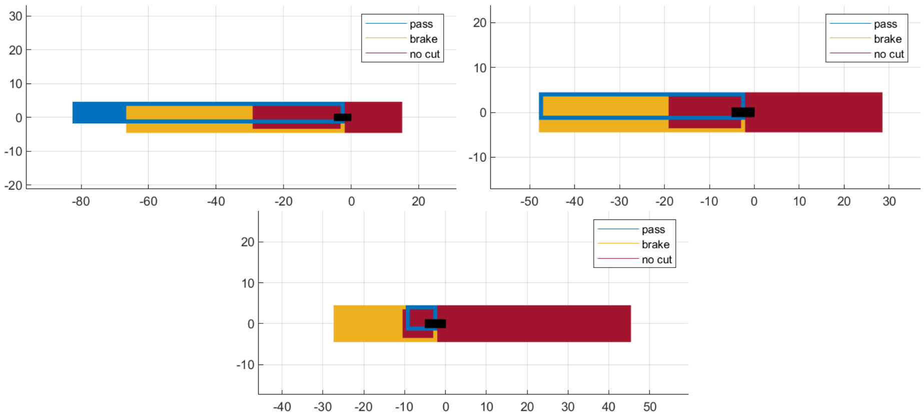


Figure 3.10: The longitudinal regions of full effect for the different object components. The distances are continuous functions of the host's and objects' velocity and acceleration. The images displays the region around the object when the object is slower(top left), at equal velocity(top right), and faster than the host(bottom).

3.3.3 Pass Component

The Pass Component is an auxiliary component that allows the host to overtake slower road-users smoothly. Except for the lateral range, the component contains no safety critical parameters, but only tunable parameters that may be adjusted for comfort. The component should only cause passing for vehicles with a lower longitudinal velocity than the desired cruise control speed. The higher the difference, the stronger the amplitude.

As the amplitude increases the desired behavior is to slowly approach the edge of the bias region until the amplitude is larger than the lane and preference force at which point the next equilibrium should be in the bias region of the target lane. The composition method allows this behavior as is visualized in the two plots of figure 3.12.

Pass Component Amplitude - Let v_o represent the longitudinal velocity component of the object. Let v_{des} be the desired velocity given as driver input. Then the amplitude of the Pass Component is given by

$$A_{pass} = 2A_{max}\, clip(\frac{v_{des} - v_o}{\Delta v_{pass}}, 0, 1). \tag{3.25}$$

When in the same lane, the longitudinal range is defined so the host can switch lanes in time as to not have to slow down. It will also have enough strength to

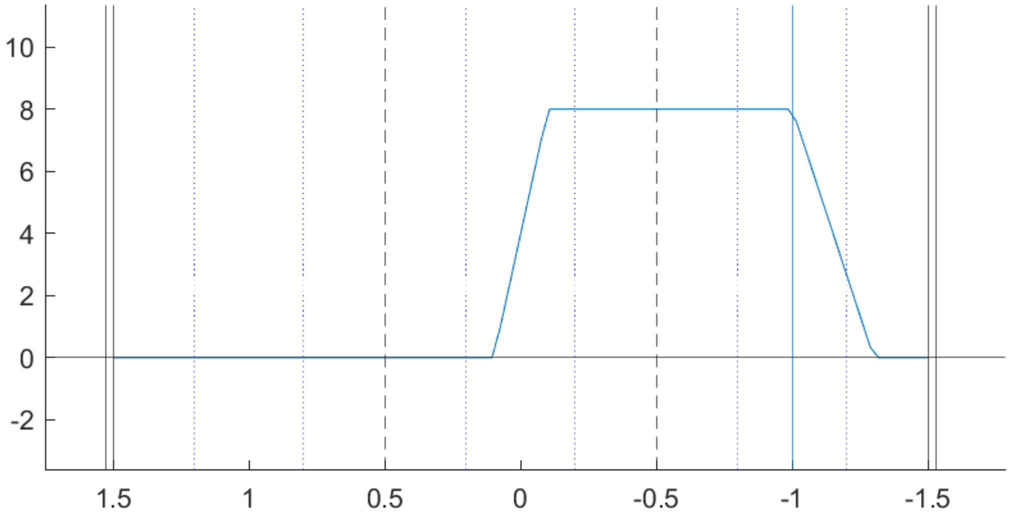


Figure 3.11: The Pass Component encourages the trajectory over the line to the left.

avoid switching to the slow lane due to lane preference if it will not have time to switch back to the fast lane before having to slow down. These distances are a function of the hosts and objects future longitudinal behavior. The prediction does not have to be exact as there is no safety critical aspect of the Pass Component. The time it takes to switch lanes is dependent on the situation and other component parameters, and could theoretically be calculated exactly. However, the calculations are simplified here by assigning a parameter called t_{switch}.

The longitudinal behavior of the host during t_{switch} is predicted as if it was able to follow only the f_{cc} component. The solution to the differential equation described by it is described in section, 2.5. The predicted host velocity t seconds into the future is denoted $v_{cc}(t)$ and the distance travelled during time t is $s_{cc}(t)$.

The behavior of other road-users is modelled by assuming it keeps a constant acceleration during some time t_a, followed by constant velocity.

Object Prediction - Let v_o, a_o be the longitudinal velocity and acceleration of another vehicle. The predicted velocity $v_o(t)$ and distance travelled $s_o(t)$ can then be given as

$$v_o(t) = v_o + a_o min(t, t_a), \tag{3.26}$$

$$s_o(t) = (v_o + a_o min(t, t_a))t - a_o min(t, t_a)^2/2. \tag{3.27}$$

Start Brake Distance - Let $d_{des}(v, l)$ be the desired trailing distance to an object, as described in equation (3.10). Let l_o be the longitudinal extent of the object. Let v, v_o be the velocity of the host and object, respectively. Then the distance $d_{start,brake}$ to the point behind the object at which the Trail Component limits the acceleration

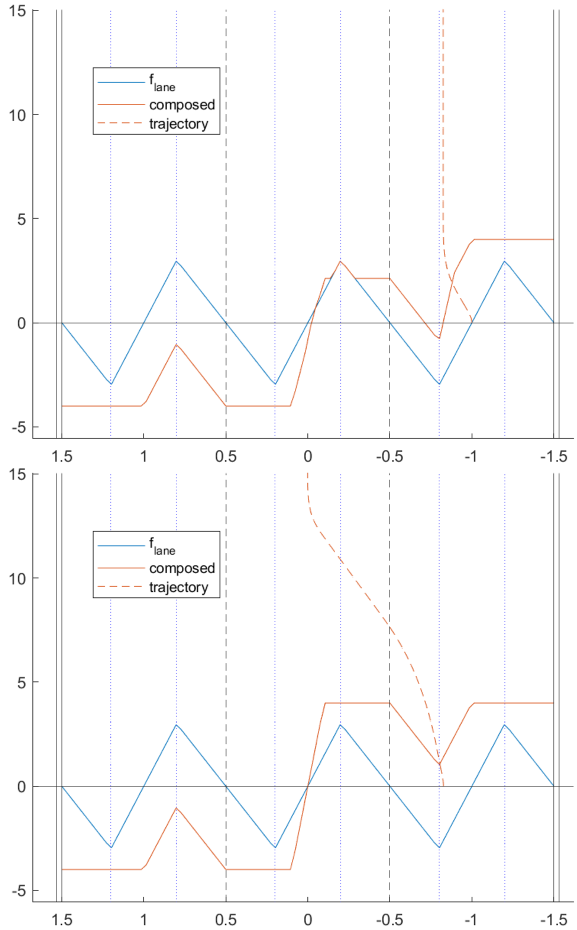


Figure 3.12: As the pass component gets stronger when approaching a slow vehicle from behind the equilibrium moves towards the edge of the bias region(top). When the pass component is stronger than the Lane and Preference Components, the next equilibrium is within the bias region of the target lane(bottom).

to f is given by

$$d_{start,brake}(f, v, v_o) = d_{des}(v_o, l_o) + 2\frac{\eta}{\omega}(v - v_o) + f/\omega^2. \tag{3.28}$$

The key distances can then be calculated by combining the predicted distance travelled by the host, the object, and the predicted brake distance.

Longitudinal Pass Regions - Let l_o be the objects longitudinal extent and let $v_o(t), s_o(t)$ be the predicted velocity at time t and distance travelled during time t of the object. Let $v_{cc}(t), s_{cc}(t)$ be the predicted velocity at time t and distance travelled during time t of the host. The longitudinal ramping of the Pass Component k_x is then given as

$$d_{pass} = d_{start,brake}(f_{cc}(v_{cc}(t_{switch})), v_{cc}(t_{switch}), v_o(t_{switch})) + s_o(t_{switch}) - s_{cc}(t_{switch}), \quad (3.29)$$

$$d^*_{stay} = d_{start,brake}(f_{cc}(v_{cc}(2t_{switch})), v_{cc}(2t_{switch}), v_o(2t_{switch})) + s_o(2t_{switch}) - s_{cc}(2t_{switch}), \quad (3.30)$$

$$d_{stay} = max(d^*_{stay}, d_{pass} + \Delta x) \quad (3.31)$$

$$k_x = min(trapezoid(x, d_{pass}, d_{stay}), trapezoid(-x, -1, 0)).$$

The component extends far enough to the left to push the vehicle into the *bias region* of the left lane as ensured by the *lateral range function*. It does not reach into the lane to the right.

Lateral Pass regions - Let y, v be the lateral position and velocity of the object. The lateral ramping of the Pass Component k_y is then given as

$$\begin{aligned} l_0 &= \Delta y_{range}(y, v) + \Delta y_{bias}, \\ l_1 &= \Delta y_{range}(y, v), \\ r_1 &= 0, \\ r_0 &= 0.5 - \Delta y_{bias}, \\ k_y &= min(trapezoid(y, r_1, r_0), trapezoid(-y, l_1, l_0)). \quad (3.32) \end{aligned}$$

Pass Component - Let x, y denote the longitudinal and lateral positions relative to the host of the object respectively. The Pass Component is then given by

$$(3.33)$$

$$f_{pass} = A_{pass} min(k_x, k_y). \quad (3.34)$$

3.3.4 No-Cut Component

The No-Cut Component is intended to keep the host from turning into the lane of other objects at an unsafe longitudinal distance. The component pushes outwards in both directions from the object, reaching past the line into the bias region of the adjacent lanes. The area of full effect is visualized in figure 3.13.

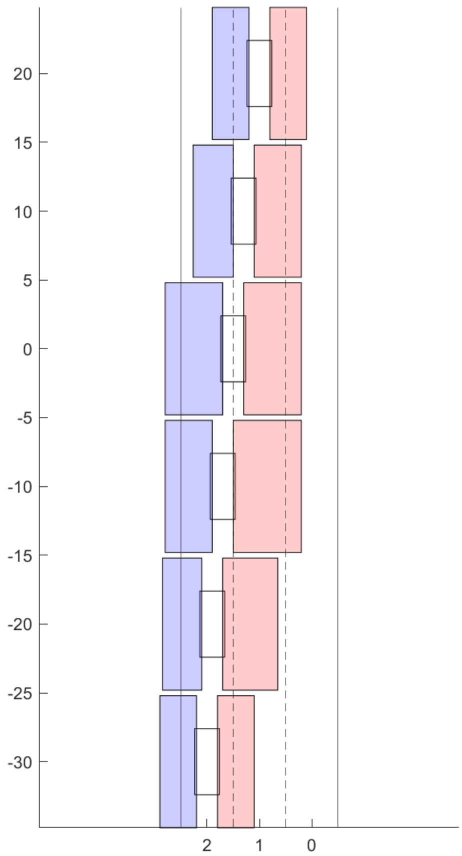


Figure 3.13: The lateral extents of the cut force are changing dynamically dependent on lateral position and velocity to prohibit the host from performing a simultaneous lane switch into the same lane as other road-users. The value of the component is $2A_{max}$ in the blue area and $-2A_{max}$ in the red.

The lateral extents of the component are such that it keeps the host from initiating a lane switch into any of the lanes the vehicle occupies. The longitudinal extents of the component are based on the longitudinal safe distance from the formal Responsibility-Sensitive Safety model RSS, described in section 2.3. This way, if the host is in the lane next to an object with an unsafe longitudinal distance the No-Cut Component will counteract any component encouraging a switch into the lane, as seen in fig 3.14.

The distances defined by the RSS system are based on expecting emergency manoeuvres. When all vehicles are at similar velocities this defines a reasonable cut-in distance, however with large differences in velocity it is bound to be uncomfortable.

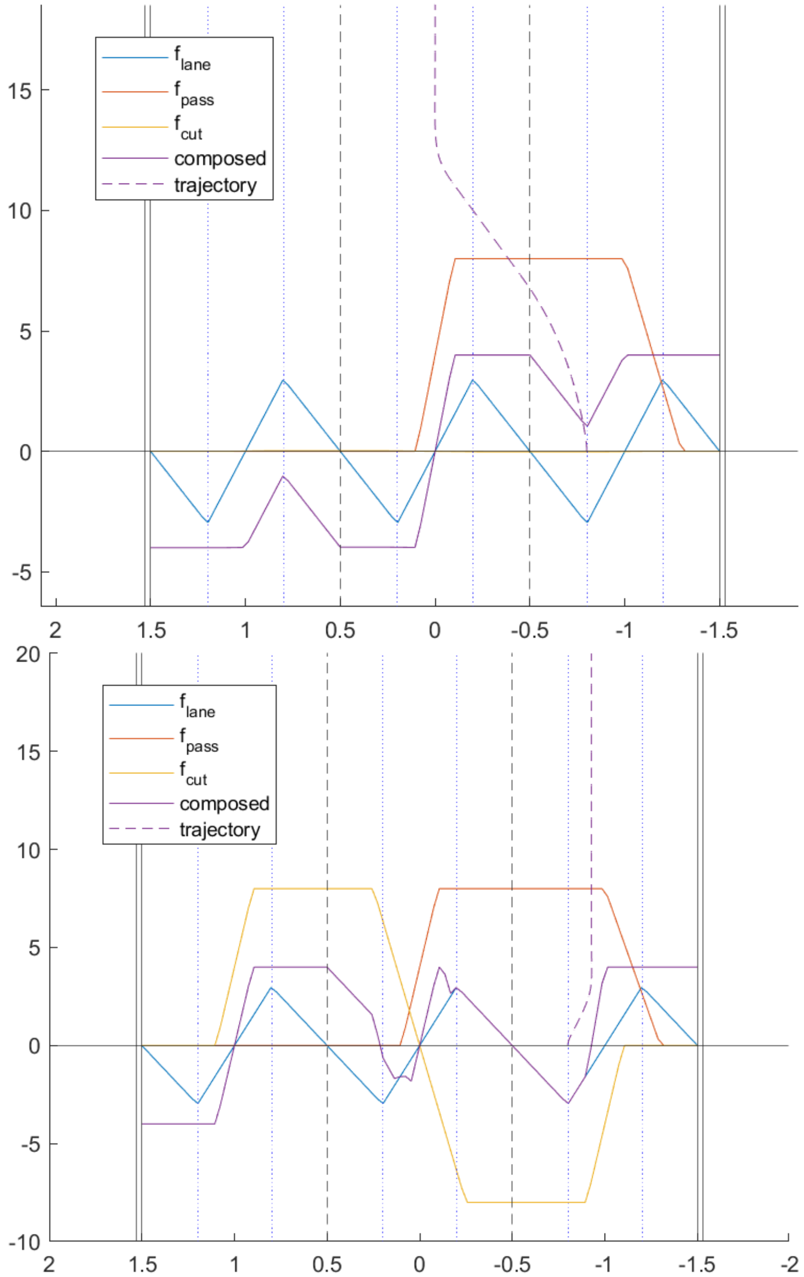


Figure 3.14: The No-Cut component stays at maximum amplitude within unsafe longitudinal distance and counteracts any component that encourages a switch into the lane of the vehicle. The lateral force with Pass component from vehicle in right lane(Top). The lateral force where the No-Cut component from a vehicle in the center lane counteracts the Pass component from the right(Bottom).

Thus, we can define the range for end of effect with respect to comfortably adjusting the speed to the vehicle ahead. By using the parameters used in the Cruise Control component for minimum comfortable acceleration a_{min} we can define a comfortable equalizing distance as the closest distance between the vehicles assuming the rear vehicle slows down with a_{min} and the front vehicle stays at current acceleration as long as its velocity is positive. The equalizing distance d_{eq} can be expressed as a closed expression to be calculated efficiently.

Equalizing Distance - Let v_r and v_f be the longitudinal velocity of the rear and front object respectively. Let a_f be the acceleration of the front vehicle. The comfortable equalizing distance is then given by

$$\begin{aligned} v_r(t) &= max(0, v_r - a_{min}(t - t_{comf})), \\ v_f(t) &= max(0, v_f + a_f t), \\ d_{eq}(v_r, v_f, a_f) &= min_\tau \int_0^\tau v_r(t) - v_f(t)\partial t. \end{aligned} \tag{3.35}$$

No-Cut Regions - Let v and v_o be the longitudinal velocity of the host and the object respectively. Let a_o be the longitudinal acceleration of the object. Let d_{min} be the minimal safe longitudinal distance, as described in equation (2.35). The longitudinal ramping of the No-Cut Component k_x is then given by

$$\begin{aligned} behind_1 &= d_{min}(v, v_o, \rho, a_{max}, b_{min}, max(b_{o,max}, -a_o)), \\ behind_0 &= behind_1 + max(\Delta x, d_{eq}(v, v_o, a_o)), \\ front_1 &= d_{min}(v_o, v, \rho_o, max(a_{max}, a_o), b_{o,min}, b_{max}), \\ front_0 &= front_1 + max(\Delta x, d_{eq}(v_o, v, 0)), \\ k_x &= min(trapezoid(x, behind_1, behind_0), trapezoid(-x, front_1, front_0)). \end{aligned} \tag{3.36}$$

The lateral distances used for ramping the component is given as

$$\begin{aligned} near_1 &= \Delta y_{bias}, \\ left_1 &= \Delta y_{range}(y, v), \\ left_0 &= \Delta y_{range}(y, v) + \Delta y_{bias}, \\ right_1 &= \Delta y_{range}(-y, -v), \\ right_0 &= \Delta y_{range}(-y, -v) + \Delta y_{bias}, \\ k_{left} &= min(trapezoid(y, left_1, left_0), trapezoid(-y, -near_1, 0)), \\ k_{right} &= min(trapezoid(-y, right_1, right_0), trapezoid(y, -near_1, 0)). \end{aligned} \tag{3.37}$$

No-Cut Component - Let x, v_0, a_o, l_o denote the longitudinal position relative to the host, velocity, acceleration and extent of the object respectively. Let v be the velocity of the host. The No-Cut Component is then given by

$$f_{no-cut} = 2A_{max} min(k_x, k_{left} - k_{right}). \tag{3.38}$$

3.4 Lateral Composition

The composition of lateral force components is done in three steps. First the auxiliary components are composed to yield the *Final Auxiliary Component.* Secondly the Final Auxiliary Component is composed with the lane force to yield the lateral force in the road coordinate system, which allows to analytically determine the required damping to avoid oscillations around stationary equilibriums. Lastly the damping and the Curvature Force is added to yield a lateral acceleration in WCS.

3.4.1 Auxiliary Composition

The auxiliary lateral components are composed by taking the sum of the maximum positive(left) value and the minimum negative(right) value. This way it is enough that there exists *some* strong force with a left(or right) value, to prohibit any final auxiliary component against that direction.

Final Auxiliary Component - Let N be the number of vehicles around the host. Let $f_{pass,i}, f_{no-cut,i},\ i \in N$ be the Pass Component and No-Cut Component from each vehicle. Let $F_{aux} = \{f_{pref,weak}, f_{pref,strong}\} \cup \{f_{pass,i}, f_{no-cut,i},\ i \in N\}$ be the set of auxiliary components. Then the final auxiliary component is given by

$$f_{aux} = clip(\max(\{0\} \cup F_{aux}) + \min(\{0\} \cup F_{aux}), -A_{max}, A_{max}). \tag{3.39}$$

Lemma 4. *If there exists at least one auxiliary component with a value of* $2A_{max}$*, then* $f_{aux} \geq 0$ *(a). Similarly, if there exists at least one auxiliary component with a value of* $-2A_{max}$*, then* $f_{aux} \leq 0$ *(b).*

Proof. - Assume $\exists f \in F_{aux}$ s.t. $f = 2A_{max}$. Since all components has a value, $f \in [-2A_{max}, 2A_{max}]$, the composed auxiliary force will be non-negative.

$$\begin{aligned} f_{aux} &= clip(2A_{max} + \min(\{0\} \cup F_{aux}), -A_{max}, A_{max}), \\ &\geq clip(2A_{max} + -2A_{max}, -A_{max}, A_{max}) \\ &= 0. \end{aligned} \tag{3.40}$$

□

3.4.2 Analytically Determined Damping

The auxiliary force is then composed with the lane force to get the lateral acceleration in the road coordinate system.

Lateral Force in RCS - Let f_{aux} be the final auxiliary component. Let f_{lane} be the lane force. The Lateral force in RCS is then given by

$$f_{rcs} = \max(0, f_{lane}, f_{aux}) + \min(0, f_{lane}, f_{aux}). \tag{3.41}$$

Since all lateral components are continuous and piece-wise linear and their composition is done by use of addition, min and max operations, so is the resulting field. By adding a lateral damping component, the solution to the system locally is given by an overdamped harmonic oscillator, described in detail in section 2.2. The important property of the system is that it will converge to an equilibrium without overshooting if the initial velocity is low enough. This means that there will be no oscillations in lane, nor overshooting into an unsafe lateral position.

The damping coefficient η of the system

$$x'' = -2\eta\omega x' - \omega^2 x \tag{3.42}$$

has to be at least 1, for the system to be overdamped. Or written differently, in the system

$$x'' = -k_{damping}x' - ax, \tag{3.43}$$

we need $k_{damping} \geq 2\sqrt{a}$. We can determine an upper bound on the value of a for all times due to the composition method.

For any two functions $f_1(x), f_2(x)$ with derivatives bounded by a_1, a_2 respectively, $\left\|\frac{\partial f_i}{\partial x}\right\| < a_i$, $\forall x$, the derivative of their min or max composition is bounded by $\max(a_1, a_2)$, $\left\|\frac{\partial g}{\partial x}\right\| < \max(a_1, a_2)$, $\forall x$, where $g(x) = \max(f_1(x), f_2(x))$ or min.

For any two functions $f_1(x), f_2(x)$ with derivatives bounded by a_1, a_2, the derivative of the sum of the functions $g(x) = f_1(x) + f_2(x)$ is bounded by the sum of the bounds, $\left\|\frac{\partial g}{\partial x}\right\| < a_1 + a_2 \, \forall x$.

Since the lateral force in RCS f_{rcs} is constructed mainly through min and max operations and only by two additions, we can determine an upper bound on the lateral derivative for all times. Since all lateral components are ramped linearly over the lateral distance Δy_{bias}, the maximum lateral derivative of any auxiliary component is bounded,

$$\left\|\frac{\partial f}{\partial y}\right\| \leq \frac{2A_{max}}{\Delta y_{bias}} \qquad \forall f \in F_{aux}. \tag{3.44}$$

The composed auxiliary component is constructed by a single addition thus $\frac{\partial f_{aux}}{\partial y} \leq 4A_{max}/\Delta y_{bias}$. The lateral force in RCS is then constructed by adding the auxiliary force with the lane force.

A case when the upper bound of the derivative is reached around a stable equilibrium is when the host in between two cars. If the No-Cut component of each car ramps down and the resulting auxiliary component is partially counteracted by the lane force, all derivatives are added together. This case is visualized in figure **??**

Lateral Derivative bound - let w be the width of the lane. At each point in time,

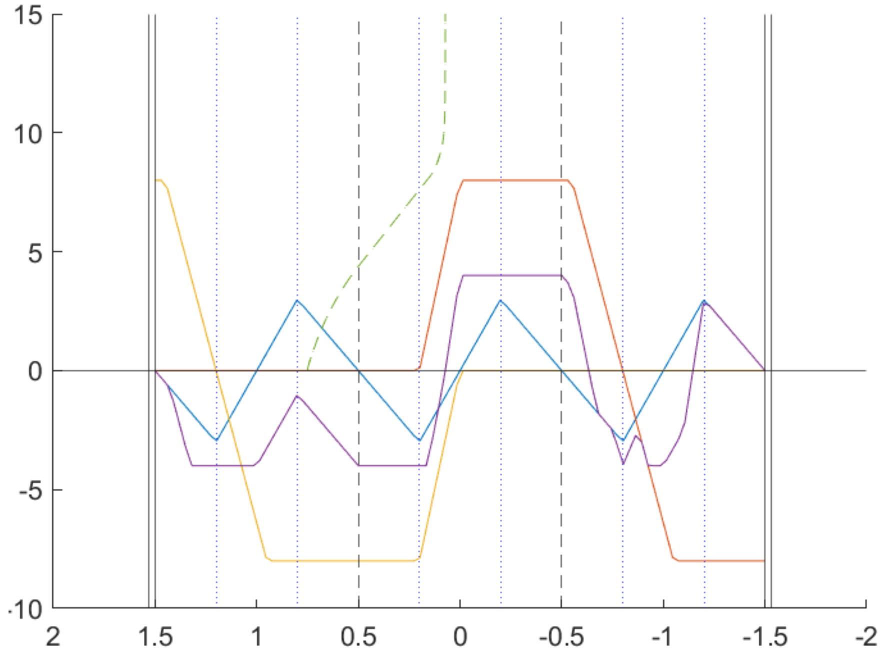


Figure 3.15: The highest lateral derivative appears when two auxiliary components ramp from the maximum value in each direction in the same position, and the lane component ramps in the same direction. This happens when there are vehicles close in each adjacent lane. Even approaching this equilibrium the trajectory does not overshoot.

the maximum lateral derivative of the lateral force in RCS is then bounded by

$$\delta_{max} = \frac{4A_{max} + A_{lane}}{\Delta y_{bias}}, \tag{3.45}$$

$$\left\| \frac{\partial f_{rcs}}{\partial y} \right\| \leq \delta_{max}. \tag{3.46}$$

Consider the differential equation describing the lateral position over time. Since the force is expressed in $[m/s^2]$, the forcing term needs to be expressed considering the lane width, w. Let v be the lateral velocity

$$y'' = -k_{damping} y' + \frac{f_{rcs}(t)}{w} \tag{3.47}$$

$$= -k_{damping} \frac{v}{w} + \frac{f_{rcs}(t)}{w}. \tag{3.48}$$

The system is made overdamped by defining the damping force based on the upper bound of $\frac{\partial f_{rcs}}{\partial y}$.

Damping Force - Let v be the lateral velocity of the host and let w be the width of the lane. The damping component is then given by

$$k_{damping} = 2\eta_{lat}\sqrt{\frac{\delta_{max}}{w}}, \tag{3.49}$$

$$f_{damping} = -k_{damping}\frac{v}{w}. \tag{3.50}$$

The final composition first combines the auxiliary component with the lane force and and damping to get the lateral acceleration relative to the road, adding the curve force.

Composed Lateral Force - Let f_{curve} and $f_{damping}$ be the Curve Force and the damping force respectively. Let f_{aux} be the final auxiliary component. Then the composed lateral force is given by

$$f_{lat} = clip(f_{curve} + f_{damping} + f_{rcs}, -A_{max}, A_{max}). \tag{3.51}$$

3.5 Control

To realize the acceleration in RCS, the acceleration vector is converted to the Vehicle Coordinate System, VCS, through rotation. In VCS, the x-axis is parallel to the wheelbase of the vehicle, *forward* for the car, and the y-axis points orthogonally to the left facing forward.

Let θ be the angle from the x-axis of the vehicle to the x-axis of the Road Coordinate system in the position of the host. The forward acceleration a_x and sideways acceleration a_y for the car is then given as

$$\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} sin(\theta) & cos(\theta) \\ -cos(\theta) & sin(\theta) \end{bmatrix} \begin{bmatrix} f_{long} \\ f_{lat} \end{bmatrix}. \tag{3.52}$$

With a kinematic bicycle model the acceleration a and steering angle δ of the vehicle relates to the VCS acceleration vector of the center of the bounding box. In simulation the expression is simplified by letting the acceleration from the forces define the acceleration for the center of the rear axle of the car, which reduces the expressions.

Let v be the velocity of car, let l_{base} be the distance from from the rear axle to the front axle of the car, also known as wheel base. Let a_x, a_y be the forward and sideways acceleration of the center of the rear axle of the car. Then the acceleration a and steering angle δ are given as

$$a = a_x, \tag{3.53}$$

$$\delta = atan(l_{base}\frac{a_y}{v^2}). \tag{3.54}$$

4
Results

The proposed algorithm for autonomous highway driving, *the Oskillator*, is evaluated by formal analysis of trajectories and by subjective analysis of the behavior in a simulation environment.

The trajectory analysis shows that all stable equilibriums are within a *bias region* of the lane centers as long as other road-users also stay near their lane centers. The host will never leave the bias region of its lane towards a lane with another vehicle at an unsafe longitudinal distance. The host is shown to stop in time to avoid crashing if the vehicle in front applies full brake from the minimal safe longitudinal distance. The results from the trajectory analysis is compared to the formal model of RSS described in section 2.3.

The trajectories will by design not cause any danger to a third part even if another vehicle were to brake the safety assumptions. This is assured by not braking too aggressively as a response to close cut-ins, and not turning into a lane of another vehicle even if another car were dangerously close to the side.

The high-level manoeuvres and comfort are evaluated by a few examples from the simulation environment. In the simulation it is apparent that the host is able to pass or slow down to the vehicle ahead. However, the lateral velocity when passing is low so the pass takes a long time. The results of safety and comfort are then discussed in chapter 5.

4.1 Trajectory Analysis

The trajectory analysis is done by proving certain statements of the trajectories of the solutions to the longitudinal and lateral differential equations respectively. The longitudinal trajectories do not crash with the leading vehicle even when the lead vehicle decelerates with the maximal emergency braking $b_{o,max}$, from the minimal safe distance, d_{min}. When approaching a slower vehicle at constant velocity, the host will converge to the desired headway distance without overshooting.

The RSS states a longitudinal response to dangerous longitudinal situations as well as a lateral response to dangerous lateral situations. The longitudinal response requires the rear vehicle to apply full brake until stand-still upon entering the unsafe longitudinal distance behind another vehicle. The proposed algorithm may reduce the braking deceleration before stand-still if the distance to the vehicle ahead allows it.

4.1.1 Longitudinal Safety

If the rear vehicle is able to respond before the reaction time ρ, the required braking deceleration is less than b_{min} to avoid danger. Thus the behavior is safe according to the modified RSS if it can guarantee a crash will be avoided upon entering the unsafe area from behind in any way using only longitudinal control.

MRSS Definition 1 (Safe Longitudinal Behavior)**.** *If a vehicle c_r comes closer than d_{min} to a the car in front c_f with an unsafe lateral distance, c_r has to apply a braking deceleration such that it will not crash given c_f never decelerates more than b_{max}, and c_f must never decelerate more than b_{max}.*

To prove longitudinal safety we must ensure the host respects the proper longitudinal response defined in MRSS Definition (1).

The proposed algorithm can ensure to never crash as long as the vehicle in front never exceeds a braking deceleration of $b_{o,max} = 7[m/s^2]$. It is shown with a simulation calculating the closest distance to the vehicle ahead if the host starts at the minimal safe longitudinal distance behind the object and the object has an acceleration of -7 until the velocity is 0 is shown below. The reaction is also delayed with 0.1s to simulate sensing times.

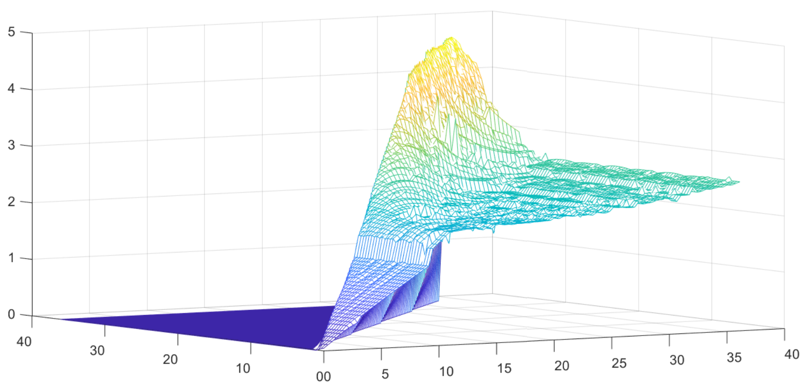


Figure 4.1: Simulation starting from the safe longitudinal distance behind another vehicle, controlled by the longitudinal forces. The initial velocity of the host on the right axis and of the object on the left axis, the minimal distance to the vehicle ahead on the z-axis.

The test is run on a grid of reasonable initial velocities, $(v_o, v) \in [0, 130]^2[km/h]$ where v_o, v are the velocities of the host and the other object respectively. The closest distance to the ahead vehicle is always non-negative, $\min_t(x(t)) \geq 0$. The shape of the plot has 4 regions of interest. It is zero when $v_0 >> v$ as that is the minimal safe longitudinal distance, the host would start at a distance of 0 and would then later increase the distance. The other flat region for $v >> v_o$, appears since the other object reaches full stop, letting the host lower the braking force until coming within Full Brake Distance. For very low velocities the same thing happens only the initial distance is less than the Full Brake Distance to start with. For higher similar velocities, $v_0 \approx v$, only the Trail Component is active as the object keeps a high deceleration until the velocity is low.

4.1.2 Additional Details of Longitudinal Behavior

As the desired trailing distance is much further than the safe longitudinal distance, it will be able to avoid crashing with larger marginal and for higher decelerations of the vehicle in front. The only time the vehicle can be at the border of the unsafe longitudinal distance is as a result of a cut-in.

Since any false trigger of emergency manoeuvres is dangerous, it is important that the host only decelerates greatly when in danger. Thus the deceleration after the host ends up closely behind another vehicle as a result of a cut-in by either the host or the other vehicle is saturated w.r.t. distance. The trail component has saturated the response with respect to distance, meaning that if the object has a constant velocity and no acceleration, the host will at most slow down with a comfortable deceleration, a_{min}, and does thus not cause any danger to the car behind.

Note, however, that the Trail Components dependency on the objects acceleration, relative velocity and the emergency distance will ensure that the host will still avoid crashing even with this modification.

4.1.3 Lateral Safety

The host will never leave the bias region of its lane towards an adjacent lane with another vehicle at an unsafe longitudinal distance. Since the No-Cut Component will cancel out all other auxiliary forces at the edge of the bias region, The lateral force can be bounded by an autonomous system and shown to keep the host at a safe distance.

Lemma 5. *If there is a vehicle within the bias region of the adjacent lane l_o within an unsafe longitudinal distance, the host, if its initial lateral velocity is 0, cannot exit the bias region of its lane l towards l_o.*

Proof. Assume there is an object at an unsafe longitudinal distance x_o in lane l_o. First assume l_o is to the left of l, and let the center of l be at lateral position $y = 0$. The lateral position of the host is then $y \in [-\Delta y_{bias}, \Delta y_{bias}]$ and the lateral position of the other vehicle is then $y_o \in [1 - \Delta y_{bias}, 1 + \Delta y_{bias}]$.

The longitudinal ramping coefficient of the No-Cut component of the object, k_x,

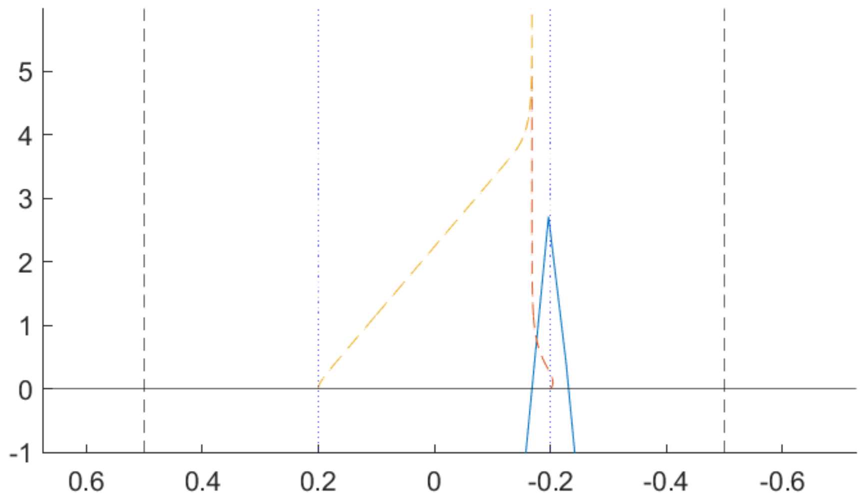


Figure 4.2: The No-Cut component of a vehicle within the bias region of the adjacent lane cancels out all auxiliary components on the edge of the bias region of the host lane. This enables the lower bound for the lateral force to be used to prove the trajectory can never engage in a lane switch into the vehicles lane. Even with a maximum velocity towards the lane it returns to the bias region of its own lane by the lower bound on the system with a vehicle at an unsafe longitudinal distance in the lane $l_o = -1$.

is 1 throughout the unsafe longitudinal distance by definition (3.36). The right lateral ramping coefficient, k_{right}, is 1 for lateral positions in the area of full effect, $[y_o - \Delta y_{range}(-y_o, -v_o), y_o - \Delta y_{bias}]$. In this region the No-Cut component is thus $-2A_{max}$ and according to lemma 4(b), the auxiliary component in this area is thus non-positive $f_{aux} \leq 0$.

The position $y_{-} = \Delta y_{bias}$ is in the area of full effect since

$$y_o - \Delta y_{bias} > 1 - 2\Delta y_{bias} > \Delta y_{bias} \tag{4.1}$$

and

$$y_o - \Delta y_{range}(-y_o, -v_o) < \Delta y_{bias} \tag{4.2}$$

by lemma 3.

Since the auxiliary component is non-positive and the lane component is $f_{lane}(\Delta y_{bias}) = -A_{lane}$ at $y = \Delta y_{bias}$, the lateral force in rcs will be less than $-A_{lane}$ there. The lateral force in rcs can then be bounded from above by the function $g(y) > f_{rcs}(t, y)$ which increases from $-A_{lane}$ with δ_{max} on either side of y_{bias} until reaching the maximal acceleration of A_{max}

$$g(y) = min(A_{max}, -A_{lane} + \delta_{max}\|y - \Delta y_{bias}\|). \tag{4.3}$$

The dynamic system can be be bounded by the autonomous system

$$y'' = -k_{damping}y' - g(y), \tag{4.4}$$

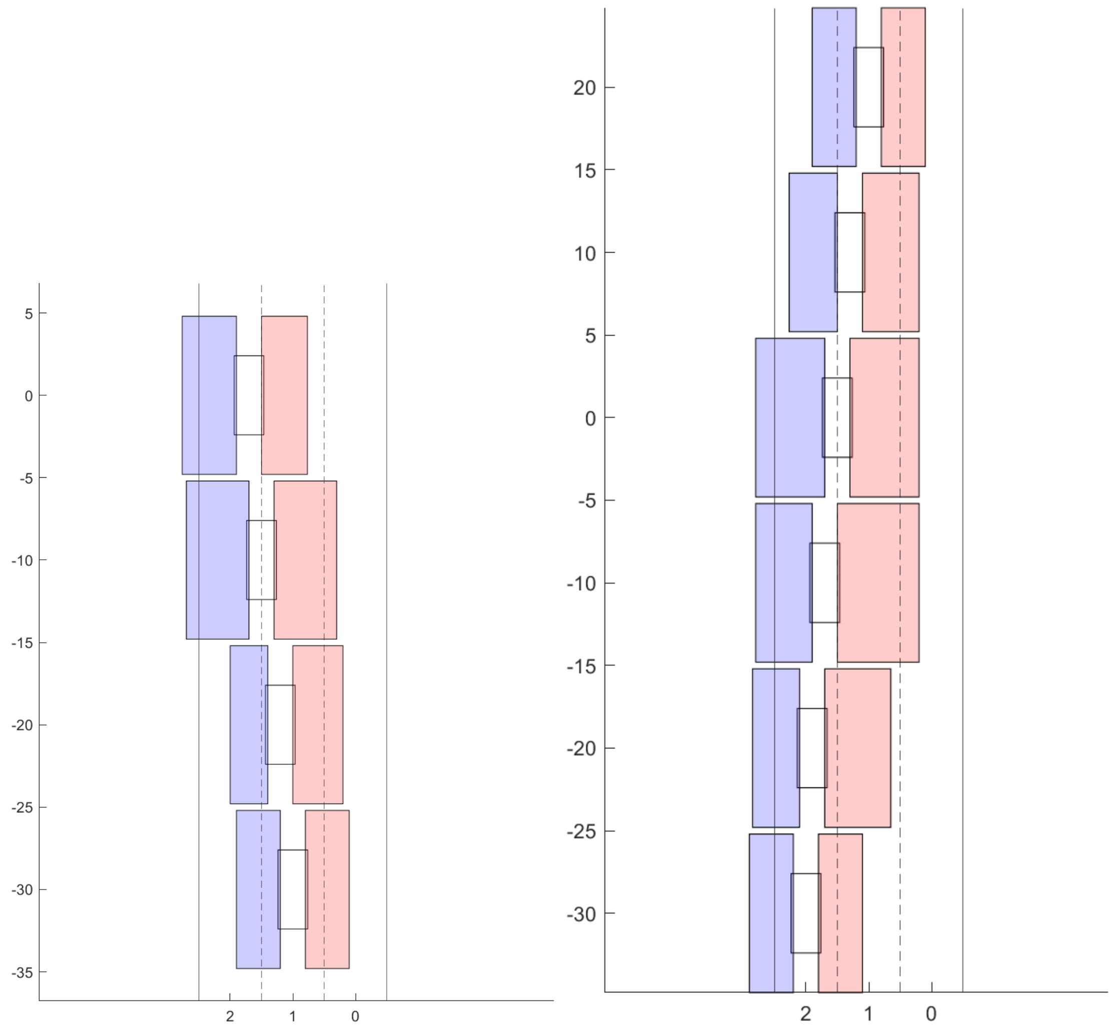


Figure 4.3: The regions where the No-Cut component is sure to counteract any auxiliary component towards the cars lane. Longitudinally it ranges the entirety of the unsafe longitudinal distance. To the left, a car in different positions with lateral velocity zero. To the right, a car in a pass scenario, velocities are $[0, 0.7, 0.8, 0.9, 0.5, 0]m/s$, from bottom to top.

which has one stable equilibrium $y_{eq} = \Delta y_{bias} - A_{lane}/delta_{max}$.

Since the system is overdamped, the solution to (4.4) will stay within the interval between y and y_{eq}, by lemma 1, and they are both inside the bias region. Thus by lemma 2, we know the actual solution will stay to the right of boundary.

The scenario when the object is to the right follows by symmetry.

□

4.1.4 Additional Details of Lateral Behavior

Even though the lateral force field is continuous, when approaching a slower car from behind the equilibrium moves from the bias region of the current lane to the first to the left. This way the system has a built in system for hysteresis for the lane change manoeuvre. The fact that equilibriums only exist near lane centers allows the proposed method to potentially comply to a modified defintion of Responsibility-Sensitive safety further discussed in section 5.2.

4.2 Simulation with Random traffic

The algorithm has been implemented in the simulation program *TrafficAI* developed by *Simteract*. *TrafficAI* provides direct access to the state of all agents and the road. The algorithm outputs a longitudinal and lateral acceleration at each point in time. Due to the kinematic model used in the simulator, the exact steering and acceleration signals can be calculated to achieve the accelerations.

Due to numeric errors and the numeric instability of a feed forward loop, we can through observation of the simulation see that the algorithm is a self-correcting control scheme. For example the curvature is approximated manually from three points on the curve of the lane, when the analytical trajectory centers in lane, so does the object even though the curvature force is incorrect.

4.2.1 Emergent Human-like Behaviors

The subjective analysis of the behavior observed in the simulator shows several signs of Human-like Behavior emergent from the forces. A description of some of them are mentioned below.

Lane Biasing - When approaching vehicles in adjacent lanes, the host will bias slightly away from the lane of the vehicle, providing extra margin.

ACC - The vehicle is able to smoothly approach the desired distance to the vehicle ahead, both when approaching from far behind and after a vehicle cuts in front of the host.

Passing - When approaching a vehicle with a lower velocity than the desired, the host will switch lanes to pass when left lane is available.

Double Switching - If there are three lanes on a road and the vehicle may want to move to the rightmost lane from the leftmost lane. When this happens the host will slow down laterally upon reaching the center of the middle lane, which is required by law.

Avoiding Danger - When a vehicle in the adjacent lane on one side shows potentially dangerous behavior with a high lateral velocity and the lane on the other side is free, the host will bias far from the dangerous lane or even switch lanes.

4.2.2 Missed Human-like Behaviors

The main issue with the lateral force field is the high required damping. The maximum lateral velocity ever achieved by the host is the limited to $\approx 0.37[m/s]$ in each direction, with the current set of parameters and a lane width of $4m$. This makes switching lanes abnormally slow. It is thus hard to be able to merge into lanes where the velocity of other vehicles are different from the host's, as it will enter the No-Cut region of the next vehicle before completing the switch and will then return to its own lane.

The host takes no consideration of the indicator signals of other road-users. In development, the idea of increasing the lateral range by a full lane in the direction of the indicator, was tested. The Idea worked, however at the cost of introducing discontinuities to the force field. Also it may cause emergency braking if a car starts indicating at the wrong times, and so it was excluded from the description of the algorithm.

The host cannot merge well into slower lanes to the right as the pass force is just as strong as the strong preference force. To increase the levels of priority without increasing the derivative of the lateral components a new composition method is needed. This topic is discussed further in section 5.3.

5

Discussion

In this chapter we discuss the view on safety of algorithms for autonomous vehicles, and potential improvements to be done to the proposed algorithm. The first section argues how a system that is formally safe for a multi-agent system may not be enough when there are rouge agents, humans, that can break the safety invariant. This way it is required that the algorithms used need a protocol for unsafe situations even though they should never occur in the formal safety system.

Secondly, an idea for modifying the definition of lateral safety on highways with respect to lanes is presented.

Lastly, the problem of the proposed algorithm, too slow lateral velocity when switching lanes is discussed. The cause is described and potential solutions are presented.

5.1 What is Safety?

In computer science a threaded program is considered safe is there is no order of execution that will cause a crash or faulty behaviour. One learns in early stages of concurrent programming theory, how methods of limiting risks by timing margins for communication is faulty by nature. However low the risk of failure is, the program will crash eventually. The causes of failure in parallel software may be very subtle, and their effects monumental. Therefore, optimising programs for safe execution instead of average execution speed is almost always preferable. In real life however, many important decisions are made unsafe as long as the probability of failure is low enough.

From a background of formal programming safety it might be tempting to look at road safety from a similar perspective. How can we make our vehicle never cause an unsafe state. This is a problem formally solved by RSS. However, in designing the algorithm and comparing to real life experience, I have noticed how real world scenarios sometimes force the driver to make actions that are not formally safe as the alternative would be worse. For example, a road-user would not brake aggressively to make a safe distance to a vehicle that cuts in directly ahead if another vehicle is directly behind. The components are therefore designed, not only to never enter potentially dangerous situations itself, but also to provide the safest behavior when in a potentially dangerous situation by the behavior of other road-users. This idea of never *causing* danger is important in an environment with human lives at stake and potentially false detections by sensing equipment.

5.2 Modified Responsibility-Sensitive Safety

The definition of safe lateral distance in RSS has no consideration for the lanes on the road. For example, it could be considered *safe* by their definition, to enter another vehicle's lane with an unsafe longitudinal distance, as long as the emergency distance is not violated. I would argue that it is not.

The algorithm does not comply to the protocol of lateral behavior in RSS as the host *can* return to the center of its lane to ensure enough distance to vehicles in the adjacent lanes, even if another vehicle has moved into it. However, it will not switch lanes if there is an unsafe longitudinal distance to the vehicles in, or attempting to switch into, the target lane. With other road-users respecting the same idea, it could be possible to prove that no vehicles can crash.

I therefore propose a modified version of Responsibility-Sensitive Safety, MRSS. The new version is based on respecting the lanes of other road-users instead of a distance. The definition of the distances and the protocol is outlined. Due to the lack of time the proof of absolute safety by induction was not completed, but left as a hypothesis.

5.2.1 New Lateral Safety

The modified lateral safety is defined w.r.t. lanes and specifically the bias leeway of the lanes defined in section 3.1. All positions are defined with respect to the center of the bounding box and the distances w.r.t. the nearest edges of the bounding boxes. The definition is dependent on the additional parameter v_{max} describing the maximal lateral velocity allowed on highways, and can be high, but is needed to put an upper bound on the lateral stop deviation. Especially $v_{max} > a_{max}\rho$.

Two vehicles cannot crash if they are in different lanes, within bias leeway, Δy_{bias}, of their respective lane center. The safe lateral distance is defined so that no vehicles can come into the same lane or too close in adjacent lanes with an unsafe longitudinal distance. The definition is less trivial the original as the lanes occupied by a certain vehicle changes discretely. The definition of the protocol for safe lateral behavior must thus make sure that a vehicle respects lanes other vehicles are *about to* enter.

The safe lateral distance is more understandably defined with the use of some additional definitions. We will define a new Lateral Stop Deviation considering v_{max}. When a vehicle at lateral position y and lateral velocity v needs to brake laterally it will be able to stay at positions $[y + \Delta y_{min}(v), y + \Delta y_{max}(v)]$.

MRSS Definition 2 (Lateral Stop Deviation)**.** *Let* v *s.t.* $\|v\| \leq v_{max}$ *be the lateral* *velocity of a car. Let w be the width of the lanes of the road. Then the outmost possible lateral deviation to stop, $[\Delta y_{max}, \Delta y_{min}]$ w.r.t. parameters ρ, a_{max}, b_{min} is given as*

$$\hat{v} = min(v_{max}, v + a_{max}\rho),$$
$$\hat{\rho} = \frac{\hat{v} - v}{a_{max}},$$
$$\Delta y_{max}(v) = \left(\frac{v + \hat{v}}{2}\hat{\rho} + \hat{v}(\rho - \hat{\rho}) + \frac{max(0, \hat{v})^2}{2b_{min}} - \frac{min(0, \hat{v})^2}{2b_{min}}\right)/w, \quad (5.1)$$

$$\Delta y_{min}(v) = -\Delta y_{max}(-v). \quad (5.2)$$

In particular $\Delta y_{max}(0)$ will be used to describe deviations for a stationary target.

MRSS Definition 3 (Occupation of Lanes)**.** *A vehicle is said to occupy all lanes containing the vehicles current position or points within the Lateral Stop Deviation from the current position.*

The definition of safe lateral distance is no longer symmetric w.r.t. the cars, as one car may be close to entering the lane of another while the opposite is not necessarily true. Consider for example two cars in adjacent lanes at the right-most position in their respective lane, the car in the left lane is then within the unsafe region of the right car, but the opposite is not true. The distance and response will therefore be defined for one vehicle to another.

MRSS Definition 4 (Safe Lateral Distance)**.** *Assume WLOG a car c_l is to the left of car c_r. let l_l be the rightmost lane occupied by c_l and l_r be the leftmost lane occupied by c_r. Let v_1, v_2 be the lateral velocities of the cars, respectively. The lateral distance c_l has to c_r is safe if*

1. *l_l and l_r are not adjacent, or*
2. *c_l has a position at least $0.5 - \Delta y_{bias}$ from l_r and vice versa, or*
3. *c_r has a position less than $0.5 - \Delta y_{bias}$ from l_l and c_l has a position at least 0.5 away from l_r .*

Correspondingly, the lateral distance c_r has to c_l is safe if

1. *l_l and l_r are not adjacent, or*
2. *c_l has a position at least $0.5 - \Delta y_{bias}$ from l_r and vice versa, or*
3. *c_l has a position less than $0.5 - \Delta y_{bias}$ from l_r and c_r has a position at least 0.5 away from l_l.*

In item 3 in the first enumeration above, c_r was in a safe attempt to switch into l_l when c_l started occupying l_l, as c_r should have stayed at a position more than than

$0.5 - \Delta y_{bias}$ from l_l otherwise. This scenario is known as a *simultaneous lane switch.* As c_l moves towards the center of l_l, c_r must move back to its lane to get a safe distance to c_l.

MRSS Definition 5 (Safe Lateral Behavior)**.** *If a car c_1 enters the unsafe area of another car c_2 from the side at time $t = 0$, then c_1 must during time $[0, \rho)$ have any acceleration a s.t. $\|a\| \leq a_{max}$, and lateral velocity v s.t. $\|v\| \leq v_{max}$, after which,*

- *if c_1 is to the left of c_2, c_1 must apply a lateral acceleration of at least b_{min} until its velocity $v \geq 0$. It must then stay at a velocity $v \in [0, v_{max}]$ until at a safe lateral distance to c_2.*
- *if c_1 is to the right of c_2, c_1 must apply a lateral acceleration of at most $-b_{min}$ until its velocity $v \leq 0$. It must then stay at a velocity $v \in [-v_{max}, 0]$ until at a safe lateral distance to c_2.*

Hypothesis: Absolute Safety - If all vehicles comply with the safe longitudinal behavior in MRSS Definition 1, and the safe lateral behavior in MRSS Definition (5), no vehicles can crash.

5.3 Lateral Priorities with Limited Damping

Due to the damping, the proposed algorithm is unable to yield a trajectory with higher lateral velocities than

$$v_{max} = \frac{A_{max}}{k_{damp}} \approx 0.37[m/s], \tag{5.3}$$

on a road with lane width $w = 4m$. This means that a lane switch from the edge of the bias region of one lane to another takes at least $6.4[s]$. This is unusually long, and makes it hard for the host to merge into lanes where the other road-users have a different velocity. Why the velocity is this low and what could potentially be done to increase it is discussed in this section.

To ensure that the vehicle avoids oscillating in lane around the equilibrium the damping coefficient k_{damp}, is analytically determined from the highest lateral derivative δ_{max} of the lateral force, as $k_{damp} = 2\sqrt{\delta_{max}}$. The maximum lateral velocity is given by $\frac{a_{max}}{k_{damp}/w}$ where w is the width of the lane, and if the damping coefficient is too large it would be impossible to perform a lane switch in a reasonable time frame, with comfortable or even feasible accelerations. One issue with the naive composition of the force components, adding them together, is that the lateral derivative becomes theoretically unbounded, and in practice too large. The proposed composition method for lateral force components reduces the required damping by reducing the lateral derivative. The lateral derivative is reduced from $\mathcal{O}(an)$ to $\mathcal{O}(a)$ where a is the highest amplitude of a force component and n is the number of vehicles, as compared to the sum of the components.

However, even though the composed auxiliary component never exceeds A_{max} the individual forces range to $A_{max} = 2A_{max}$. And since the forces may decrease in distance Δy_{bias} and the auxiliary composition makes a single addition, the derivative of the auxiliary component is bounded by $4A_{max}/\Delta y_{bias}$.

The reason why some forces need an amplitude larger than A_{max} is to be able to have a force that can allow passing in one direction $f_{pref,weak}$ with the appropriate speed, whilst allowing a switch to be done in the opposite direction by f_{pass}. We can say that the system has 2 levels of priority. Say that we would like the strong preference component to allow switching against the pass force, it would then need an amplitude of $3A_{max}$, a third level of priority. With the current method of composition the lateral derivative is proportional to the levels of priority. The current damping is close to being too high, with a third or more levels of priority it would no longer be possible to perform lane switches in reasonable time and still guarantee non-oscillatory trajectories, without increasing the saturation limit of acceleration.

The maximum lateral velocity scales with the amplitude of the forces, since $v_{max} \sim a_{max}/\sqrt{\delta_{max}} \sim \sqrt{a_{max}}$. However, increasing the acceleration leads to less comfortable trajectories. For a more sophisticated model with more levels of priority a new method of lateral composition, or different type of damping is needed that does not scale with levels of priority.

This could potentially be done by first composing the components without ramping, in the range of any effect. And subsequently ramping between the values with a constant derivative of $2A_{max}/\Delta y_{bias}$, with post-processing. This way all values will be in the range $[-A_{max}, A_{max}]$ before ramping, and can thus be done within Δy_{bias}.

The issue with this approach, is that it is hard to find a closed expression for the lateral force in a single lateral point. It would require the host to calculate the force in adjacent, if not all, lateral positions on the road.

A simpler solution would be to allow slight oscillations. Any damping will set a maximal velocity and an exponential decrease in oscillations over time for an autonomous(stationary) system. If the oscillations are small enough the same assertions of safety can be made. By readjusting accelerations to the *underdamped* system, it might be possible to achieve a safe, feasible and comfortable trajectory that performs the lane changes in a better way.

Another approach would be to interpret the resulting lateral force field as a velocity field instead. This way the solutions will be free from oscillations by default. The acceleration could then be determined through Euler Backwards to achieve the desired oscillation free behavior. With this approach the behavior could be further tailored to the application.

6

Conclusion

By deciding controls in a system for autonomous driving on highways on force fields, the host can be controlled smoothly and assertions of safety can be made. The proposed design is able to yield a good longitudinal behavior in all scenarios tested, including but not limited to cut-ins, approaching slower vehicles and sharp turns. The proposed design of the lateral force field allows to prove that the host will never engage in a lane switch to an occupied lane with a dangerous longitudinal distance to the other vehicle. It also guarantees that all stable equilibriums will be close to the center of the lanes if other road-users are near their respective lane center. Both the longitudinal and lateral force fields are continuous with respect to all inputs but can still avoid flickering between lanes. The emergent passing behavior is made with a very low lateral velocity. It would be preferable to be able to increase lateral speeds and lower lateral accelerations for a smoother experience. Possible solutions to this problem are analysed and discussed. The formal framework Responsibility-Sensitive Safety is discussed and a modified version is proposed.

Bibliography

[1] M. T. Wolf and J. W. Burdick, "Artificial potential functions for highway driving with collision avoidance," in *2008 IEEE International Conference on Robotics and Automation*, pp. 3731–3736, May 2008.

[2] C. W. Reynolds, "Steering behaviors for autonomous characters," in *Game developers conference*, vol. 1999, pp. 763–782, Citeseer, 1999.

[3] S. Shalev-Shwartz, S. Shammah, and A. Shashua, "On a formal model of safe and scalable self-driving cars," *CoRR*, vol. abs/1708.06374, 2017.