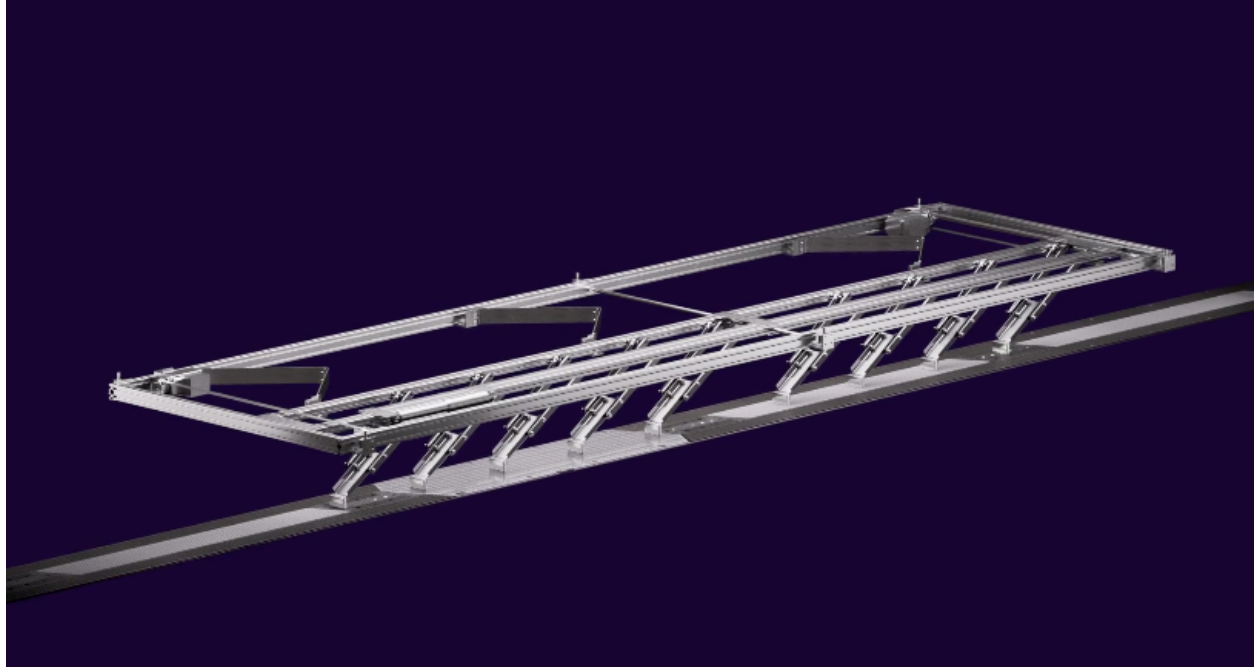




CHALMERS
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Analyzing the Impact of Road Irregularities on Electrical Connectivity

A Comprehensive study of Pickup Mechanism for Continuous Vehicle Charging

Master's thesis in Automotive Engineering

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DEPARTMENT OF MECHANICS AND MARITIME SCIENCES

CHALMERS UNIVERSITY OF TECHNOLOGY
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Department of Mechanics and Maritime Sciences
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Abstract

In today's world, electrical vehicles are growing in demand for obvious reasons. To support this growth, countries around the world are trying different strategies to develop the right infrastructure for electrification. One such system is electric roads where the battery electric vehicles are energized when in motion through the transport infrastructure.

Pickups are a mechanical extension to the chassis of a vehicle that is used to "pick up" electrical current. This pickup uses a spring system to extend on to the rail below and draw traction current. Since this pickup is placed between the chassis and the road, and also experiences forces from both ends i.e., due to the surface irregularities in road and due to suspension dynamics of the vehicle, it is important to model the pickup to determine the contact force existing between the two conductive surfaces.

A relationship established between the contact force and contact resistance was extended to the vehicle-pickup setup modelled in this project. For higher contact forces, it was seen that the contact resistance reduces drastically (at any given traction current). Lower contact resistance is good and is needed to get a better quality of electrical connection (i.e., continuous energy flow). This can be achieved by having less deflections on the pickup which depends on a number of parameters, including the construction of the pickup itself and then the external conditions i.e., type of roads on which the vehicle is in motion.

As expected, the integrated model of the vehicle and the pickup performs better on expressways than regional roads due to the difference in amplitudes on both the roads. Further, a relationship on the contact force and resistance helped in securing the parameters that have an impact on the quality of the electrical connection which is primarily the pickup itself (material, construction setup, stiffness).

Keywords: pickup, contact force, contact resistance, electric roads.

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Varun Bharadwaj, Gothenburg, December 2023

Below is the list of abbreviations and nomenclature that have been used throughout this report.

Abbreviations

EV	Electrical Vehicle
PSD	Power Spectral Density
ERS	Electrical Road System
BEV	Battery Electrical Vehicle
IRI	International Roughness Index
QCM	Quarter Car Model
HCM	Half Car Model

Nomenclature

R_c	Contact resistance
f	Contact force
v_x	Velocity of the vehicle
m_s	Sprung mass
m_{us}	Unsprung mass
m_{pickup}	Mass of the pickup
k_s	Spring stiffness
k_t	Tire stiffness
k_{sf}	Spring stiffness on the front axle
k_{sr}	Spring stiffness on the rear axle
k_{tf}	Tire stiffness of the front wheels
k_{tr}	Tire stiffness of the rear wheels
k_{pickup}	Stiffness of the pickup
$k_{veh-pickup}$	Stiffness of the link arm that connects the vehicle to the pickup
$k_{pickup-road}$	Stiffness of the link arm that connects the pickup to the road
k_{brush}	Stiffness of the steel brush that is the final extension of the pickup
c_s	Dampening coefficient of the spring
c_t	Dampening coefficient of the tire

c_{sf}	Dampening coefficient of the spring on the front axle
c_{sr}	Dampening coefficient of the spring on the rear axle
c_{tf}	Dampening coefficient of the tire- front wheels
c_{tr}	Dampening coefficient of the tire- rear wheels
z_s	Displacement of the sprung mass
z_u	Displacement of the unsprung mass
z_r	Excitation due to road irregularities
z_{pickup}	Displacement experienced by the pickup
ω	angular frequency
Ω	spatial frequency
tf	transfer function
t_{sim}	Time period for simulation
I	Current
l_1	Distance from CG to front axle
l_2	Distance from CG to rear axle



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1

Introduction

1.1 Background

Electrification of vehicles, both commercial and passenger, is a phenomenon we have seen this decade. To support this rapid growth of Electrical Vehicles (EV), governments have been accelerating the development of the needed infrastructure. One such development is the inclusion of chargers in (on) roads i.e., **electrified roads**. The development of such roads have been initiated to compensate for the long charging times usually experienced in a conventional charging setup. Electrified roads accommodate technologies that are used to charge the vehicle when they are moving.

Electrified roads, today, exist in three different forms. One, the pantograph i.e., overhead charging system; two, induction charging i.e., through magnetic transmitters placed under the road segment and, three, conduction-based charging through pickup (similar to an inverted pantograph) connecting to the rails on road. Though all these are in an experimental stage and are still under investigation with experimental road segments set across different parts of the country, the focus in this thesis was on the third method i.e., electrified roads through conduction.

A study by the Swedish National Road and Transport Research Institute revealed that electric roads appear to provide a cost-effective means to significantly reduce carbon emissions from heavy trucks [1]. In a scenario where the expansion connects the three biggest cities in Sweden, emissions will be cut by one-third of the overall emissions from heavy duty trucks in Sweden. Also, in terms of passenger vehicles, electrified roads could significantly increase the range and reduce the cost of EVs, making them more attractive to consumers. A big advantage of using electrified roads instead of relying on stationary charging is the possibility of using smaller batteries that will reduce the extraction of rare earth minerals. However, the research on electrified roads across the globe has been very less compared to the study on stationary charging points. Countries like Sweden and Germany have been leading the way when it comes to real-world studies on electric roads [2].

1.2 Problem Statement

The contact between the electric road and the vehicle takes place through the stripper or the pickup. This contact will be affected by how the vehicle moves vertically. The movement of the vehicle relative to the roadway will determine whether the pickup is in contact with the wire in the road (i.e., the rail), as well as its quality (resistance). Hence, this contact force between the pickup and the rails (in the road) is crucial as studying this will show

1. how the influence of the vehicle's suspension and dampening characteristics (including the trailer's in case of a truck) affect the electrical connection,
2. the effect of uneven roads on the vehicle, the pickup, and hence, on the electrical connection.

1.3 Objectives

The aim of this project is to study and understand how the vertical dynamics (*parameterized based on the suspension/dampening characteristics of the vehicle and road unevenness*) of a traveling vehicle are influencing the movement of the pickup and in effect, the electrical connection. Further, breaking down the objectives:

- Develop a model for the vehicle considering all the necessary parameters of a 4x2 truck
- Develop a model for the pickup system by assuming the necessary information based on the ELON setup
- Integrate the vehicle and the pickup model and run the simulation over a given road profile (regional roads and expressways)
- Determine the contact force of the pickup and hence the quality of the electrical connection transferred

1.3.1 Envisioned outcome

The expected outcome is to have a deeper understanding of the vehicle and the pickup response to the road (rail) surface by means of frequency response using transfer functions. Further, try to make use of the determined contact force to understand the losses in electrical connection and the decisive factors that play a role in such losses when the vehicle moves over the given surface. A stretched target will be to determine the roughness index of the modified vehicle (i.e., with the pickup attached as an extension to the chassis).

1.3.2 Deliverables

- Power spectral density (PSD) plots of the given road profile to understand the power spectrum of the frequencies occurring on the given surface
- An integrated model of the vehicle and pickup in matlab
- Comparison of the modelled system for different design scenarios and simulation conditions

1.4 Assumptions and Limitations

One of the pre-requisites for conducting the thesis was that there is no product/component or "ready hardware" for the pickups. Hence, the modelling for the transfer mechanism is based on the photos shared by *Elon Road*. The parameters for the spring stiffness of the pickup are assumed based on the construction geometry and its installation on the vehicle. Further, the pickup is assumed to be attached to the tractor i.e., the driving body in a truck (4x2 configuration). If the pickup is attached to a trailer (trailing body attached to the tractor via a fifth wheel), the complexity of the model has to be increased to include additional parameters such as the coupling forces, momentum, etc between the tractor and the trailer.

The entire model (i.e., the vehicle and the pickup moving on a given surface) is assumed (and limited) to be linear in nature based on the simplicity in scenarios considered i.e., the path traversed (regional and expressways), where small perturbations are given as input around a specific point i.e., the end of the pickup. Further, the output is used to determine the contact force based on the known fact that the force changes linearly with respect to the deflection of the pickup amplified by a constant which is the spring stiffness. Also, to add to the linearity, the installation of the pickup (since) is assumed to be very simple i.e., point location at CG of the vehicle. This implies that the pickup will not be affected by a moment around the CG i.e., (pitch). Further, any other large vertical forces (for example, heave) on the pickup apart from the suspension forces of the vehicle and the road irregularities is not considered in modelling the system.

2

Theory

2.1 Literature study

According to [1], as mentioned above, there are basically 3 types of electric roads that are considered for studies in Sweden; conductive overhead, conductive in/on-road rails and inductive technology. There are numerous advantages and disadvantages for each technology and it all comes down to maintenance activities, all weather conditions and installment costs when making the judgement on laying out the type of Electric Road System (ERS).

Perfect sliding contact that occurs during conduction to transfer electric charge from the rail to the vehicle through the pickup requires appropriate contact force between the pickup and the third rail (i.e., similar to the pantograph and the contact overhead wire). Contact force and traction current are variables that has been used to develop a fitting formula for contact resistance through some experiments that have been explained in [3]. The formula is:

$$R_c = k/f^n \quad (2.1)$$

where R_c is the contact resistance, f is the contact force, k and n are constants determined by contact materials, contact form, surface conditions.

Based on the experiments carried out by Wang, et al., the contact resistance is almost a constant when contact force reaches a constant value. Therefore, the above formula is revised to define a fitted formula based on least squares method.

$$R_c = af^n + b \quad (2.2)$$

where a , b and n are fitting constants.

Further, for different traction currents, ranging from 20A to 60A with a step of 10A, a , b and n are determined by

$$b = R_c - af^n \quad (2.3)$$

$$b = 337/I - 2.42 \quad (2.4)$$

2.2 EVs & Charging Infrastructure

Electrical vehicles are gaining traction all over the globe, in fact, the number of new passenger EVs sold in the first half accounted to around 8.3% (according to <https://www.ev-volumes.com/>) globally and about 18% in Europe thanks to growing technologies, increased awareness on climate change, etc. The passenger segment has been driving this switch to EVs. However, the case is different in the commercial segment of medium and heavy duty electric trucks where the trend has started to shape up since 2020. In this paper the focus is primarily on charging infrastructure for medium and heavy duty electric trucks, a market that is slated to reach about 25000 trucks in volume before 2026. To put things in perspective, we have about 4000 electric trucks on the road today [4].

The number of truck owners and fleet owners making a move to electric trucks are very low due to some significant hurdles in the way. Based on a survey carried out by Land Line Media [5], the biggest reason for the resistance to adopting EVs is range anxiety and battery life followed by charging time. This resistance is further fueled by the initial high costs and most likely subscription costs to accommodate good charging infrastructure. From the government's perspective, charging stations also occupy additional space if not built within the existing fuel stations. There are multiple ways that address the problems mentioned above to accelerate the switch to EVs. One such way is the development of continuous charging systems that can be incorporated within the existing network of roads, i.e., e-roads.

Electric roads or e-roads are transport infrastructure systems developed to provide a constant source of supply to battery electric vehicles (BEV) when they are in motion. The energy transfer happens between an element that acts as an interface between the vehicle and the road. This element is usually an extension from the chassis of the vehicle. The energy can be transferred by the following methods:

- **Induction (wireless)**- In this setup, inductive electrical infrastructure beneath the asphalt charges vehicles as they drive, via an electrical receiver fitted underneath the vehicle.
- **Conduction**- Energy can be harnessed in two different ways i.e., either through overhead (pantographs) or through the road (using a rail). Pantographs are devices that are mounted on top of the vehicle (similar to an electric train) to pick up current from one or several contact wires. On-road electric systems are the second way to charge vehicles through conduction. In this setup, a mechanical pickup (similar to pantograph) that is attached to the chassis of a vehicle is dropped down to the road to pick up current from a rail that built into the existing asphalt section of the road. One such "on-road conductive rail technology" is currently being developed by ELON Road [6] and is in rigorous testing phase in a highway section south of Sweden.

2.2.1 Conductive Rail Technology- ELON Road

As discussed above, conductive rail technology utilizes embedded conductive rails or strips within the road surface, allowing EVs equipped with specialized extensions to connect and draw power directly from the rails. The system operates similarly to how trams or trolleybuses receive electricity from overhead wires. However, in this case, the road is a rigid surface and has irregularities unlike the overhead wires which is relatively flexible.

The conductive rails are typically segmented and activated only when a vehicle passes over it, thus minimizing energy loss. The pickup makes contact with the rail segments, transferring electricity to the vehicle's onboard battery thus reducing range anxiety. Truck and fleet owners' biggest concern i.e., downtime (due to charging stops) is also reduced with the help of this setup.



Figure 2.1: The mechanical construction setup of a ELON pickup system

Regardless of the several advantages to on-road electric system, as any other technology, this too has a flip side to it. In this case, as touched upon earlier, the road is a rigid surface consisting of irregularities. The surface roughness of the road varies locally and has a specific profile globally (i.e., ups, downs and turns due to geographical reasons). The mechanical construction of the pickup becomes very crucial to handle/ counteract this varying profile of the road as it is subjected to

- Upward forces experienced by the contact element passing over the rail
- Downward forces experienced by the contact element due to the suspension kinematics of the vehicle

Although, there will be some horizontal forces (specifically, sliding forces) acting on the pickup element, vertical forces dominated and will be the focal point of studies in this thesis. In this case, the construction of the pickup is narrowed down to a mechanical spring system that is balanced between the chassis and the road rail. The construction of a pickup is shown below.

Based on the construction shown, it is clear that the varying vertical forces have an effect on the quality of electric connection picked up when the vehicle is moving continuously over the electrical strips. The extent of influence is determined by the contact force of the pickup with respect to the rail that has been built into the road. If the current flowing through the coils in the electric strip within the rails is assumed to be constant, the contact resistance changes based on the contact force. This relationship will help us determine the impact of geometry of the pickup on the quality of electric connection.

2.3 Vertical Dynamics

Vertical dynamics is the "vertical" interaction between the vehicle's suspension system, tires, and the road surface, influencing critical aspects such as handling, stability, and passenger comfort. Optimizing these elements is essential for designing vehicles that offer a seamless and enjoyable driving experience [7].

The introduction of a pickup, positioned between the chassis and the road, introduces additional complexities to this interaction, altering existing factors and impacting the quality of energy transfer between the source and the vehicle. This added element necessitates a comprehensive understanding of its influence on vertical dynamics.

Irregularities and imperfections in the road profile can induce vibrations and disturbances that challenge the pickup's ability to maintain a continuous distribution of charge. Understanding the characteristics of different road surfaces is essential for designing the pickup that can effectively adapt to varying conditions. This means that the pickup's structure and material properties can affect the absorption and dissipation of vibrations and impacts, potentially influencing the overall pickup of current.

2.4 Road Surface as an influencing parameter

As discussed in the parent section, road surface characteristics play a crucial role in determining vehicle performance, ride quality, overall driving experience and specifically, in this paper, the pickup behaviour and the quality of electrical energy transferred to the vehicle. Road surfaces can vary significantly in terms of texture, roughness, and material composition, each influencing vehicle dynamics in distinct ways.

Road texture refers to the fine-scale irregularities on the road surface, typically measured in millimeters or micrometers. The surface roughness, on the other hand, refers to the larger-scale deviations from a smooth surface, typically measured in centimeters or meters. Excessive roughness can cause vibrations, impacting passenger comfort and potentially causing damage to vehicle components including the pickup.

2.4.1 International Roughness Index (IRI)

The International Roughness Index (IRI) is a standardized measure of road surface roughness used to assess road quality and plan or prioritize maintenance activities. It provides a quantitative representation of the variations in elevation along a traveled wheel path (the longitudinal road profile)[8].

IRI values are typically expressed in units of meters per kilometer (m/km), representing the accumulated suspension motion of a standard passenger vehicle traveling over one-kilometer segment of the road. Higher IRI values indicate a rougher road surface, while lower values indicate a smoother surface [8].

The IRI is calculated using a quarter-car simulation model, which mathematically simulates the response of a standard vehicle suspension system to the longitudinal road profile. The model takes into account the vehicle's suspension characteristics and the speed at which it is traveling [9].

The "golden car" model serves as a reference vehicle for calculating the IRI. It represents a standard passenger vehicle with typical suspension characteristics and varies based on regions, conditions, etc and is usually managed and updated by the governing transport authority in their respective nations. This reference model allows for comparable IRI measurements across different road profiles and/or simulation conditions[10].

For a truck, quarter car model with the golden car parameters are as below:

Table 2.1: Parameters for reference quarter truck and golden car

Symbol	Value	Unit	Symbol	Value	Unit
m_s	3400	kg	$c = c_s/m_s$	6	s^{-1}
k_s	270000	N/m	$k_1 = k_t/m_s$	653	s^{-2}
c_s	6000	Ns/m	$k_2 = k_s/m_s$	63.3	s^{-2}
m_t	350	kg	$\mu = m_u/m_s$	0.15	-
k_t	950000	N/m			
c_t	300	Ns/m			

2.4.2 Road Data

For this paper, road profile data from both expressways and regional roads in Sweden, collected from the Östergötland region in 2022 were analyzed. The expressway data is segmented into data segments, i.e., structural arrays within Matlab identified by longitudinal profiles on the left (V) and right (H) sides. For regional roads, data is organized into similar sections but split apart (based on how the sample data collected). These were further concatenated into a single longitudinal profile.

PSD plots are further used to compare the two roads and also get a better understanding of the profile. PSD is used in signal processing and data analysis to quantify how the energy (power) of a signal is distributed across different frequencies. In the context of road profiles, it is used to analyze the frequency components of the surface roughness (road amplitudes along the longitudinal profile). A PSD plot typically has frequency (x-axis) on a logarithmic scale and energy or power (y-axis) on a linear or logarithmic scale.

2.5 Quarter & Half Car models

Quarter Car Model (QCM) is a simplified representation of a vehicle suspension system consisting of two masses, sprung and unsprung. The sprung mass represents the portion of the vehicle that is supported by the suspension, such as the body and the load on it. The unsprung mass represents the portion of the vehicle that is not supported by the suspension, such as the tires.

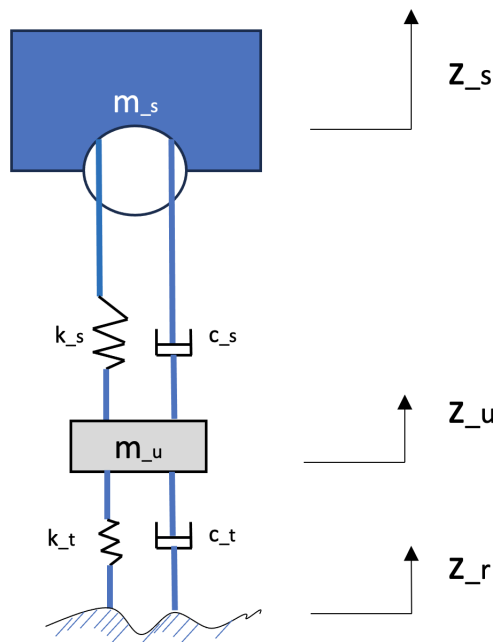


Figure 2.2: Free body diagram of a quarter car model

The two masses are connected by a spring and damper, which represent the suspension system. Spring stiffness and damping coefficient are two main parameters that determine the behavior of the suspension system. The QCM is a system with two degrees of freedom, meaning that it has two independent motions; the vertical displacement of the sprung mass and the unsprung mass.

The equation of motion for QCM [11] is as follows:

$$m_s \ddot{z}_s + c_s (\dot{z}_s) + k_s (z_s) = 0 \quad (2.5)$$

$$m_u \ddot{z}_u + c_s (\dot{z}_s - \dot{z}_u) - c_u (\dot{z}_u - \dot{z}_r) + k_s (z_s - z_u) - k_u (z_u - z_r) = 0 \quad (2.6)$$

Equation 2.5 indicates the equation of motion for sprung mass and equation 2.6 for unsprung mass.

Half Car Model (HCM) is a relatively more complex representation of a vehicle suspension system than the QCM. It consists of three masses; sprung mass, unsprung mass i.e., further divided into front and rear unsprung mass.

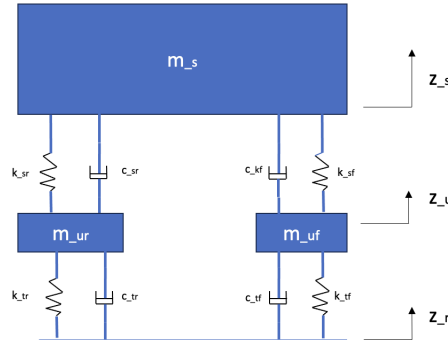


Figure 2.3: Free body diagram of a half car model

The equations for HCM [bengt] are shown below:

$$\begin{aligned}
 m_s \ddot{z}_s + c_{sf}(\dot{z}_s - \dot{z}_{uf} - l_2 \dot{\theta}) + c_{ur}(\dot{z}_s - \dot{z}_{ur} + l_1 \dot{\theta}) \\
 + k_{sf}(z_s - z_{uf} - l_2 \theta) + k_{ur}(z_s - z_{ur} + l_1 \theta) = 0
 \end{aligned} \tag{2.7}$$

$$\begin{aligned}
 I \ddot{\theta} - l_1 c_{uf}(\dot{z}_s - \dot{z}_{sf} - l_1 \dot{\theta}) + l_2 c_{ur}(\dot{z}_s - \dot{z}_{sr} + l_2 \dot{\theta}) \\
 - l_1 k_{sf}(z_s - z_{uf} - l_1 \theta) + l_2 k_{sr}(z_s - z_{ur} - l_2 \theta) = 0
 \end{aligned} \tag{2.8}$$

$$m_{uf} \ddot{z}_{uf} - c_{ur}(\dot{z}_s - \dot{z}_{uf} - l_1 \dot{\theta}) + c_{rf}(\dot{z}_{uf} - \dot{z}_{rf}) - k_{uf}(z_s - z_{uf} - l_1 \theta) + k_{rf}(z_{rf} - z_{uf}) \tag{2.9}$$

$$m_{ur} \ddot{z}_{ur} - c_{uf}(\dot{z}_s - \dot{z}_{ur} - l_1 \dot{\theta}) + c_{rr}(\dot{z}_{ur} - \dot{z}_{rr}) - k_{ur}(z_s - z_{ur} - l_1 \theta) + k_{rr}(z_{rr} - z_{ur}) \tag{2.10}$$

Similar to the QCM, three masses are connected by springs and dampers, which represent the suspension system. The HCM is a four degree of freedom system, meaning that it has four independent motions; the vertical displacement of the sprung mass, the vertical displacement of the front and rear unsprung mass, and the pitch rotation of the sprung mass. HCM is a more accurate representation of a vehicle suspension system than the QCM as it can be used to analyze the effects of different suspension parameters on ride comfort, handling, as well as the effects of pitch rotation on vehicle dynamics.

2.5.1 Golden Car model as a reference

As discussed earlier, the golden car's IRI value will be used as a reference to determine how the roughness index is affected when a pickup is used by the vehicle to "pick up" current from the rail (within the road).

In this section, we will compute the IRI value of a standard model defined. IRI, in simpler terms, is the accumulated suspension motion divided by the distance travelled by the vehicle [9].

$$H_{IRI} = \frac{-\omega^2 k_1}{(k_2 + i\omega c)(k_2 + k_1 + i\omega c - \omega^2 \mu) - (k_2 + i\omega c)} \quad (2.11)$$

where, $\omega = \Omega v_x$ is the angular frequency having units *rad/s*. An assumption is made for the spatial frequency i.e., $\Omega = 1$ based on standard road waviness value according to the ISO standard 8608 (ISO 8608, 1995). The IRI is usually determined at a constant speed of 80kmph. Using the parameters from table 2.1 and the equation given above, the standard IRI value for a reference truck based on the quarter car model is **2.21 units** i.e., metres/kilometre. This indicates that the standard values lie in the "fair road" spectrum.

3

Approach and Implementation

3.1 Construction of a pickup

The pickup, also a mechanical system, constitutes a spring arm, an extended shoe to the pickup and the wiring needed to transfer the current to the vehicle. In addition to these primary components, the system may also include other features such as a damper, sensor, etc..

The spring arm is a flexible metal arm that attaches the pickup to the truck chassis. It is designed to absorb the bumps and vibrations of the road while keeping the pickup in contact with the ground. The extended shoe is part of the pickup that actually makes contact with the road and is typically made of a conductive material.

In this thesis, a spring mass system is used to model the pickup. The link arm (shown below) extended from the chassis is stimulated as a spring.

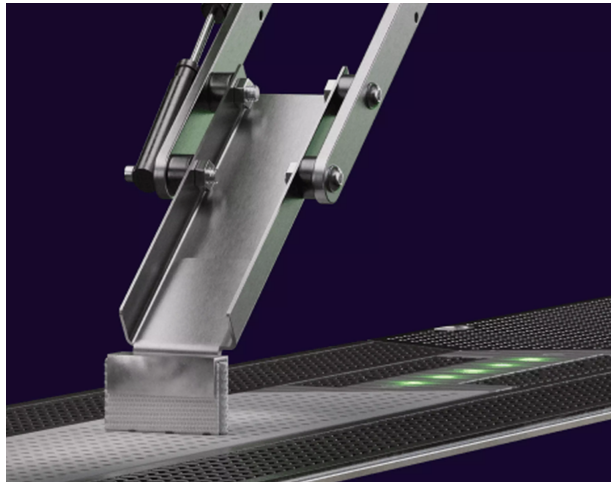


Figure 3.1: Spring system holding the pickup at either ends

Since the exact parameters of the "elon-road" pickup is not known, two scenarios to model the system is used.

- The stiffness of the spring that is connected to the chassis has a value equivalent to the vehicle's stiffness. This indicates that the extended arm (i.e., below the unit pickup mass) has a lower stiffness value and bears the grunt of the surface

irregularities of the road. This spring is responsible for the contact force of the pickup relative to the road.

- The stiffness of the spring that is connected to the chassis is equivalent to the spring arm of the pickup and the second spring is an indication of the extended steel brushes that are in contact with the rail when the vehicle is in motion.

3.1.1 Determining the stiffness of the spring

The setup used to pickup current from the road at "Elon Roads" is a system of ten spring systems placed 35cm equidistant from each other. Further, the total mass of the pickup system is around 1-1.2 kilos with unsprung mass = 200g.

Using the above details and some assumptions (shown in figure below), the spring stiffness can be determined for two scenarios.

3.1.1.1 Scenario "A"

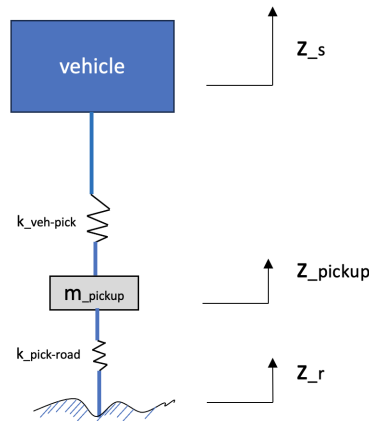


Figure 3.2: Spring system holding the pickup at either ends

Assuming certain angles between each of the pickup (given they are placed 35cm apart), the deflection of the pickup is determined. Using the below equation, stiffness= 52 Nm.

$$z_{pickup} = (F_{load} - F_{unload}) / (k_{pick-road}) \quad (3.1)$$

3.1.1.2 Scenario "B"

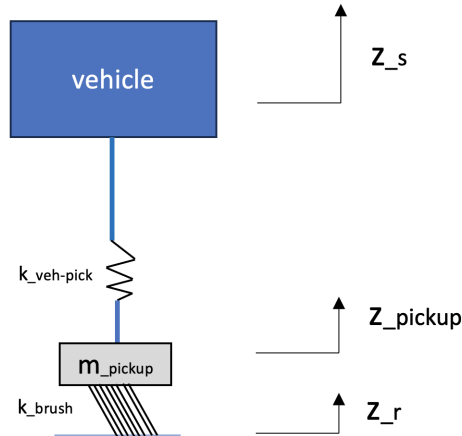


Figure 3.3: Spring system holding the pickup at the vehicle end and the brush

According to [12], the spring stiffness of steel brushes can be considered to be 30-40 N-m.

3.2 Construction of the vehicle car model

In this thesis, matlab is used to model the vehicle and the pickup. Construction of a mathematical model for the vehicle in matlab consists of different steps:

- The vehicle body is represented as a rigid body with six degrees of freedom and the equations of motion for the vehicle body can be derived using Newton's second law of motion.
- The suspension system is modeled as a set of springs and dampers that connect the vehicle body to the wheels.
- The tires are modeled as flexible bodies that interact with the road surface.
- The powertrain is modeled as a system that converts engine power into torque at the wheels. The powertrain model includes components such as the engine, transmission, and differential.

In this report, we will not model the powertrain setup of the vehicle and make an assumption that the truck is travelling at a constant speed of 80 kmph.

3.2.1 Quarter car

First, the quarter car model i.e., just one wheel of the truck is modelled and then a few more parameters are added to increase the complexity of the model to get a more accurate simulated output. Since most of the trucks are usually rear wheel

driven (assumed truck configuration is 4x2 tractor), the rear suspension and dampener system will be the spring-dampener setup as shown below (in figure).

The equations for modelling the excitation of a single wheel in the truck due to surface irregularities in the road is as below:

$$f_{vehicle} = tf \frac{m_s}{(m_s + m_u)s^2 + (c_s + c_t)s + (k_s + k_t)} \quad (3.2)$$

The above equation indicates the response of the truck with respect to dynamic inputs given to the sprung and unsprung mass of the vehicle through its suspension system. s is a complex variable which has a real and an imaginary part. The real part represents the decaying response of the damping to the input whereas the imaginary part represents the frequency response.

The vehicle frequency function is received as an output on solving the above equation in matlab. This function is then fed with the road input for a given time. The time vector (t_{sim}) for simulation is derived based on the vehicle's travel speed which is then interpolated with the given road data to determine the vehicle's response to the surface irregularities (road profile $array_R$).

$$t_{sim} = 0 : \frac{1}{s_{rate}} : 100 \quad (3.3)$$

where $s_{rate} = v_x/0.1$ and 0.1 is the pitch between each road amplitude measured. Further,

$$z_r = interp1(t, array_R, t_{sim}) \quad (3.4)$$

Now, with z_r as the excitation at a given time interval t_{sim} , the vehicle response is calculated through a matlab function called $lsim$. As shown below, this function simulates the effect of the system i.e., the vehicle frequency model when it is subjected to external forces for a given amount of time. The output is the displacement experienced by the truck.

$$[z_s, t_{sim}] = lsim(f_{vehicle}, z_r, t_{sim}) \quad (3.5)$$

3.2.2 Half car

In addition to the parameters considered above in QCM, the front wheel and suspension system of the truck is also considered in the HCM (see figure below). The transfer functions for the front half and the rear half of the truck are modelled separately. With this, the road excitation is modelled and finally, the response of the whole truck is observed.

The equations for modelling the excitation of the front and rear part of the truck due to the road surface irregularities in the road is shown below.

$$f_{vehicle} = tf \frac{[m_{sf}s^2 + (c_{sf} + c_{tf})s + (k_{sf} + k_{tf})][m_{sr}s^2 + (c_{sr} + c_{tr})s + (k_{sr} + k_{tr})]}{[m_{tf}s^2 + (c_{sf} + c_{tf})s + (k_{sf} + k_{tf})][m_{tr}s^2 + (c_{sr} + c_{tr})s + (k_{sr} + k_{tr})]} \quad (3.6)$$

In the above equation, m_{tf} and m_{tr} represents the total mass on the front and rear axle respectively. According to section 2.5, the sprung mass is considered same for both front and rear axles (in total), which will not be the case in reality. In Matlab, the front and rear system of equations are solved separately due to the simulation time encountered in a 16 GB memory computer. Further, as mentioned for QCM, the frequency of the vehicle as a function is received as an output on solving the above equation in matlab. This function is then fed with the road input for a given time.

$$[z_s, t_{sim}] = lsim(f_{vehicle}, z_r, t_{sim}) \quad (3.7)$$

where u is the road excitations and is obtained according to equation 3.4 and $f_{vehicle}$ includes frequencies of the truck in both front and rear ends.

The pitch of the truck is not modelled as the truck is moving at a constant speed of 80 kmph i.e., the truck witnesses $0m/s^2$ of acceleration and deceleration. Further, this will not impact the pickup system as the entire pickup is modelled as a single spring dampener system placed in the center of the truck i.e., distance from COG to truck front and rear axles is equal ($l_1 = l_2$).

3.3 Integration of the models

The integration of vehicle model with the pickup is done for both expressway and regional roads. While the procedure is same for the two, road excitations, as expected, differ from each other. The pickup experiences forces from two ends (see figure below) i.e., the vehicle itself and then also due to the surface irregularities in the road.

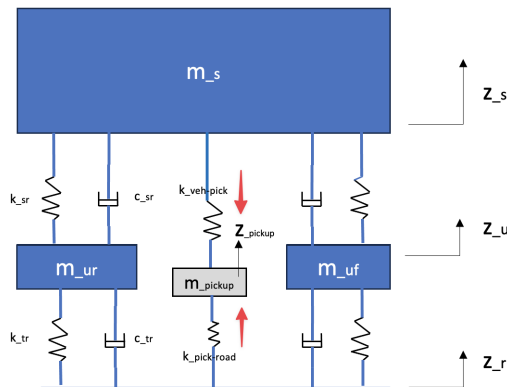


Figure 3.4: Spring system holding the pickup at either ends

The combined functions in Matlab for the QCM integrated with a pickup for regional roads is a below:

```
y= interp1(t_sim, y, linspace(0, t_sim(end), length(array_R)), 'linear', 'extrap')
```

y is the resampled vehicle response, this acts on the pickup in a direction opposite to the road excitation input (see figure 3.4). Hence, the sum total load acting on the pickup (i.e., the pickup response) is determined by:

```
pickup_response = lsim(pickup_sys, -y'+array_R, t)
```

The above equation simulates the pickup response (*pickup sys*) based on the excitation input from the road irregularities and the vehicle suspension.

3.4 Contact Force

Contact force is determined using the standard spring force equation i.e., $F = kx$ where the spring stiffness is a constant value and the deflection is the pickup response. The pickup response is determined as shown in previous section. Spring stiffness can have two values as discussed earlier i.e., based on the assumption made on the construction of the pickup.

$$f = k_{pickup} * z_{pickup} \quad (3.8)$$

3.5 Contact Resistance

The contact resistance is basically an indication of the extent of resistance shown by the conductive materials to transfer current between the interface. According to [3], the relation between contact force and contact resistance can be derived based on experiments where the conducting material runs over the conductive surface to transfer energy. Although the experiment is conducted for trains where the conductive wire is a bit more flexible and overhead, the parallels to the ELON road setup hold good to get an understanding of the relationship between contact force and contact resistance.

From the research paper [3], equation for contact resistance is as below:

$$R_c = (-3.29I + 230)f^{0.0106I-0.847} + \left(\frac{337}{I} - 2.42\right) \quad (3.9)$$

where R_c is the contact resistance, f is the contact force and I is current. For loads of 20-60A in steps of 10A, the contact resistance is determine for the contact force simulated over a given time period.

3.6 Correlation with IRI

To determine the IRI when the truck is installed with the pickup system, the stiffness values of the pickup and the vehicle are added in series. Referring to table 2.1, the stiffness value of the pickup i.e., either 520 Nm (scenario A or B) will not make a difference to the ratio of parameters that is needed to calculate the roughness index.

Further, given the spatial frequency has the same value as the standard road i.e., 0.1 with a road waviness of 2, the IRI value for the truck with the pickup will almost be equal to **2.21 units** when all the parameters are solved using equation 2.11.

4

Results

4.1 Performance of the pickup

The performance of the pickup is determined by a few parameter i.e., the contact force and the quality of the electrical energy transferred to the vehicle's energy storage system. And this, of course, as seen in earlier sections, depends on the road profile.

The vehicle is simulated on different road conditions, specifically, expressways and regional roads from Östergötland county in Sweden.

4.1.1 Road profile

The PSD plots (Energy vs frequency) for regional roads and expressways are plot considering the given amplitudes which are recorded with a pitch of 10cm.

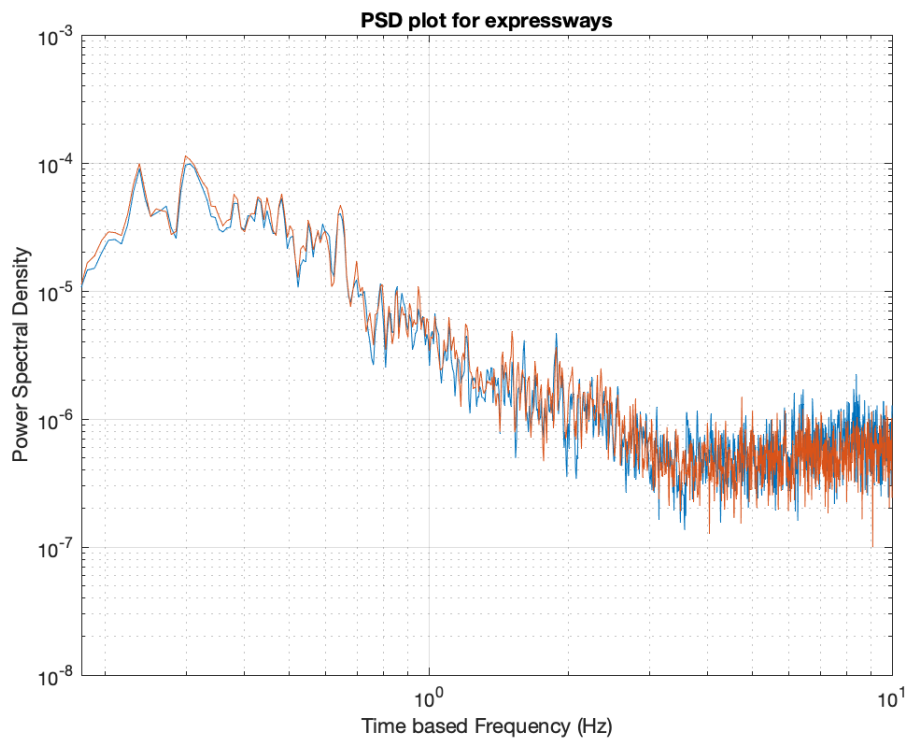


Figure 4.1: The power vs frequency curve for expressways

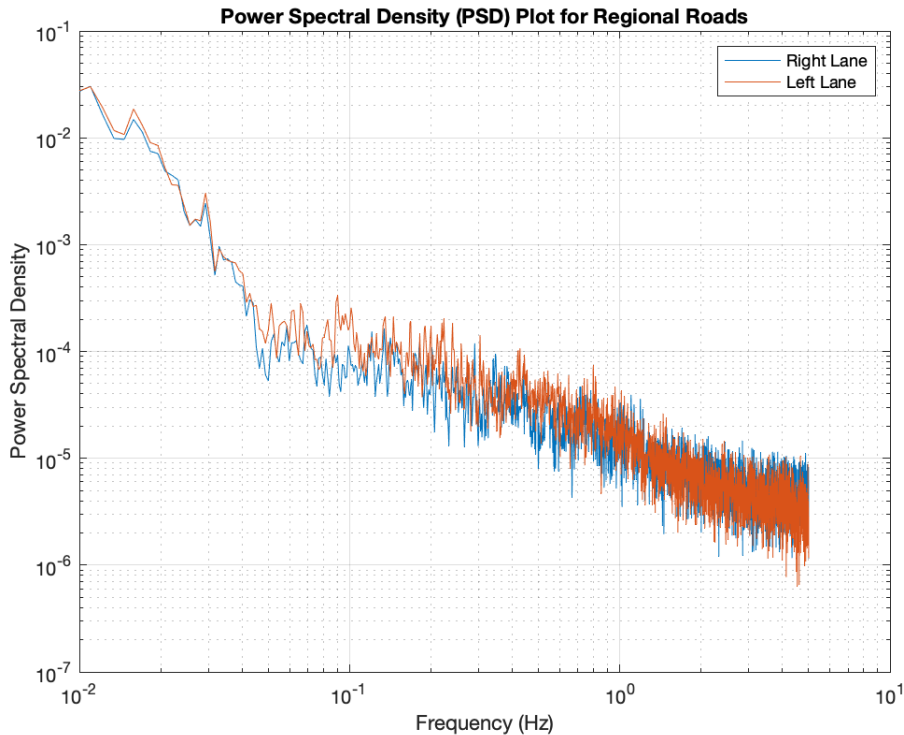


Figure 4.2: The power vs frequency curve for regional roads

The decrease in power (y-axis) with increasing frequency (x-axis) in the graph above suggests that the road profile's roughness is more pronounced at lower frequencies and gradually diminishes as we move to higher frequencies. This pattern is common in road profiles (both expressways and regional roads) where the lower frequencies correspond to larger-scale irregularities such as large undulations or potholes, and higher frequencies represent smaller-scale details like texture, etc.

Also, on observing the y-axis i.e., power values in the plots, it is evident that the surface irregularities in regional roads are much higher based on the higher power values. This means that the pickup system will experience more energy transfer hiccups if the rail is installed in regional roads.

4.1.2 Vehicle and Pickup response

The vehicle and pickup response are observed based on the modelled system of equations for both QCM and HCM. The vehicle response without the pickup is shown first. Further, the pickup system is added/ integrated with the vehicle and the response of the individual pickup system is analyzed.

The x axis denotes the simulation, which was considered 100s initially. This simulation time changed when the data recorded as vehicle response was extrapolated to fit in based on the road input. The sum excitations considering road and vehicle is then given to the pickup. The deflection observed is seen in y axis. As expected, the truck experiences very low excitations but the individual pickup system placed between the chassis and the road undergoes a fair bit of displacement.

4.1.2.1 Scenario "A" for stiffness value of pickup

In the first scenario, the pickup is modelled assuming there are no steel brushes at the end (refer figure). This means that the vehicle stiffness applies to the first spring and the second spring has a stiffness of 52 Nm. For this, the plots are shown below.

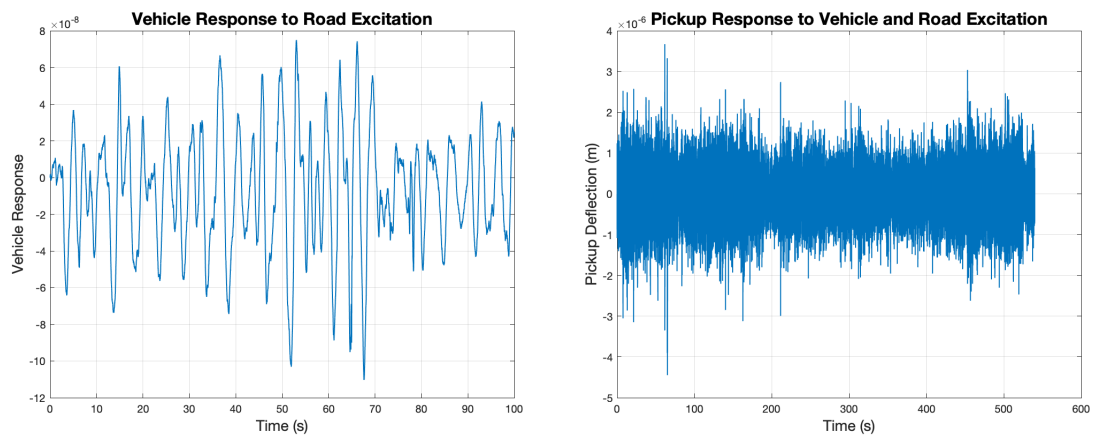


Figure 4.3: Vehicle response to road excitation (expressways) observed based on QCM

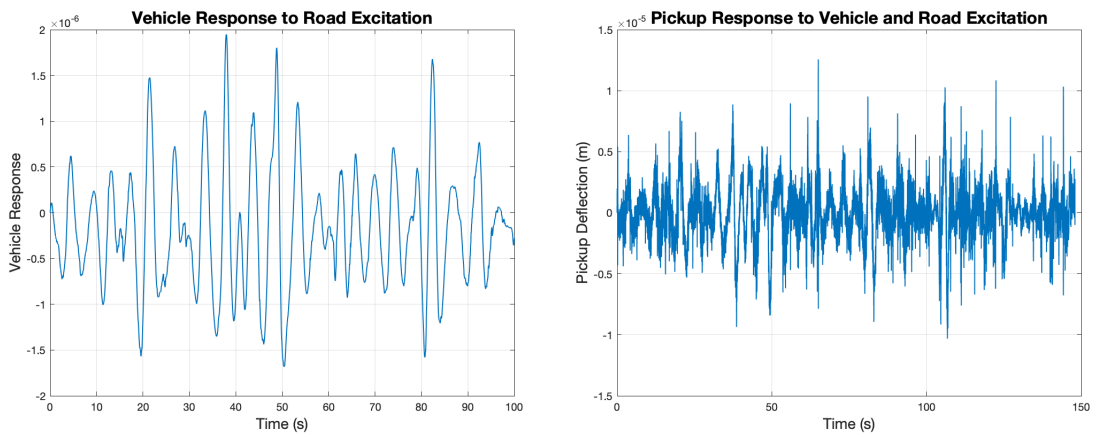


Figure 4.4: Vehicle and Pickup response to road excitation (regional) observed based on QCM

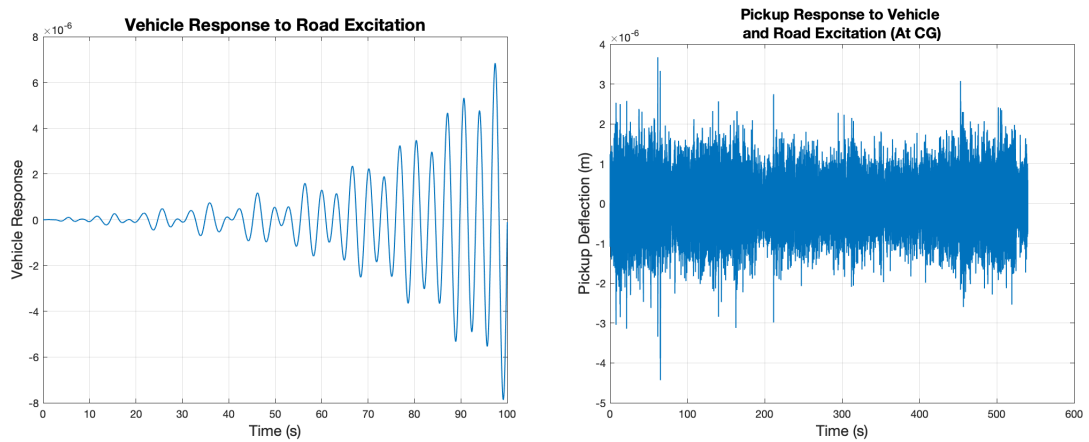


Figure 4.5: Vehicle and Pickup response to road excitation (expressways) observed based on HCM

4. Results

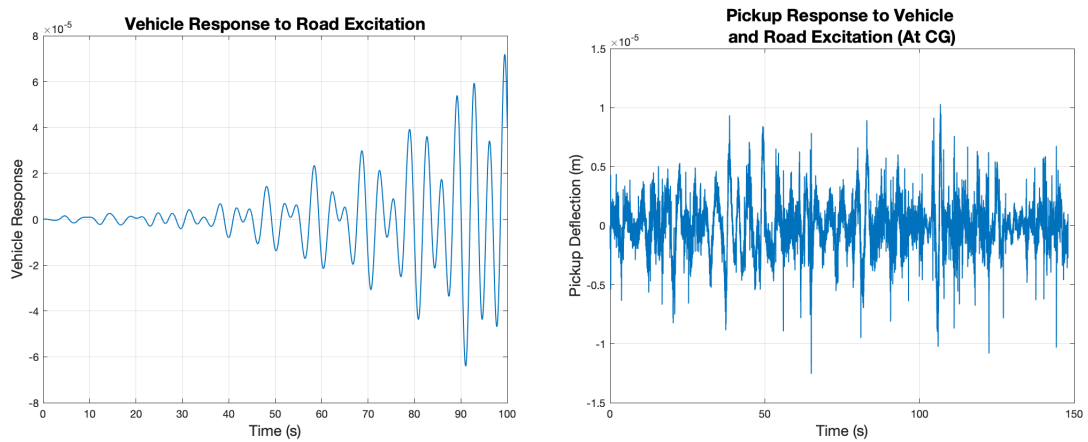


Figure 4.6: Vehicle and Pickup response to road excitation (regional) observed based on HCM

As seen above, the resultant deflections do not change much whether the system of equations is modelled using QCM or HCM. This is because of the assumptions made due to "known" data of how the pickup is installed on the truck chassis i.e., 10 pickups are concentrated as a single system at the CG of the truck. However, the deflections of the pickup (and the vehicle) clearly vary based on the road conditions. The deflection observed by the pickup is way higher in regional roads.

4.1.2.2 Scenario "B" for stiffness value of pickup

In the second scenario, the pickup is modelled assuming there are steel brushes at the end (refer figure). This means that stiffness of the first spring is 52 Nm and the second spring has a stiffness of 52 Nm. For this, the plots are shown below.

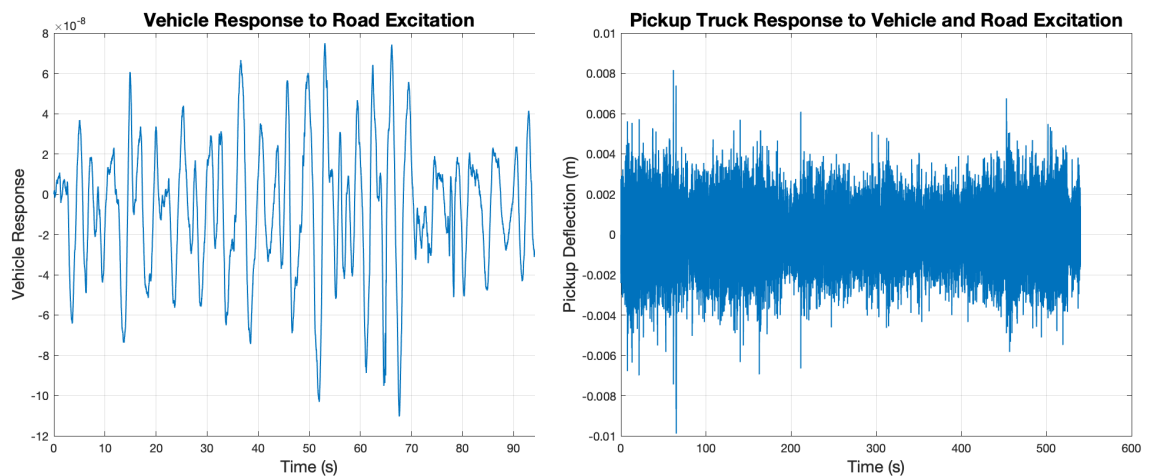


Figure 4.7: Vehicle and Pickup response to road excitation (regional) observed based on QCM

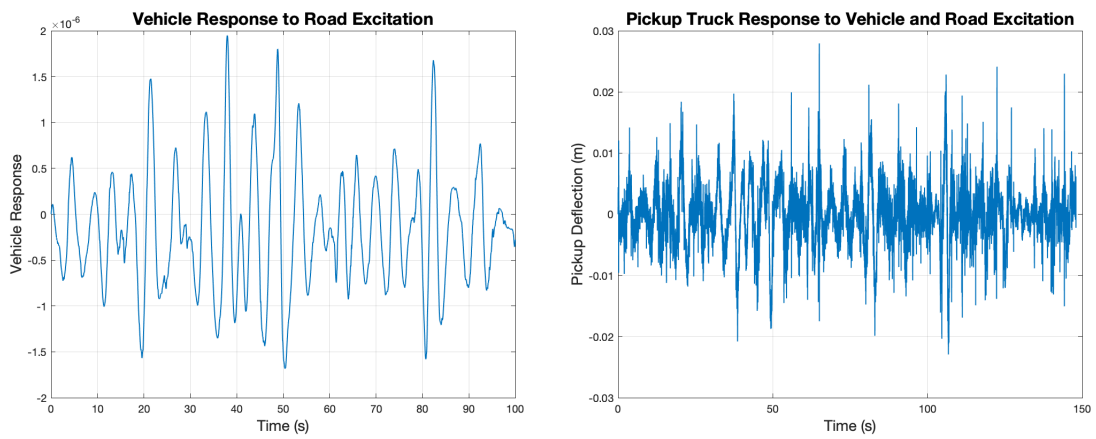


Figure 4.8: Vehicle and Pickup response to road excitation (regional) observed based on QCM

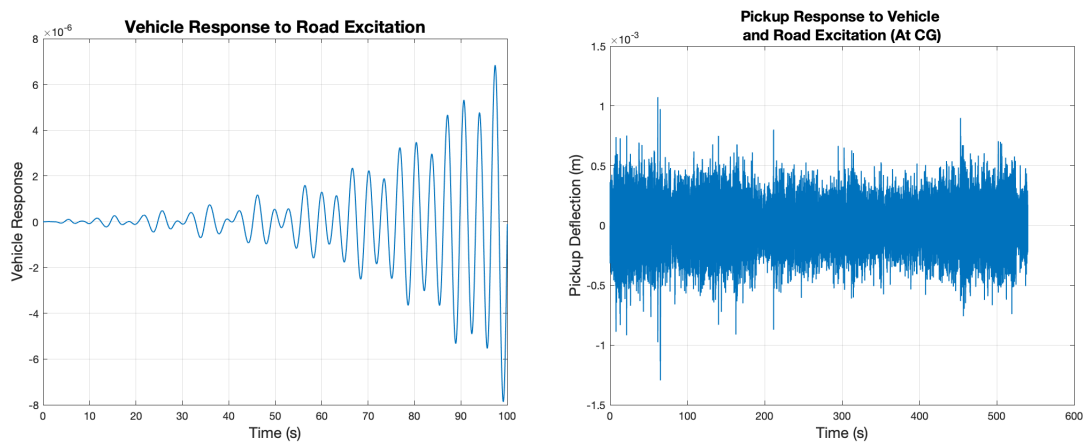


Figure 4.9: Vehicle and Pickup response to road excitation (expressways) observed based on HCM

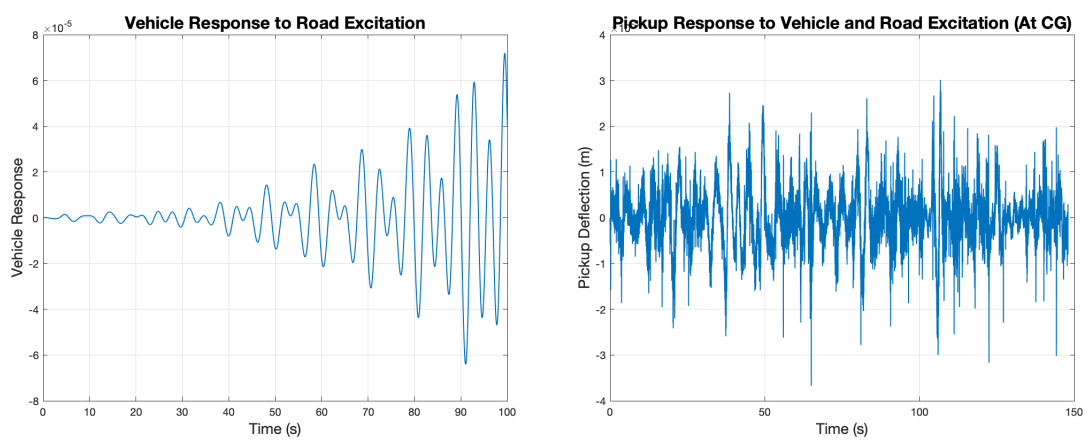


Figure 4.10: Vehicle and Pickup response to road excitation (regional) observed based on HCM

4. Results

As observed previously, the pickup (and the vehicle) has more deflections in regional roads than expressways and the differences in response isn't much affected if the system of equations is modelled using HCM or QCM. However, there is a very interesting anomaly when the truck is travelling on regional roads with this specific stiffness value of the pickup system i.e., the pickup deflection reaches almost 3m at peaks. This can be due to the "selection" of peaks during extrapolation in Matlab.

Further, in all the cases seen above, the pickup deflection is way higher compared to scenario A. This indicates that the assumption made for the pickup system might be faulty. However, this conveys the importance of the mechanical construction of the pickup.

In addition to the time plots shown above, Fourier transform plots i.e., spectrum plots have been used to visualize the energy density of the deflection. Fourier transform plots are valuable because they provide insights into the frequency components of a signal, helping analyze its composition in the frequency domain. This gives a better understanding of the energy distribution for the frequency band i.e., higher energy indicates higher deflections. The plots shown below exemplify it by indicating a higher energy density value for regional roads in comparison with expressways and also a higher value for scenario A of the pickup with respect to B.

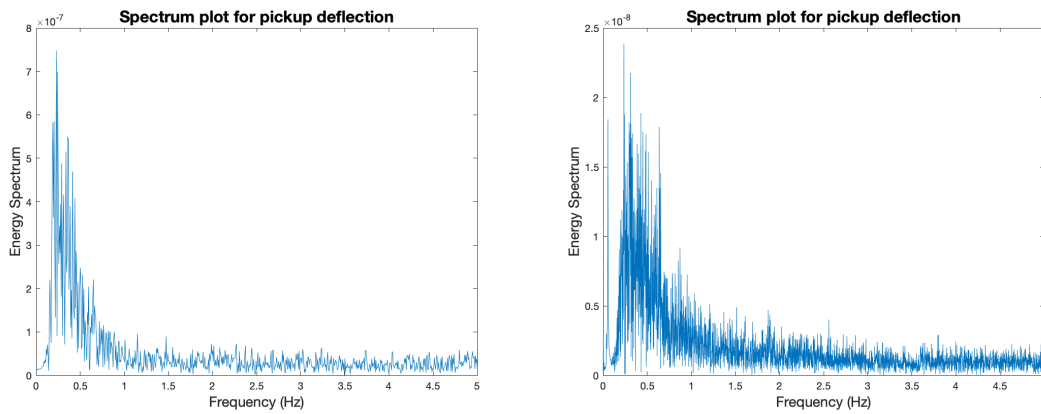


Figure 4.11: Vehicle and Pickup response to road excitation for regional roads and expressways- scenario A

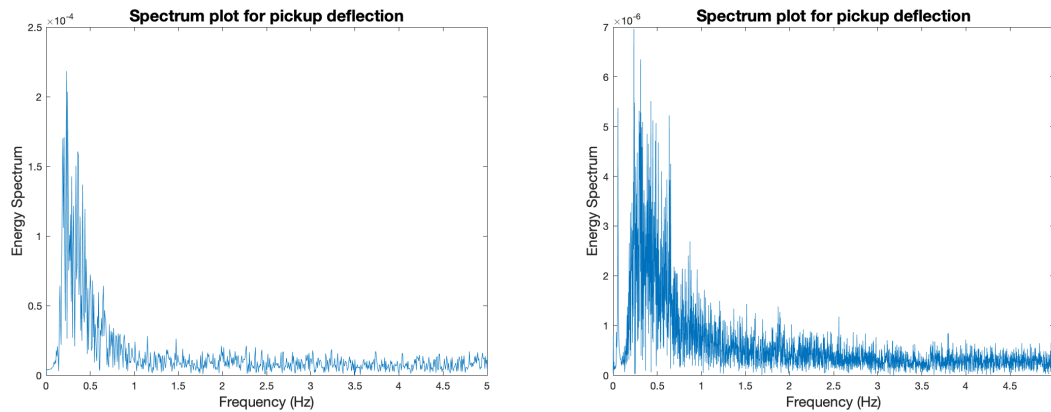


Figure 4.12: Vehicle and Pickup response to road excitation for regional roads and expressways- scenario B

4.2 Quality of Electrical connection

The quality of electrical connection is determined by the contact resistance of the pickup as it travels over the rail. Contact resistance is a parameter used to determine the quality of an electrical connection. It refers to the "resistance" in flow of electric current at the interface between two conductive materials in contact. The lower the contact resistance, the better the quality of the electrical connection.

First, the contact force of the pickup wrt the rail (and the road) is determined using the standard spring force equation. As seen above, results obtained through QCM or HCM doesn't make a huge impact based on the assumptions made in this thesis. Hence, the contact force variations over the road profile considering system of equations modelled through HCM are shown below.

4.2.1 Scenario "A" for stiffness value of pickup

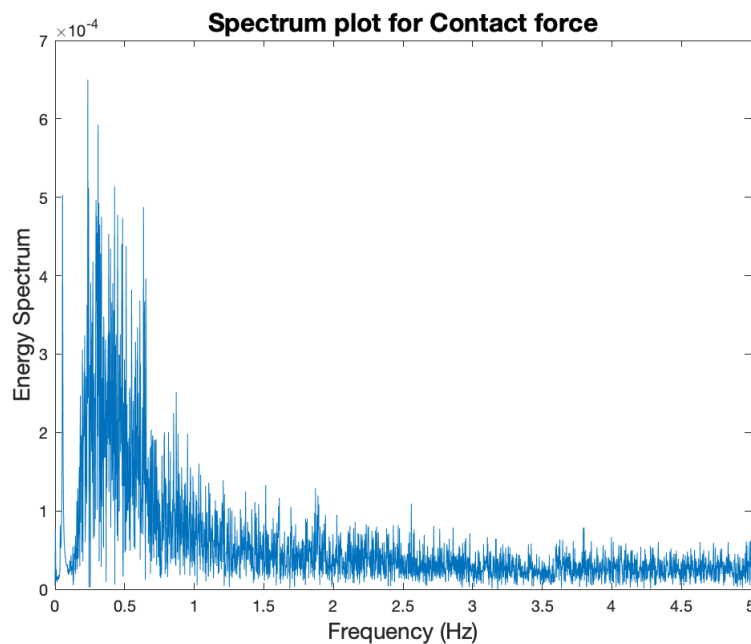


Figure 4.13: Energy spectrum for contact force where pickup stiffness = 520Nm

Further, the contact resistance is determined based on the relation with contact force (as discussed in previous section). The plots depicting contact force vs contact resistance is shown below:

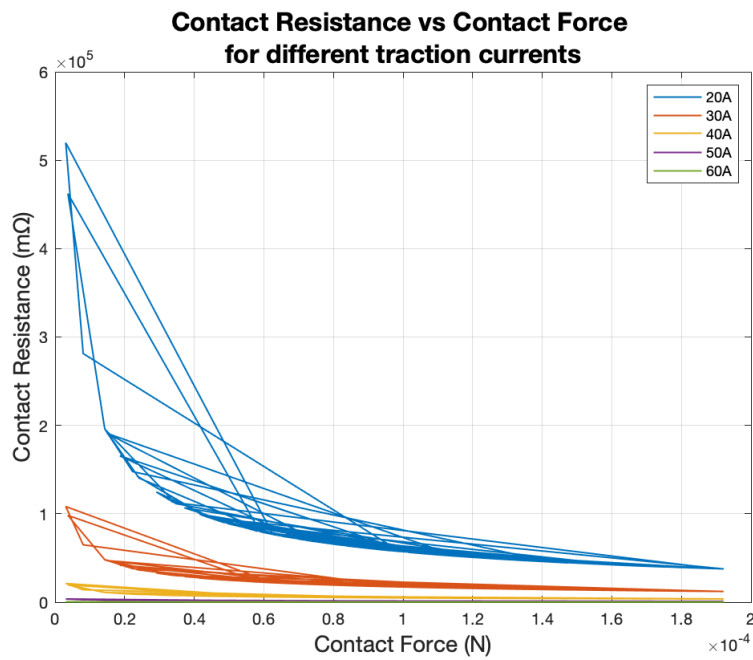


Figure 4.14: Contact resistance plotted for pickup stiffness = 520Nm at different loads

4.2.2 Scenario "B" for stiffness value of pickup

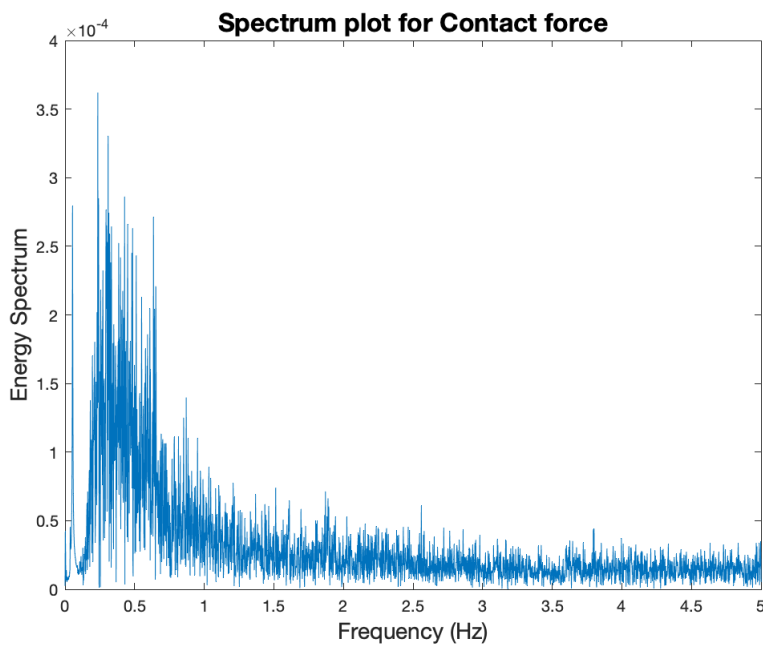


Figure 4.15: Energy spectrum for contact force where pickup stiffness = 300Nm

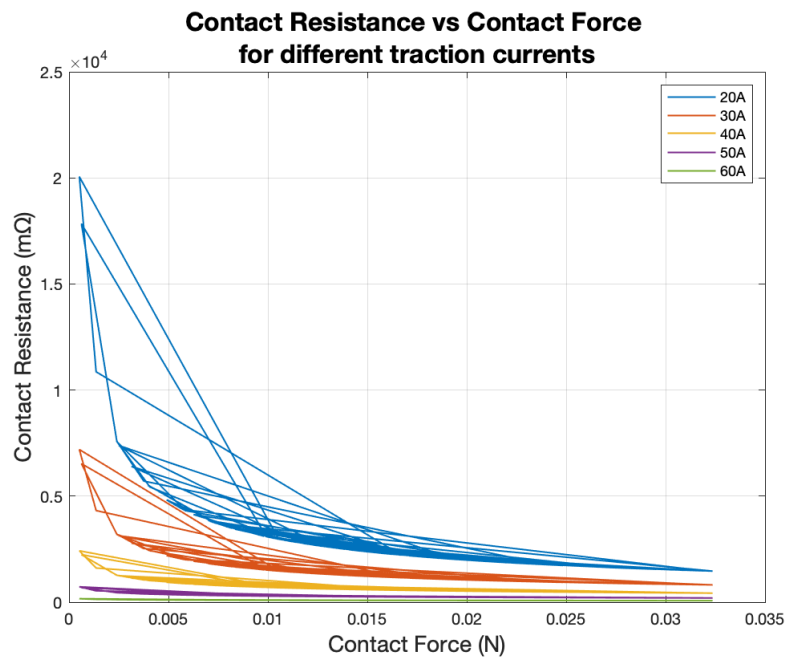


Figure 4.16: Contact resistance plotted for pickup stiffness = 300Nm at different loads

From the graphs, it is clear that the contact force plots follow the pickup deflection and is just increased by a scale which is the stiffness of the pickup. The relationship between contact force and resistance also is as expected, i.e., contact resistance reduces with increasing contact force.

An interesting thing to notice is the contact force for the two different scenarios changes i.e., higher contact force for scenario B. On comparing the graphs, it is evident that the contact resistance is lower in scenario B. It is to be noted that this contradicts the fact that having a stiffer spring will lead to an improved quality in current transfer. This is due to the fact that the contact force is determined based on the deflections and the contact resistance correlation is directly dependent on the contact force and does not consider the deflections and behaviour of the pickup.

5

Conclusions and future work

5.1 Conclusion

The integrated model consisting the vehicle and the pickup parameters using an equation based approach in Matlab was validated and simulated successfully, although with quite a lot of assumptions made for the parameters of the pickup. The entire model was built up on the premise that the pickup is placed in the center of the truck i.e., exactly where the CG is ($l_1 = l_2$). Further, the simulation was done for two different scenarios of the pickup, based on the spring stiffness and the construction of the pickup.

The pickup, after being installed on the chassis, is given excitation from two ends; one, irregularities in the road surface and two, the vehicle's dynamic behaviour. On simulating the pickup response for excitations in regional roads and expressways, it is evident that the pickup reaction follows the vehicle response (as expected) and varies based on the road surface, showing higher deflections on regional roads compared to expressways. As observed in the plots shown in section 4.1.2, the pickup response always gets worse when the vehicle runs over regional roads. To expound, the scale of deflection is almost 5-10% higher in regional roads compared to expressways although the vehicle deflection doesn't increase much. This is due to the reason that the vehicle displacement exemplifies the response experienced by the pickup. A good way to solve this would be having a very stiff spring that connects to the pickup mass and then having a lesser stiff spring as an extension to the pickup mass that runs over the rail. This is discussed in scenario "B" and can be seen through the plots shown above.

The quality of energy transfer depends on the contact resistance which is further dependent on the contact force that exists between the interface of the two conductive surfaces. The pickup response (deflection) is used to determine the contact force and as seen in section 4.2, follows the deflection but is scaled up by the factor of the stiffness of the pickup.

The most important parameter that influences the quality of the energy transfer is the mechanical construction of the pickup (including the materials used) as it is seen that even small changes in the assumption made to the stiffness of the pickup (scenario A and B) has a huge impact on the response and hence the contact resistance. Moving on, the contact force between the conductive surfaces need to be maximized irrespective of the traction voltage on the rail, this reduces the resistance and hence, helps a continuous flow of energy throughout the route. And, how do we increase contact forces? By the end of this report, it is clear that this can be achieved by reducing the deflection of the pickup.

5.2 Future work

An establishment that was clearly made with this thesis report is that the quality of electrical connection indeed depends on the construction of the pickup and the stiffness of the springs used in the setup. This was validated through a mathematical model of the vehicle and pickup and

5. Conclusions and future work

was simulated with excitations when run on different types of roads. Further, the model can be improved in several ways, some of which are discussed below.

In terms of vehicle modelling, the half car model can be improved to show pitch and heave, and also include longitudinal forces (speed changes) that will affect the wear of the pickup at the ends where it comes in contact with the rail. The pickup model can be made more accurate once the mechanical construction of the pickup is clearly known. A good way to reduce deflections of the pickup is by adding a dampener to the system (both physically and virtual modelling). Further, digging into the details of the steel brush by modelling the bristles helps with visualizing the pickup's response to each surface irregularity on the road and determining the wear and fatigue life of the brush. Also, the details on how the pickup is attached to the chassis of the truck is important to understand the influence of the vehicle's suspension system on the pickup. An interesting thing to model would be the fifth wheel coupling on the tractor when the pickups are connected to the trailer instead of the truck.

The road data used in this thesis can be extended to fit in more irregularities (peaks) like potholes, bumps, etc. that will affect the quality of the electrical connection drastically due to the amount of deflection the pickup will experience. Today, the fitting formula was determined through experiments conducted for pantograph connected to an overhead wire which was extended to the pickup in this report. This experiment can be conducted for the similar setup of the pickup discussed in this report to determine the different fitting constants which will affect the contact resistance observed. Further, through the experiment and the modelling, a correlation between the virtual and the physical model can be established. This will help in improving the construction of the pickup which is the most critical parameter to have a good electrical connection.

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